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Identification and Control of an Unstable SOPTD system with positive zero

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Abstract

The work deals with the identification and control of unstable Second Order plus Time Delay (SOPTD) system with positive zero. Presence of positive zero complicates the performance of the control system dynamics. There are many unstable systems which exhibit the second order plus time delay with positive zero such as drum boiler, distillation column. No work has been reported in the literature on identification of unstable SOPTD process with positive zero. In this work, a subspace based method and an optimization method are proposed to identify an unstable SOPTD model with positive zero followed by the PID controller design which can handle set-point changes and disturbance rejection. The subspace-based method uses input-output measurements to estimate the state space model. This method uses projections of block Hankel matrices followed by a singular value decomposition to determine the order of the system. It offers the key advantages on providing low parameter sensitivity with respect to perturbations for higher order systems. The model parameters are also identified using optimization technique by matching the closed loop responses of the process and the model. In any optimization technique, the initial guess plays an important role for proper convergence. A method is suggested to obtain the initial guess values for process gain, poles, zeros and delay. The parameters identified by subspace based method are compared with that obtained using optimization technique. For the models identified by the above two methods, controllers are designed and implemented. Simulation studies on linear and nonlinear systems are demonstrated to evaluate the performance of the proposed methodologies. The closed loop performances comparison can be made in terms of time integral errors and total variation in input variable.

Keywords: Unstable systems, SOPTD with positive zero, subspace identification, optimization method

1. Introduction

Unstable systems are common in chemical industries. Examples include isothermal CSTR (Liou and Chien, 1991), Nonlinear Bioreactor (Agarwal and Lim, 1989), Dimerization reactor (Alio and Al-humaizi, 2000), Fluidized bed reactor (Kendi and Doyle, 1996), polyolefin reactor (Seki et al., 2001). Padma Sree and Chidambaram (2006) have given an excellent review on the control of unstable systems. Unstable SOPTD systems with a zero are difficult to control due to the presence of an overshoot or inverse response. Some of the examples reported in literature include the Klien's unrideable bicycle (Klien, 1989), Jacketed CSTR (Bequette, 2003).The presence of zero in the unstable transfer function imposes a difficulty in controlling such systems. For the purpose of designing controllers, identification of the model parameters are required which helps in improved tuning of the controllers. Open loop identification cannot be applied to unstable systems. The systems with positive zeros are slow in response because of their undershoot response at the beginning of the response. Presence of this positive zero complicates performance of control system dynamics. One of the prominent problems with this kind of system is internal stability. Ram et al. (2014) have identified unstable SOPTD systems with negative zero using optimization method. In this work, the method is extended to unstable SOPTD systems with positive zero. Recently Sankar Rao and Chidambaram (2017) reported subspace identification for unstable systems. In this work, the unstable system under consideration is stabilized by a PI/PID controller. A second order time delay model with a positive zero is identified from the closed loop response using a step change in the set point. The process model is identified using optimization method and subspace method separately. The identified models by the above two methods are used for designing controllers and the closed loop performance are evaluated for servo and regulatory response.

2. Proposed Methodology

2.1 Identification of Unstable SOPTD systems with positive zero by subspace identification method

Subspace identification is used to get a linear time invariant state space models directly from the input and output measurement data. The various forms of the subspace based identification methods have attracted much of interest. Subspace identification methods use the concepts of systems theory and linear algebra. Subspace identification methods consist of two steps. In first step, it determines the extended observability matrix and state sequences from the row and column spaces of certain matrices, which are formed from the input output data. Second step estimates the state space model using either the knowledge of extended observability matrix or state sequences.

2.2. Identification of Unstable SOPTD systems with positive zero by optimization method

Consider the unstable second order open loop transfer function with a positive zero given by $G_p(s) = \dfrac{k_p(1-ps)}{a_1 s^2 + a_2 s + 1} e^{-\theta s}$. In order to identify the process parameters (kp, p, a_1, a_2), system is stabilized by a PID controller and the closed loop response is noted. The system is modelled as second order process with a positive zero

$$\frac{y_{cl}(s)}{y_r(s)} = \frac{k_p(1-ps)}{(\tau_e s^2 + 2\zeta\tau s + 1)} e^{-\phi s} \tag{1}$$

For a unit step change in the setpoint the closed loop response is given by (Seborg and Millichamp, 2006)

$$y(t) = k_p\left[\frac{p}{\tau_e\sqrt{(1-\zeta^2}} e^{-\zeta t'/\tau_e} \sin(\sqrt{(1-\zeta^2}\frac{t'}{\tau_e})\right] + \left(1 - e^{-\zeta t/\tau_e}\left[\cos(\sqrt{(1-\zeta^2}\frac{t'}{\tau_e}) + \frac{\zeta}{\sqrt{(1-\zeta^2}} \sin(\sqrt{(1-\zeta^2}\frac{t'}{\tau_e})\right]\right) \tag{2}$$

Where $t'=t-\Phi$. Ananth and Chidambaram (1999) have defined the formulas for obtaining ζ and τ_e

$$v_1 = \frac{y_{\inf} - y_{m1}}{y_{p1} - y_{\inf}}; v_1 = \frac{y_{p2} - y_{\inf}}{y_{p1} - y_{\inf}} \tag{3}$$

$$\zeta_1 = \frac{-\ln(v_1)}{\sqrt{\Pi^2 + (\ln(v_1))^2}}; \zeta_2 = \frac{-\ln(v_2)}{\sqrt{4\Pi^2 + (\ln(v_2))^2}} \tag{4}$$

$$\zeta = \zeta_1 + \zeta_2 \tag{5}$$

$$\tau_e = \frac{\Delta T \sqrt{1-\zeta^2}}{\Pi} \tag{6}$$

$$a_1 = \tau_e^{\,2}; a_2 = -2\zeta\tau_e^{\,2} \tag{7}$$

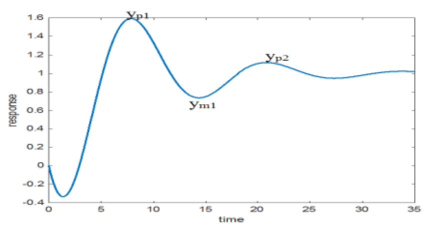

Figure 1: Closed loop response of a system to a step change in setpoint

From the closed loop response the value of y_{p1}, y_{p2}, y_{m1}, y_{inf} are noted. The sign of final steady state value of u (deviation value) is negative and the sign of k_p is positive, then there is one unstable pole and one stable pole present in the system. In order to get the initial guess values for identifying the process parameters from the closed loop response, the method given by Ananth and Chidambaram (1999) is used to obtain the initial guess values of a_1 and a_2. The initial guess value for k_p is taken as the ratio of the final steady state value of the closed loop response to the final steady state value of the manipulated variable u. Equations (3) to (6) are solved to obtain the initial guess values of a_1 and a_2. Equation (2) is solved to obtain the initial condition for the p from undershoot. To evaluate the proposed methodology a known unstable system is taken. The system is stabilized using PID controller and the closed loop response is obtained by giving a unit step change. The model to be identified is also stabilized using the same controller settings. All simulations are carried out on Matlab and Simulink. In order to solve the least square optimization problem, the Matlab routine *lsqnonlin* is used. The objective function is formulated to minimize the sum of the square of the errors between the closed loop response of the model and the process.
Using the methodology explained in Section 2.1 and 2.2, the model identification of the unstable SOPTD system with positive zero is carried out. The identified model parameters by both the methods are stabilised by designing controllers and the results compared.

3. Simulation studies

3.1. Example 1

Consider the transfer function given by $G(s)=\frac{3.5(5-0.7s)}{(s^2-6.867)}$. The system is stabilized by the PID controller k_c=0.4708; τ_I = 8; τ_D =0.58. The response of the system for unit step change in set point is given in Figure 2. The data obtained are y_{p1}=3.5090; y_{p2}=1.3478; y_{m1}=0.0658; y_{inf} =1; ΔT=12.25; u=-0.3884; ζ=0.0683; τ_e =1.9451. The system has an inverse response which shows the presence of a positive zero in the transfer function. The initial guess values for kp is taken from the closed loop response as the ratio of final steady state value to initial steady state value k_p=1/(-0.3884) = -2.575. To obtain the initial guess values of a_1, a_2 and p the methodology proposed above is applied. The initial guess values are obtained as a_1 =3.7834; a_2=-0.2658; p= -0.9121. To identify the model parameter optimization technique is carried out (Section 2.2). The converged final model parameters identified are k_p=2.5484; p=-0.14; a_1=0.1456; a_2=0. Figure 2 shows that the closed loop response of the identified model matches with the actual response for the same controller settings. Using the identified model parameters the PID controller settings are designed using IMC method. Table 1 shows the controller settings obtained and the ISE, IAE and the TV values. The transfer function model is simulated and the output data is generated. Pseudo Random Binary Signal (PRBS) is used as the exogenous input to excite the process. The order of the system is estimated by inspecting the singular values. The number of dominant singular values will give the information of the order of the system. The model parameters estimated based on subspace based method are K_p = 2.34, p = 0.12, poles are 2.618 and 2.532. From the residual analysis, it is confirmed that the identified model is capable of explaining the dynamic relationship between the cause and effect. Based on the identified model, a PID controller is designed by IMC method and the obtained settings are listed in Table 1. Using the controller settings designed based on the identified model (by optimization and subspace methods), the closed loop performances are evaluated by introducing a unit step change in the set point at time, t = 0 sec and a negative step change in disturbance variable at time, t = 25 sec which is shown in Figure 3. Figure 3 shows the comparison of the closed loop servo and regulatory performance for the identified model by optimization method and subspace identification method.

Table 1: Performance evaluation of the identified models

Examples	Models	Controller settings			Time integral errors		Total Variation in u (TV)
		Kc	τ_I	τ_D	ISE	IAE	
Ex1	Model 1	0.4914	14.6224	0.4141	214.5	42.68	48.3
	Model 2	0.5383	14.3915	0.4143	126.3	38.19	59.2
Ex2	Model 1	0.6493	30.4435	13.2848	220.6	244.3	1170
	Model 2	0.6093	30.3920	13.2903	219.2	258.4	1076

Model 1: Identified by optimization method; Model 2: Identified by subspace method

3.2. Example 2

Consider the isothermal Continuously Stirred Tank Reactor given by Rao and Chidambaram (2006). The transient nonlinear CSTR model is linearized around the unstable operating point and the resulted transfer function relating concentration of A to to the feed concentration is given by $\frac{\Delta C_A(s)}{\Delta C_f(s)} = \frac{-0.2679(1-41.67s)}{(279.3s^2 - 2.7981s + 11)} e^{-20s}$

The system is stabilised to obtain an underdamped closed loop response of the form as shown in Figure 1 by using the controller settings given by K_c=0.3471;τ_I=-100, τ_D =0.429. From this the value of y_{p1}=1.647; y_{p2}=1.08; y_{m1}=0.7682; y_{inf}=1; ΔT=197.3; u=-3.614 are noted. The values of ζ and τ_e are obtained as ζ =0.0708; τ_e = 31.3224. Using equation (2) and (7) the initial guess values of kp, a_1, a_2 and p are obtained and these are kp=-0.2767.;p=-236.2121;a1=222.3402;a2=-9.3381.Closed loop time constant is assumed as 2-3 times of open loop time constant. The initial guess value for open loop time delay is taken same as the closed loop time delay. Using optimization the process parameters are identified and the converged parameters are obtained as k_p=-0.2679.; p=-41.6667;a_1=279.03;a_2=-2.9781, θ =20. The identified model is used for designing the controllers. In the present work, the controller settings proposed by Sree and Chidambaram (2002) by synthesis method is used. The controller settings given by them are in the form $Gc(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)\left(\frac{1}{\alpha s + 1}\right)$. For the present model parameters obtained by optimization, the controller settings are enlisted in Table 1. Table 1 also shows the IAE, ISE and TV values. Model parameters obtained from the subspace identification method are Kp = -0.285, p = -41.67, a_1 = 279.2, a_2 = -2.953 and θ =20, PID settings are determined by synthesis method and given in Table 1. The time integral error are also calculated and presented in Table 1. Figure 4 shows the closed loop performance for the identified models by the two proposed methodologies.

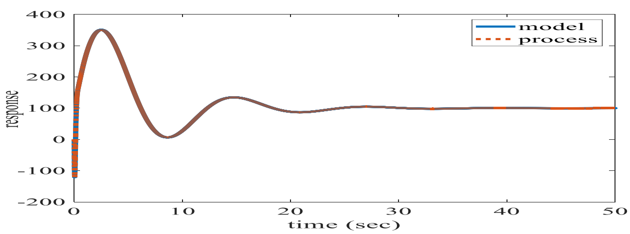

Fig 2: Closed loop servo response of the process and identified model for the same controller settings (optimization method-example 1)

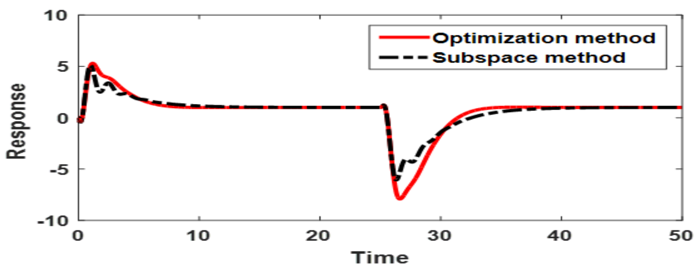

Figure 3: Closed loop response of the controllers designed based on the identified model by optimization method and subspace methods by IMC method for example 1

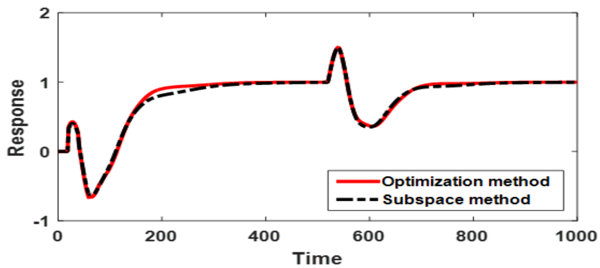

Figure 4: Closed loop response of the controllers designed based on the identified model by optimization method and subspace methods by synthesis method for example 2

4. Conclusions

A closed loop identification method based on optimization and subspace identification for unstable SOPTD systems with zero is proposed. Two case studies were demonstrated in order to evaluate the identification methods. The model parameters obtained by the proposed method were found to match with the actual values. Based on the identified model controllers were designed using methods available in literature. A good set-point tracking and disturbance rejection was obtained. The ISE, IAE and TV values are evaluated. It is found that the closed loop responses obtained by both the identified models are almost similar which emphasis that the proposed methods give good model parameters for designing suitable controller for unstable SOPTD system with zero.

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