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Design Charts for Optimal Design of T-beam Floors

A. V. SUBRAMANYAM*
S. R. ADIDAM†

Mathematical programming techniques have been used to minimize the cost of T-beam floors. One-way continuous slabs and simply supported T-beams have been designed by the limit state approach in accordance with CP 110-1972 using M25 grade concrete and cold worked high yield reinforcement. Cost of one of the bays has been minimized using the DFP algorithm. Parametric studies have been carried out for various span ratios, cost ratios of materials and magnitudes of imposed loads. Use of the design charts developed has been explained.

INTRODUCTION

T-BEAM floor is one of the most common structural forms in the field of reinforced concrete construction. Therefore any saving that can be made will be highly beneficial. Cost of T-beam floor system can be considerably reduced using optimization techniques[1]. With the availability of high strength materials, sufficient care must be taken to see that serviceability requirements like maximum allowable deflection, maximum crack width are not violated. This is best done using the limit state approach considering all the relevant limit states.

In order to minimize the cost of T-beam floors, influence of variation in the cost of materials and magnitudes of imposed loads for different spans of slab and beam have to be taken into account. Unless the results are available in the form of design charts, optimization will not be of much use to the designer.

SCOPE

Materials used are M25 grade concrete and TorBar (cold worked high yield reinforcement). Mild condition of exposure has been assumed to arrive at the cover for reinforcement. The problem has been solved for the following cases:

(1) l_s, span of slab in mm: 2500, 3000, 3500, 4000, 4500, 5000;
(2) r, ratio of span of beam l_b to l_s: 2.00, 2.25, 2.50;
(3) q_k, imposed load in kN/m^2: 2, 3, 4;
(4) cost ratio R_1: 600, 1200, 1800; and
(5) cost ratio R_2: 20, 50, 80,

where R_1 = ratio of cost of concrete per m^3 to cost of reinforcement N^{-1},
and R_2 = ratio of cost of form work per m^2 to cost of reinforcement N^{-1}.

*Department of Applied Mechanics, Karnataka Regional Engineering College, Surathkal-574157, India (currently Research Scholar, Indian Institute of Technology, Kanpur-208016, India).

†Department of Civil Engineering, Indian Institute of Technology, Kanpur-208016, India.

FORMULATION OF THE PROBLEM

The slab is designed as a one-way continuous slab using the moment coefficients given in CP 110[2] and the beam designed as a simply supported beam. Figures 1 and 2 show details of slab and beam reinforcement respectively. Minimum shear reinforcement has been provided for the whole span of the beam and additional shear reinforcement has been provided wherever required. Curtailment of main reinforcement has been done after checking for bond requirements. Side reinforcement has been provided whenever the depth of beam exceeds 750 mm, to control crack widths. Detailed computations, for the long term deflection of the beam including effects of creep and shrinkage and for the flexural crack widths of the beam, have been carried out as given in Appendices A2 and A3 of CP 110.

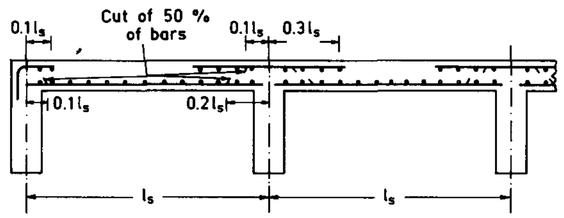

Fig. 1. Details of slab reinforcement.

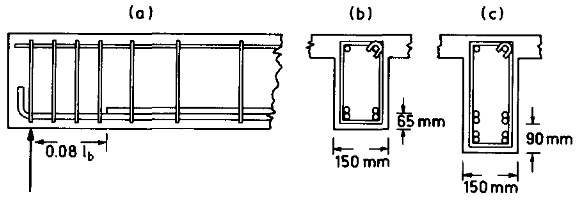

Fig. 2. Details of beam reinforcement.

The design variables

The design variables chosen are:

(1) x_1, thickness of slab;

(2) x_2, maximum area of main reinforcement per unit width of slab;

(3) x_3, total depth of beam; and

(4) x_4, area of tension reinforcement for beam.

The objective function

The objective function is the cost of the floor from the centre of end span to the centre of next span. This includes the following:

(1) cost of concrete including cost of materials, mixing, placing and curing;

(2) cost of main, secondary, shear and side reinforcements; and

(3) cost of form work.

The objective function F can be written as,

$$F=\{l_s x_1+(x_3-x_1)b_w\}R_1+\Big\{0.4\,l_s x_2\left(1+\frac{c_1}{c_2}+\frac{c_3}{c_2}\right)+1.6\,c_4 x_1 l_s+A_{sd}+c_{10}b_w(2x+2y)(1+c_6c_7)+c_5x_4\Big\}\gamma_s+\{l_s+2(x_3-x_1)\}R_2, \quad (1)$$

where A_{sd} is the area of side reinforcement; b_w the width of rib of beam; c_1 the moment coefficient for maximum positive moment in the end span; c_2 the moment coefficient for maximum negative moment at support; c_3 the moment coefficient for maximum positive moment in the intermediate span; c_4 the coefficient for minimum secondary reinforcement for slab; c_5 the ratio of average length of tension reinforcement to the span of beam; c_6 the ratio of additional shear reinforcement to the nominal shear reinforcement; c_7 the ratio of length of beam for which additional shear reinforcement has been provided to l_b; c_{10} the coefficient for minimum shear reinforcement; x the width of shear reinforcement link; y the depth of shear reinforcement link; and γ_s is the specific weight of steel.

The constraints

The first constraint ensures that m_u, the ultimate moment of resistance of slab per unit width, is not less than m, the bending moment due to ultimate loads per unit width

$$m-m_u\leqq 0. \quad (2)$$

The second constraint makes sure that the maximum deflection in the slab, at service loads, does not exceed the prescribed value, by limiting the ratio of span l_s to the effective depth of slab d_s to 26ξ for a continuous slab, where ξ depends on the stress in steel at service loads

$$l_s/d_s-26\xi\leqq 0. \quad (3)$$

The third constraint ensures that M_u, the ultimate moment of resistance of the beam, is not less than M, the bending moment due to ultimate loads in the beam

$$M-M_u\leqq 0. \quad (4)$$

The fourth constraint checks that the shear capacity V_c of the beam is at least equal to V, the shear force due to ultimate loads

$$V-V_c\leqq 0. \quad (5)$$

The fifth constraint relates to the bond capacity of the beam. Even after curtailment, the maximum bond stress f_{bs} should be less than the permissible value f_{dbs}

$$f_{bs}-f_{dbs}\leqq 0. \quad (6)$$

The sixth constraint checks that the maximum long term deflection in the beam, including effects of creep and shrinkage is not greater than the permissible value

$$y_{max}-l_b/250\leqq 0. \quad (7)$$

The seventh constraint ensures that the maximum width of flexural cracks in the beam, w_{max}, is not greater than the permissible value

$$w_{max}-0.3\leqq 0. \quad (8)$$

For effective T-beam action, the area of transverse reinforcement through the full width of the flange should not be less than 0.3% of the gross cross-sectional area of flange. Therefore, the eighth constraint can be stated as

$$0.003\,x_1-x_2\leqq 0. \quad (9)$$

The ninth constraint checks that the area of tension reinforcement for the beam A_{st} is not less than the minimum prescribed by the code

$$0.0015\,b_w d_b-x_4\leqq 0, \quad (10)$$

where d_b is the effective depth of beam.

The tenth constraint makes sure that the minimum area of main reinforcement provided in the slab is at least equal to the minimum prescribed by the code

$$0.0015\,d_s-c_3x_2/c_2\leqq 0. \quad (11)$$

To limit crack widths in beams, ε_{max}, the maximum strain in steel, at service loads, should not exceed $(0.8\,f_y)/E_s$; thus, the eleventh constraint is

$$\varepsilon_{max}-\frac{0.8\,f_y}{E_s}\leqq 0, \quad (12)$$

where f_y is the characteristic strength of reinforcement, and E_s the modulus of elasticity of reinforcement.

Solution of the problem

This is a constrained nonlinear programming problem and has been solved using the interior penalty function method. Davidon Fletcher Powell algorithm[3] with cubic interpolation technique for one dimensional minimization has been adopted for the solution of this problem. Details of the solution are given in the Appendix. Computations were carried out on a DEC-1090 system at the Indian Institute of Technology, Kanpur and required an average cpu time of 15 seconds for each set of data.

RESULTS AND DISCUSSION

Table 1 gives the optimal thickness of slab x_1 in mm and optimal maximum main reinforcement for slab x_2 in mm^2/m for different spans and different magnitudes of imposed loads. It has been observed that these values are independent of the cost ratios R_1 and R_2. Hence, only one design chart has been prepared to find x_1 (Fig. 3) and one to find x_2 (Fig. 4).

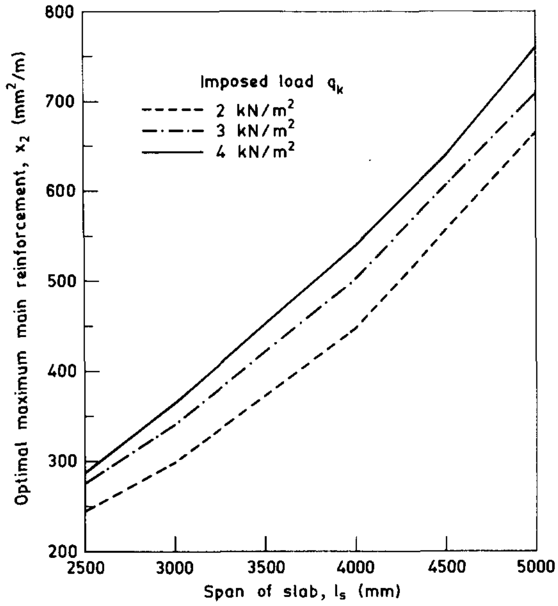

Fig. 4. Design chart for optimal maximum main reinforcement for slab.

Table 1. Optimal thickness and reinforcement for slab

q_k (kN/m^2)	2.0		3.0		4.0	
l_s (mm)	x_1 (mm)	x_2 (mm^2/m)	x_1 (mm)	x_2 (mm^2/m)	x_1 (mm)	x_2 (mm^2/m)
2500	82	245	87	275	93	285
3000	95	300	101	340	107	365
3500	108	370	115	420	121	450
4000	122	445	128	500	135	540
4500	134	550	142	605	150	640
5000	147	665	156	705	164	760

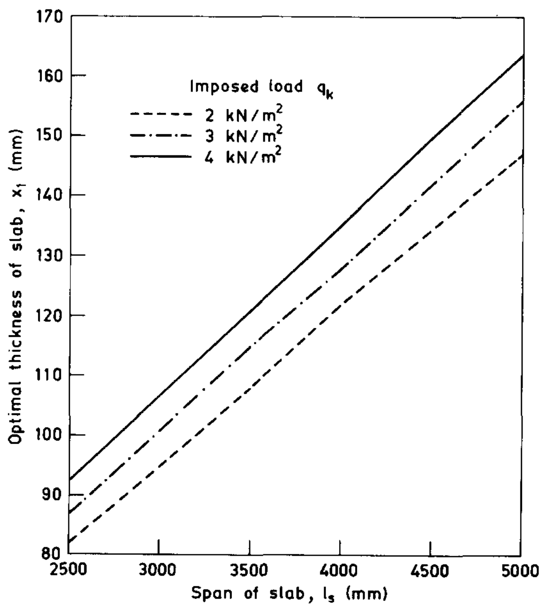

Fig. 3. Design chart for optimal thickness of slab.

Characteristic strength for TorBar is 460 N/mm^2 for bars up to and including 16 mm diameter which is the range normally employed for slabs. If the area of main reinforcement is computed on the basis of this design stress, the stress in steel at service loads would be about 267 N/mm^2. Such an approach leads to a fairly thick slab. If the quantity of main reinforcement provided is more than that required from the strength point of view, steel stress at service loads gets reduced. This would result in a shallower slab which needs lesser quantity of concrete as well as secondary reinforcement for the slab in addition to reducing the dead load on the beam, thus leading to an overall reduction in the cost of the system.

Effect of the variation in values of cost ratios R_1 and R_2 on optimal values of x_3 and x_4, other parameters remaining constant, is shown in Tables 2 and 3. As can be intuitively guessed, depth of beam decreases with the increase in the relative cost of concrete or form work. As a consequence area of tension reinforcement increases in both the cases.

It has been found that in the design of simply supported T-beams, it would be advisable to keep the

Table 2. Effect of variation of R_1 on optimal values of x_3 and x_4

	R_1	x_3 (mm)	x_4 (mm^2)
q_k = 3.0 kN/m^2			
R_2 = 20	600	650	975
l_s = 3500 mm	1200	555	1180
l_b = 7000 mm	1800	490	1370
q_k = 4.0 kN/m^2			
R_2 = 50	600	1130	2700
l_s = 5000 mm	1200	990	3150
l_b = 11,250 mm	1800	910	3460

Table 3. Effect of variation of R_2 on optimal values of x_3 and x_4

	R_2	x_3 (mm)	x_4 (mm^2)
q_k = 3.0 kN/m^2			
R_1 = 600	20	650	975
l_s = 3500 mm	50	585	1110
l_b = 7000 mm	80	525	1290
q_k = 4.0 kN/m^2			
R_1 = 600	20	1340	2270
l_s = 5000 mm	50	1130	2700
l_b = 11,250 mm	80	1020	3015

width of rib as small as practicable[4]. This would reduce the quantity of concrete, value of minimum shear reinforcement as well as the perimeter of the shear reinforcement link. The width of cracks would also be less as the reinforcement will be spaced at a closer interval. At the same time, it will not lead to any stability problems as the compression zone will be in the flange only where the effective width would be more than adequate. Functionally, the rib should be wide enough to place the reinforcement suitably. A width of 150 mm is chosen here which is sufficient to accommodate 4 bars of 25 mm in two rows as shown in Fig. 2(b). When 32 mm bars are required to be used, the width is increased to 160 mm. When the area of reinforcement exceeds 3217 mm^2 (area of 4 bars of 32 mm), the arrangement of reinforcement is as shown in Fig. 2(c).

Figures 5–7 give the optimal depth of beam x_3 in mm and Figures 8–10 give the corresponding optimal area of tension reinforcement x_4 in mm^2 for different cost ratios R_1 and R_2. The requirement that side reinforcement has to be provided whenever the depth of beam exceeds 750 mm is responsible for the peculiar discontinuities in these figures. The beam depth stays constant at 750 mm even with an increase in the value of M, the bending moment due to ultimate loads, till the cost of extra tension reinforcement required exceeds the cost of side reinforcement, extra concrete and additional form work required due to any increase in the depth of beam.

It has been observed that x_3 is not a very sensitive parameter and slight departure (maximum about 5%) from the values reported here does not significantly affect the value of the objective function.

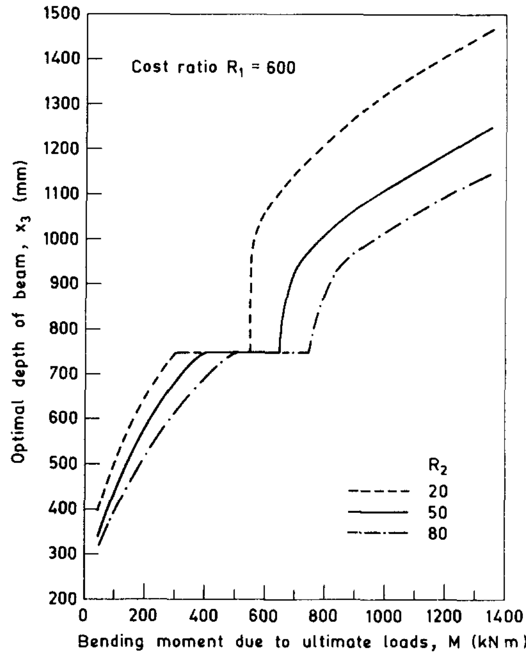

Fig. 5. Design chart for optimal depth of beam.

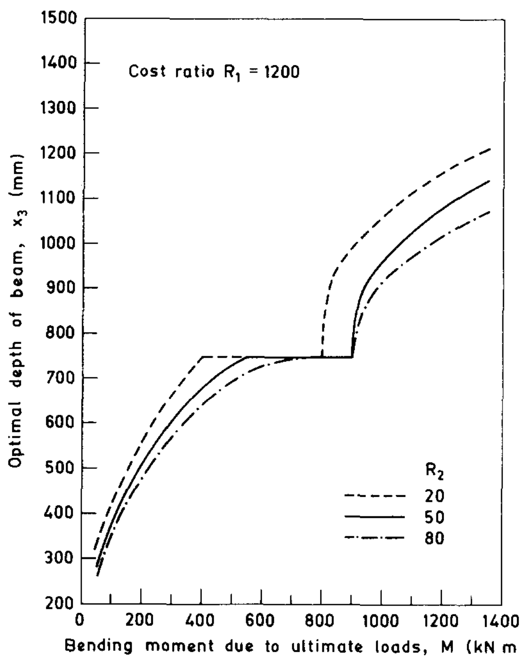

Fig. 6. Design chart for optimal depth of beam.

CONCLUSIONS

(1) Optimal values of the thickness and main reinforcement for the slab are independent of cost ratios R_1 and R_2.

(2) When the characteristic strength of reinforcement is high, it would be an optimal policy to provide more main reinforcement for the slab than is required from

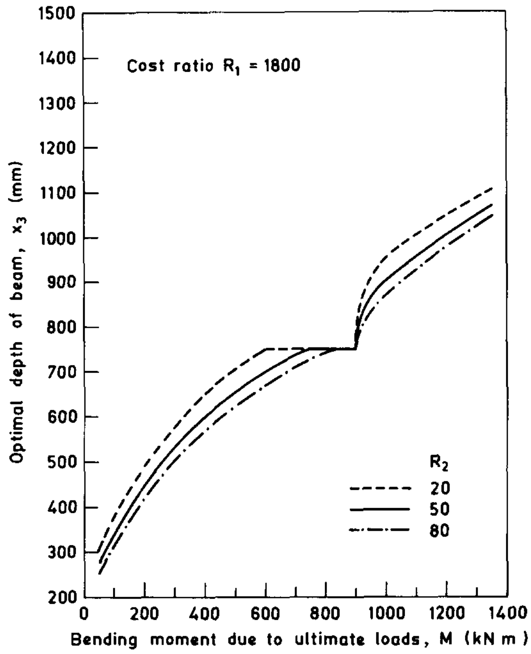

Fig. 7. Design chart for optimal depth of beam.

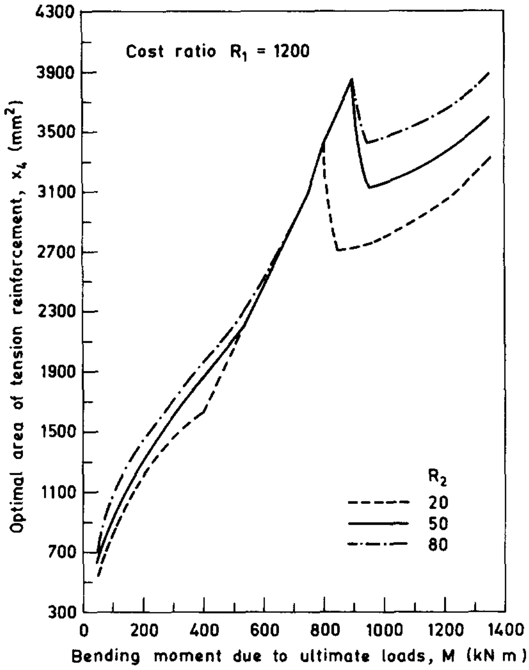

Fig. 9. Design chart for optimal tension reinforcement for beam.

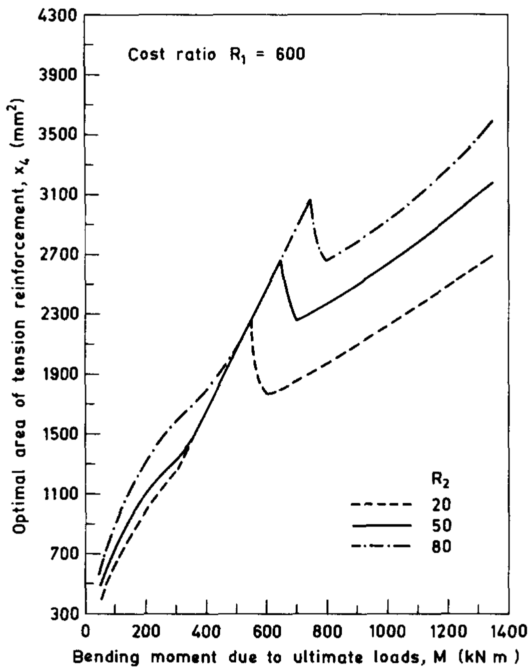

Fig. 8. Design chart for optimal tension reinforcement for beam.

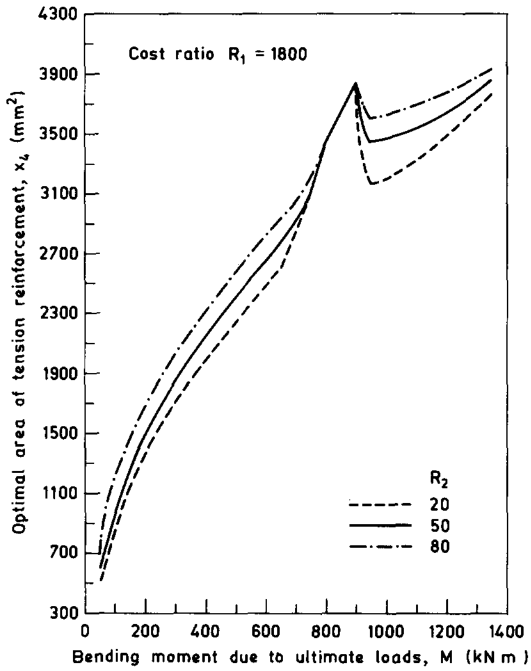

Fig. 10. Design chart for optimal tension reinforcement for beam.

the strength point of view in order to reduce the cost of the system.

(3) The width of rib should be kept as small as practicable. Thus, it would be advantageous to group the bars vertically and to provide more than one layer when required.

(4) The requirement that side reinforcement has to be provided when the depth of beam exceeds 750 mm leads to discontinuities in the design charts.

(5) Considerable reduction in the cost of T-beam floors can be achieved using the design charts developed here.

METHOD OF USING THE CHARTS

(1) Identify the span of slab l_s, span of beam l_b, magnitude of imposed load q_k, cost of finished concrete

per m^3 cost of reinforcement per newton and cost of form work per m^2.

(2) For the given values of l_s and q_k, determine the optimal value of x_1 (mm) from Fig. 3 and optimal value of x_2 (mm^2/m) from Fig. 4.

(3) Determine the cost ratios R_1 and R_2.

(4) With the known value of x_1 and an assumed value for x_3, determine M, the maximum bending moment in the beam due to ultimate load in kN m.

(5) Choose the appropriate chart for the known value of R_1 and find the optimal value of x_3 (mm) and x_4 (mm^2). Interpolate, whenever it is necessary.

(6) Having fixed the main design variables, work out the other details.

EXAMPLE

Data: $l_s = 3900\,\text{mm}$, $l_b = 8780\,\text{mm}$, $q_k = 3\,\text{kN/m}^2$. Cost of finished concrete = 480 Rupees/m^3; cost of reinforcement = 0.4 Rupees/N; and cost of form work = 20 Rupees/m^2.

From Figs. 3 and 4, for $l_s = 3900\,\text{mm}$ and $q_k = 3\,\text{kN/m}^2$, optimal thickness of slab = 125 mm, and optimal value of maximum area of main reinforcement = 485 mm^2/m. Now, $R_1 = 480/0.4 = 1200$ and $R_2 = 20/0.4 = 50$.

Assuming depth of beam as 750 mm, the ultimate load will be

$$\{125 \times 3900 + (750 - 125) \times 150\}$$
$$\times 24 \times 10^{-6} \times 1.4 + 3 \times 10^{-3} \times 3900 \times 1.6$$
$$= 38.25\,\text{N/mm}.$$

Maximum bending moment in beam at ultimate load

$$M = \frac{38.25 \times 8780^2}{8} = 3.686 \times 10^8\,\text{N mm}$$
$$= 368.6\,\text{kN m}.$$

From Fig. 6, corresponding to $R_1 = 1200$, $R_2 = 50$ and $M = 368.6$

Optimal depth of beam = 650 mm.

From Fig. 9, corresponding to $R_1 = 1200$, $R_2 = 50$ and $M = 368.6$,

optimal area of tension reinforcement = 1770 mm^2.

APPENDIX

Details of the solution of the optimization problem are given below.

There are many techniques available for the solution of a constrained nonlinear programming problem. The interior penalty function method is one of the indirect methods. Penalty function methods transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. If the original problem is to minimize a function $f(X)$ subject to constraints $g_j(X) \leq 0$, $j = 1, 2, \ldots m$ where m is the number of constraints, this problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\phi_k = \phi(X, r_k) = f(X) + r_k \sum_{j=1}^{m} G_j[g_j(X)],$$

where G_j is some function of the constraint g_j, and r_k is a positive constant known as the penalty parameter. In this study G_j has been taken as equal to $-[1/g_j(X)]$. If the unconstrained minimization of the ϕ-function is repeated for a sequence of values of the penalty parameter r_k ($k = 1, 2, \ldots.$), the solution may be brought to converge to that of the original problem.

The procedure may be summarized as follows.

(1) The solution is commenced from an initial feasible point X which satisfies all the constraints with strict inequality sign and an initial value of $r_1 > 0$. k is set equal to 1.

(2) $\phi(X, r_k)$ is minimized to get X_k^* using the DFP algorithm to find the search direction and cubic interpolation method to find the optimal step length.

(3) X_k^* is tested for optimality. If it is optimal, the process is terminated.

(4) Otherwise the penalty parameter is modified as $r_{k+1} = cr_k$ where c is less than 1.

(5) The new value of k is set equal to $k+1$, the new starting point X_1 set equal to X_k^* and the next minimization cycle commenced from step (2).

DFP algorithm and cubic interpolation method used in the minimization procedure are briefly described below.

(1) The method starts with an initial point X_1 in the design space and a $n \times n$ positive definite symmetric matrix H_1 where n is the number of design variables. The iteration number i is set equal to 1.

(2) The gradient of the modified objective function, $\nabla\phi_i$, at the point X_i is computed and the search direction S_i is computed as

$$S_i = -H_i \nabla \phi_i.$$

(3) The optimal step length λ_i^* in the direction S_i is found using the cubic interpolation method in four stages. First the search direction S_i is normalized so that a step size $\lambda = 1$ is acceptable. Then the derivative of the function ϕ is used to establish bounds on λ^* as the slope has to change from negative to positive at the optimum point. In the third stage an approximate value of λ^* is found by approximating $\phi(\lambda)$ by a cubic polynomial. If the value of λ^* found in the third stage does not satisfy the convergence criteria, the cubic polynomial is refitted in the fourth stage. A better point in the design space is then found as

$$X_{i+1} = X_i + \lambda_i^* S_i.$$

(4) The new point X_{i+1} is tested for optimality. If X_{i+1} is optimal, the iterative procedure is terminated.

(5) Otherwise, the H matrix is updated as

$$H_{i+1} = H_i + M_i + N_i,$$

where

$$M_i = \frac{S_i S_i^T}{S_i^T Q_i},$$

$$N_i = -\frac{(H_i Q_i)(H_i Q_i)^T}{Q_i^T H_i Q_i}$$

and

$$Q_i = \nabla\phi(X_{i+1}) - \nabla\phi(X_i).$$

(6) The new iteration number i is set equal to $i+1$ and the new iteration is commenced from step (2).

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