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COMPUTATIONAL EFFICIENCY OF IMPROVED MOVE LIMIT METHOD OF SEQUENTIAL LINEAR PROGRAMMING FOR STRUCTURAL OPTIMIZATION

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Abstract—The imporved move limit method of sequential linear programming is briefly explained. Comparison of computing efficiencies is made between the improved method and the conventional move limit method with six test problems. The usefulness of the method in the context of structural optimization is shown with the help of four examples.

INTRODUCTION

Sequential linear programming is one of the powerful methods for solving nonlinear programming problems. This method is more suitable for shape optimization of the continua[12] in which time taken for analysis is quite high. The move limit method of sequential linear programming, suggested by Stewart and Griffith[3] has been presented in the form suitable for structural problems by Pope[4, 5]. In an earlier paper the authors[6] have suggested improvements in the move limit method and presented preliminary results in context of a few mathematical problems. In this paper, a brief resume of the improvement suggested and the results for six test problems are presented. The comparison study between the improved method and conventional method[4, 5] is presented in the context of four structural optimization problems. The structural problems considered are optimum shape design of four mechanical components which need finite element method for stress analysis.

IMPROVED MOVE LIMIT METHOD

A general nonlinear programming problem may be defined as

$$\min z = F(\mathbf{x}) \qquad (1)$$

subject to

$$G_j(\mathbf{x}) \leq 0, \quad j = 1, 2, \ldots, m \qquad (2)$$

where $\mathbf{x}$ is a design vector of n dimensions, $F(\mathbf{x})$ is the objective function and G_j's are constraints. In the neighbourhood of the design vector $\mathbf{x}^k$ using Taylor's series expansion and retaining only upto linear terms, the objective function and constraints are approximated as

$$F(\mathbf{x}^{k+1}) \simeq F(\mathbf{x}^k) + \nabla\mathbf{F}^T(\mathbf{x}^k)(\mathbf{x}^{k+1} - \mathbf{x}^k) \qquad (3)$$

$$G_j(\mathbf{x}^{k+1}) \simeq G(x^k) + \nabla\mathbf{G}^T(x^k)(\mathbf{x}^{k+1} - \mathbf{x}^k) \qquad (4)$$

with these approximations, the problem reduces to a linear programming problem. This linear programming problem is solved with the following additional constraints on the movement of design variables

$$|(x_i^{k+1} - x_i^k)| \leq M_i^k \qquad (5)$$

where x_i^{k+1}, x_i^k and M_i^k are the ith components of $\mathbf{x}^{k+1}$, $\mathbf{x}^k$ and $\mathbf{M}^k$. The vector $\mathbf{M}^k$ prescribes the move limits on the design variables.

If $\mathbf{x}^{k+1}$ is a feasible point, the objective function is checked for improvement $[F(\mathbf{x}^{k+1}) < F(\mathbf{x}^k)]$. The sequence of linear programming is continued from $\mathbf{x}^{k+1}$ if improvement is found in objective function. Otherwise, the new design point is selected by quadratic interpolation between the design points $\mathbf{x}^k$ and $\mathbf{x}^{k+1}$. In Fig. 1, A represents the design point $\mathbf{x}^k$ and B represents the design point $\mathbf{x}^{k+1}$. Assuming the objective function to vary quadratically along AB, the objective function at any point P, along AB, can be written as

$$F(\mathbf{x}) = F(\mathbf{x}^k + \alpha\mathbf{S}) \simeq F(\alpha) = a + b\alpha + c\alpha^2 \qquad (6)$$

where

$$\alpha = AP/AB \qquad (7)$$

and

$$\mathbf{S} = \mathbf{x}^{k+1} - \mathbf{x}^k. \qquad (8)$$

Hence

$$F(0) = F(\mathbf{x}^k) = a$$

$$F(1) = F(\mathbf{x}^{k+1}) = a + b + c$$

$$\text{and } F'(0) = \nabla F^T(\mathbf{x}^{k+1}) = b.$$

The point $\mathbf{x}^+$ corresponding to the minimum $F(\alpha)$ along

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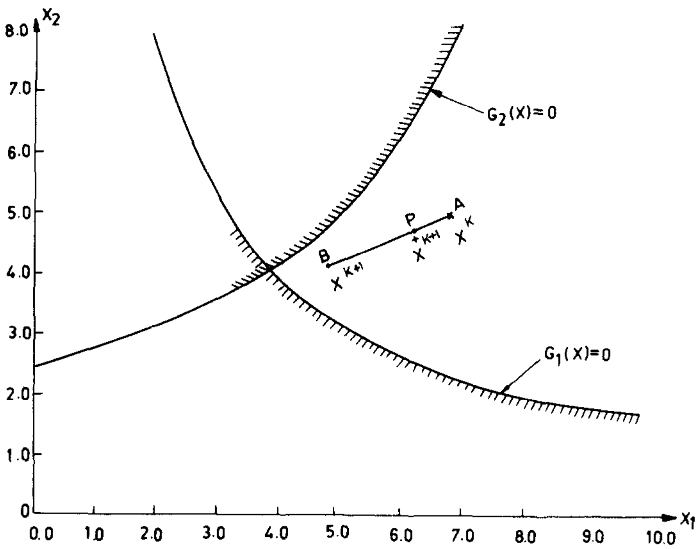

Fig. 1. Quadratic interpolation along a line.

the line is obtained by $dF/d\alpha = 0$. The corresponding value of α is

$$\alpha^+ = -\frac{b}{2c} = -\frac{1}{2}\left[\frac{\nabla F^T(\mathbf{x}^k)\mathbf{S}}{F(\mathbf{x}^{k+1}) - F(\mathbf{x}^k) - \nabla F^T(\mathbf{x}^k)\mathbf{S}}\right]. \quad (9)$$

Hence the new design point is given by

$$\mathbf{x}^+ = \mathbf{x}^k + \alpha^+(\mathbf{x}^{k+1} - \mathbf{x}^k). \quad (10)$$

The move limit is also reduced as

$$\mathbf{M}^+ = \alpha^+\mathbf{M}^k. \quad (11)$$

If a linear programming solution enters infeasible region, it is steered to feasible by moving in the gradient direction of most violated constraint. At any point, distance β from $\mathbf{x}^{k+1}$ along the gradient direction, the most violated constraint (say jth) can be linearized as

$$G_j(\mathbf{x}^{k+1} + \beta\nabla G_j(\mathbf{x}^{k+1})) = G_j(\mathbf{x}^{k+1}) + \beta\nabla G_j^T(\mathbf{x}^{k+1})\nabla G_j(\mathbf{x}^{k+1}) \quad (12)$$

since a point which just satisfies the constraint is sufficient, equating eqn (12) to zero, the value of β is obtained as

$$\beta = -\frac{G_j(x^{k+1})}{\nabla G_j^T(\mathbf{x}^{k+1})\nabla G_j(\mathbf{x}^{k+1})} \quad (13)$$

and such a point is given by

$$\mathbf{x}^* = \mathbf{x}^{k+1} + \beta\nabla G_j(\mathbf{x}^{k+1}) \quad (14)$$

since the evaluation of constraint derivatives at a point is very expensive in the optimum shape design of continua, it is preferable to use the gradient direction of previous point. Thus the eqns (13) and (14) can be modified as

$$\beta = -\frac{G_j(\mathbf{x}^{k+1})}{\nabla G_j^T(\mathbf{x}_k)\nabla G_j(\mathbf{x}^k)} \quad (15)$$

and

$$\mathbf{x}^* = \mathbf{x}^{k+1} + \beta\nabla G_j(\mathbf{x}^k). \quad (16)$$

In some problems the above technique of steering to feasible region is to be used repeatedly to reach feasible region. If number of repetitions required are large, recalculation of constraint derivatives is done after a predefined repetitions.

After steering the design vector to feasible region, if no improvement is found in the objective function, the usability of the direction $\mathbf{S}^* = \mathbf{x}^* - \mathbf{x}^k$ is checked. The quadratic interpolation is resorted only if the direction is usable. Otherwise quadratic interpolation leads to $\mathbf{x}^k$ as optimum erroneously. In such cases quadratic interpolation is to be done between $\mathbf{x}^k$ and $\mathbf{x}^{k+1}$ (Fig. 2) and then optimization is continued.

The above method has the following three improvements over the one used by Pope[4, 5]:

1. Instead of interval halving, quadratic interpolation is used.

2. For steering the infeasible design vector to feasible region, gradient direction (at previous) is used instead of the direction jointing the origin and the design vector.

3. Checking the usability of the direction $S^* = \mathbf{x}^* - \mathbf{x}^k$ before going for quadratic interpolation is included.

COMPARISON STUDY WITH THE TEST PROBLEMS

Six test problems have been studied by the improved method and conventional method[6]. Table 1 shows the numerical examples considered. Table 2 shows the comparison between the two methods. First four examples are constructed by the authors and last two problems are from Himmalblau[7]. The last two problems are five variable problems which have been used in comparison studies already published[8, 9].

In the first example, where optimum lies at the corner of the constraints, both methods are equally efficient. In the second example, effectiveness of quadratic interpolation is observed. In the third example, in which almost every linear programming solution enters infeasible region, the effectiveness of the method of steering to feasible region is found. The need for checking the usability is found in the fourth example. Fifth and sixth examples indicate in general the efficiency of suggested improvements in large variable problems.

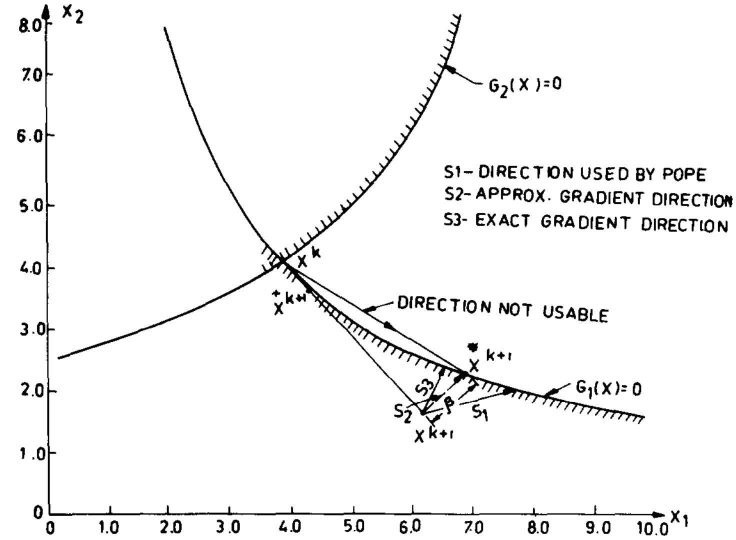

Fig. 2. Steering design vector to feasible domain and need for checking the usability of new direction.

Table 1.

Example No.		Problem
1		$\text{Min } Z = x_1^2 + x_2^2$
	Subject to	$g_1(\bar{x}) = 16 - x_1x_2 \leq 0$
		$g_2(\bar{x}) = (10 - x_1)x_2 - 14 \leq 0$
2		$\text{Min } Z = x_1^2 + x_2^2 - 11x_1 - 9x_2$
	Subject to	$g_1(\bar{x}) = 16 - x_1x_2 \leq 0$
		$g_2(\bar{x}) = (10 - x_1)x_2 - 25 \leq 0$
3		$\text{Min } Z = x_1^2 + x_2^2$
	Subject to	$g_1(\bar{x}) = 16 - x_1x_2 - 44 \leq 0$
		$g_2(\bar{x}) = (10 - x_2)x_2 - 44 \leq 0$
4		$\text{Min } Z = x_1^2 + x_2^2$
	Subject to	$g_1 = 16 - x_1x_2 \leq 0$
		$g_2 = (10 - x_1)x_2 - 25 \leq 0$
5		Prob. No. 10, page 404, Ref. [7]
6		Prob. No. 11, page 406, Ref. [7].

Table 2. Comparison study with test problems

Example No.	Method	No. of iteration	No. of function evaluation	No. of derivative evaluation
1	(a) Conventional	3	4	3
	(b) Improved	3	4	3
2	(a) Conventional	9	20	9
	(b) Improved	5	10	5
3	(a) Conventional	11	29	11
	(b) Improved	9	22	9
4	(a) Conventional	Does not reach exact optimum		
	(b) Improved	6	13	6
5	(a) Conventional	6	12	6
	(b) Improved	5	7	5
6	(a) Conventional	3	18	3
	(b) Improved	3	15	3

STRUCTURAL PROBLEMS

Optimum shape design of four mechanical components has been carried out. For stress analysis of the components finite element method is used. Quadratic isoparametric elements are used. The integrations encountered are carried out by using 2×2 Gaussion numerical integration. Stresses are calculated accurately at Gaussian points of the elements in critical zone and the stresses in the boundary sampling points are obtained by extrapolation. The derivatives of the stresses with respect to design variables are evaluated by the efficient method as used by Francavilla, Ramakrishnan and Zienkiewicz[10]. Details of the investigations about optimum shape design of the mechanical components are published elsewhere[11, 14]. Here comparison studies of the two methods are carried out. The problems selected are briefly explained below.

1. *Optimum shape design of fillets in tension bars*

Figure 3 shows the problem situation. The fillet shape is defined by sixth order polynomial (Fig. 4), which is completely determined from end conditions and from design variables. The four design variables selected are the ordinates at equal intervals along the fillet length. Minimization of volume is taken as objective function. Constraints are imposed on stress concentration factor at sampling points.

2. *Optimum shape design of rotating disks*

The problems situation is shown in Fig. 5. The shape of the cross section of a rotating disk is defined by a polynomial. The variables selected are semi thicknesses at four predefined radii. The objective function selected is minimization of weighted function in which equal weightage is given for volume minimization and stress levelling. No behavior constraints are imposed. Side constraints imposed are about the minimum value of the thickness.

3. *Optimum shape design of flanged and flued expansion joint*

Figure 6 shows the geometry of a typical flanged and flued expansion joint. The design variables x_1, x_2, x_3 and x_4 selected are

$$x_1 = r_0, \quad x_2 = r_1 = r_2, \quad x_3 = h, \quad x_4 = t$$

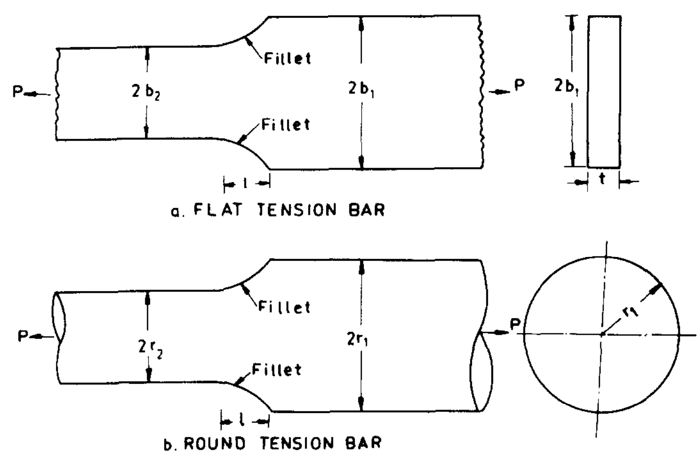

Fig. 3. Problem situation in fillet shape optimization.

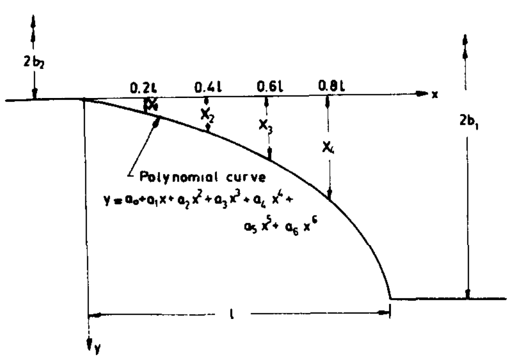

Fig. 4. Geometric description of fillet.

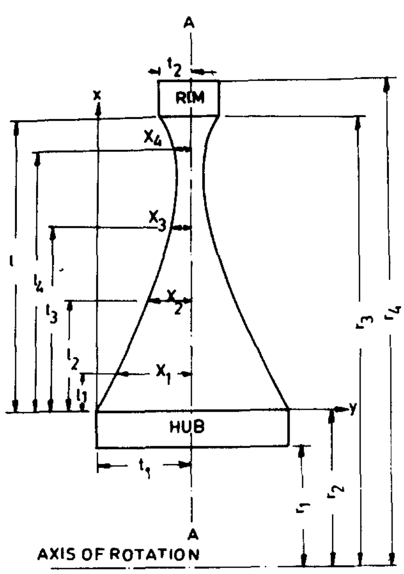

Fig. 5. A typical rotating disk.

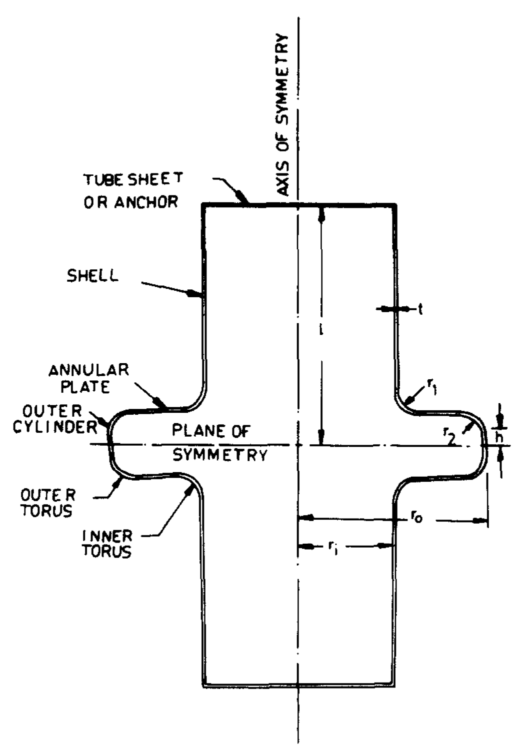

Fig. 6. A typical flanged and flued expansion joint.

where r_0, r_1, r_2, h and t are as defined in Fig. 6. The multiple loadings considered are: (i) Internal pressure only; and (ii) Internal pressure + expansion. Minimization of reaction transferred to tube sheets is taken as objective function. The constraints imposed are the design criteria of Wolf and Mains[15] on stresses at critical points in curved portions.

4. *Optimum shape design of pressure vessels and nozzle junctions*

The junction between a spherical pressure vessel and a cylindrical nozzle is defined by four curves of third degree polynomial which are completely determined by end conditions and four design variables as shown in Fig. 7. The objective function is minimization of maximum stress concentration factor. Only side constraints are imposed.

The nature of these four problems in context of suggested improvements is shown in Table 3. A sample case is selected in each problem and with the same initial point and initial move limits optimization is carried out by both methods. No attempts have been made to solve more number of cases in each problem since the nature of the problem and progress of optimization was not affected as a result of parametric variations.

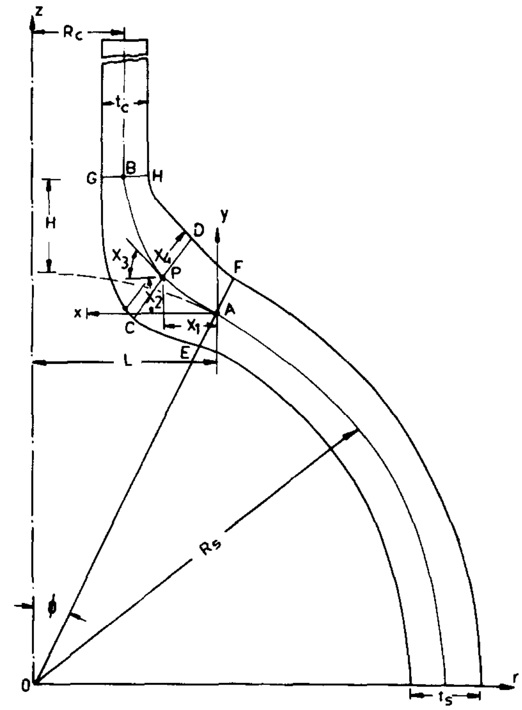

Fig. 7. A typical spherical pressure vessel and nozzle junction.

Table 3. Nature of problems in context of suggested improvements in the method

S. No.	Problem	Quadratic inter- polation	Steering to feasible region	Checking the usability
1	Fillet	Not useful	Very useful	Not useful
2	Rotating disk	Very useful	Not useful	Not useful
3	Expansion joint	Useful	Useful	Useful
4	Pressure vessel nozzle junction	Very useful	Useful	Not useful

COMPARISON STUDY WITH STRUCTURAL PROBLEMS

Table 4 shows the number of function and derivative evaluations required in optimization by the two methods. Computational time required in ICL 1909 is also tabulated for each case.

In the fillet shape optimization problem, the conventional method and the improved method have yielded the same optimum. In both the methods the constraint violation has taken only once during optimization. However in the case of improved method the feasible point obtained, after steering infeasible point, is better and hence optimum has been reached with fewer iterations.

In the optimum design of rotating disk, quadratic interpolation takes place frequently before optimum is reached. The conventional method has reached optimum after 21 function evaluations and 7 derivative evaluations whereas improved method has taken only 17 functions evaluations and 7 derivative evaluations. The reduction in number of function evaluations in improved method is due to the capability of the method in selecting a better point in one step whereas conventional method needs a number of interval halvings.

In the optimum design of expansion joint, the number of behavior constraints is very large. During optimization the design point frequently violates constraints. In this problem the constraints are highly nonlinear and hence for steering design vector to feasible domain the improved method requires fresh calculation of design derivatives just after every two cycles. The method has been found to be powerful enough to steer the design point to feasible region. The conventional method failed to steer the infeasible point to feasible region. Similar breakdown in the progress of optimization is observed with another test example also. In the optimum design of expansion joint, the need for checking the usability of new direction has been felt. The improved method has worked very satisfactorily in such a situation.

In case of pressure vessel and nozzle junction problem, quadratic interpolation occurs frequently just before optimum is reached. The nature of this problem is such that quadratic interpolation gives values very close to interval halving values. Hence here no difference is found in the progress of optimization by the two methods.

CONCLUSIONS

In all the four structural problems considered, the optimization has progressed very smoothly. The sug-

Table 4. Comparison between improved method and conventional method in the context of structural problems

S. No.	Problem	No. of iteration	No. of function evaluation	No. of derivative evaluation	Computing time in ICL 1909
1	Fillet				
	(a) Conventional method	4	9	4	33 min
	(b) Improved method	3	7	3	36 min
2	Rotating disk				
	(a) Conventional method	7	21	7	72 min
	(b) Improved method	7	17	7	64 min
3	Expansion joint				
	A. (a) Conventional method		Fails to obtain feasible design point		
	(b) Improved method	7	20	8	112 min
	B. (a) Conventional method		Fails to obtain feasible design point		
	(b) Improved method	8	32	12	149 min
4	Pressure vessel–nozzle junction				
	(a) Conventional method	8	18	8	59 min
	(b) Improved method	8	18	8	59 min

gested improvements have made the method more efficient and reliable.

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