OPTIMUM MATERIAL DISPOSITION IN 2D IN-PLANE BENDING PROBLEMS – NODES IN MOTION STRATEGY

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D E C L A R A T I O N

by the Ph.D. Research Scholar

I hereby *declare* that the Research Thesis entitled

Optimum Material Disposition in 2D In-plane Bending Problems – Nodes in Motion Strategy

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I dedicate this dissertation
to
My wife Anaja
for her consistent encouragement and endless patience
and
to
My daughter Malu
for her unconditional Love

ABSTRACT

Structural Optimization is the process of making high performance structures by identification and removal of un-necessary elements and material without affecting its functional , safety, serviceability and durability requirements. An optimized structure naturally leads to savings in cost and time.

The present research work is n the direction of devising an optimum structure that adopts the best use of material at its best location in its best form under the given conditions.

Finite Element Method has opened up ways to analyse complicated structures subjected to arbitrary loading with the required amount of accuracy demanded by an analyser. Realizing the limitations of FEM, a technique called Moving Polynomial Moving Least Square (MPMLS) has been formulated for smoothing and interpolation of stress values at any location in 2D continuum structures subjected to in-plane bending.

Optimum material disposition is achieved by relocation of material to its best position by the assessment of material utilization at any given location and using the required quantity to just satisfy the conditions. A novelty called 'Nodes in Motion' strategy has been conceptualised to facilitate guided movement of under- utilized material to its best location in the optimum quantity.

The conceptualisation, mathematical formulation, implementation and verification have been presented at every milestone of development. The results obtained have shown adoptability of the procedure for the optimum design of 2D structures subjected to in-plane bending.

The potential uses of the present research findings and scope for future work have been presented.

Keywords : Structural Optimization, Smoothing, Interpolation,2D Continuum, in-plane Bending , Material Disposition.

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Chapter 1

INTRODUCTION

1.1 GENERAL

Any manmade object in the world is a result of two important efforts. The First and foremost is a vision of the product well in advance and the second, the knowledge to make it. While the vision is more related to art, the knowledge is more connected to science.

Structural Engineering is the art of visualising assemblages of elements to satisfy the functional requirement of a user. As science, it is the knowledge of predicting its behaviour and proportioning it to satisfy the requirement of stability, strength, serviceability and durability.

Arriving at a solution that fulfils the foresaid at affordable cost offers tremendous scope for optimization which aims at getting the most, expending the least.

Structural optimization as an area of research interest has attracted the attention of investigators to propose means, modes and methods to make the designs true to the mantra ‘minimum effort - maximum effect’.

Analysis and design are iterative processes seeking better description of a system providing direction for searches. The availability of reliable analysis methods coupled with ever increasing speed of digital computers has led to remarkable progress in development of optimization tools. Enhanced capabilities of computations have paved the way to visualisation, conceptualization, formulation and solution of optimization problems in structural engineering, never attempted in the past.

From single objective formulations like maximizing the strength or minimizing the weight, matters have grown to multi-objective optimization routines wherein real time lexicographic and adaptive searches are attempted and are being exploited.

1.2 STRUCTURAL OPTIMIZATION PROBLEMS – FORMULATION

In general, a structural optimization problem typically involves

a) The identification of a set of parameters called design variables to describe design alternatives

b) The selection of an optimality criterion called 'objective', as a function of design variables, that is being minimized or maximized

c) The establishment of a set of predefined rules called 'constraints' as functions of design variables which must be satisfied by any acceptable design. Constraints are conditions that are to be met like physical, geometric, material, mandatory codal clauses, legal, etc.

d) The knowledge of procedures to determine the values of design variables which minimize or maximize the objective while satisfying all the constraints. The type of objective function and constraints decide the solution technique.

1.3 MOTIVATION AND PRINCIPAL OBJECTIVE

Traditionally, shape was never considered as fundamental variable at the level of the structure or the element, though it needed serious consideration. These days, shape optimization is being attempted to arrive at the best configuration for the structure through which the most efficient load flow can be accomplished.

Topology and shape optimization of articulated structures have been successfully attempted, methods have been proposed and are being used. Extension of these techniques to continuum like plates poses problems associated with geometric modelling in FEM. Conventional formulations miserably fail in addressing compatibility, high stress and strain gradient issues.

Present work is an attempt to overcome these unresolved issues in application of FEM in structural engineering. The issue of incompatibility at element edges have been addressed by a newly proposed smoothing formulation adopting a newly proposed Moving Polynomial Moving Least Square technique.

Available literature also suggests that there is tremendous scope to revisit, redefine and refine optimization techniques which find application in elastic continuum.

The present work envisages to propose a novelty called 'Nodes-in- Motion' which is a strategy that works as an attractor of materials towards highly stressed zones from zones where the material does not contribute for structural performance.

This works as a self governing adaptive technique.

The combination of these two approaches ensures right solution to the famous Navier's requirement of satisfaction of equilibrium, compatibility and force-deformation relationship, combined with the objectives of structural optimization.

1.4 OBJECTIVES OF THE PRESENT STUDY

The objective of the work is to evolve an iterative procedure to arrive at the best shape by way of the most efficient material disposition for in-plane plate bending problems.

The following are the steps identified to accoplish the end objective.

1. To develop a code in Visual Basic Environment to solve 2D (in-plane) plate bending problems using Finite Element Method.
2. To propose an efficient stress smoothing technique.
3. To formulate a strategy for appropriate material disposition for attainment of design objectives
4. To suggest an adaptive technique to ensure convergence.

1.5 ORGANIZATION OF THE THESIS

Chapter 1 emphasizes the need for optimization specific to the field of structural engineering, elaborates the need and motivation for the current work and outlines the objectives and scope.

An account of structural optimization using mathematical, analytical and experimental formulations available in the literature including the recent & ongoing developments and advancements is presented in Chapter 2.

Chapter 3 explains basic steps followed in a generalized Finite Element Analysis (FEA), theory and applications of 2D in-plane bending. The finer details of the code developed for FEA of 2D in-plane plate bending are explained including the verification and validation.

Unresolved issues in stress smoothness with FEA formulation and the need improving accuracy of stresses specific to structural optimization are highlighted in Chapter 4. A novel approach ‘Moving Polynomial Moving Least Square Technique’ proposed and developed is explained with illustrations.

Chapter 5 presents the concept of element based and node based evaluations of material utilization that has been suggested and developed for shape optimization with optimum material disposition.

The development of an algorithm combining the Moving Polynomial Moving Least Square technique and Nodes-in-Motion strategy for the shape optimization of plates subjected to in-plane bending is presented in Chapter 6. Several problems are solved , the results are compared and the efficiency of the formulations is verified.

Chapter 7 wraps up the entire research effort carried out so far. Mile stones achieved, results accomplished and the performances of methodologies developed are reiterated both individually and as a combination. The potential of the research findings for its use in various fields are highlighted here. The scope for future work in this specific area of research is also briefly presented.

Appendices are given, containing general derivations, the results of which are used in the development of source code.

Several documents were referred to for the achievement of objectives. Research publications both in journals and online, dissertations, books, conference proceedings and course materials that have contributed to this work are listed in References.

Chapter 2

REVIEW OF SELECTED LITERATURE

2.1 GENERAL

The description of any design process must contain the recognition of a need and a selection of alternatives. Traditionally, the selection of the "best alternative" is the phase of optimization (Schoofs,1993). The increasing necessity for lightweight, high-performance and low-cost structures, drives the considerable current research in the field of structural optimization.

Plenty of literature is available for reference in the field on methods of analysis, solution techniques in generalised and specific areas of structural optimization. A few selected works are highlighted here which have contributed to the present work in achieving the objectives.

2.2 METHODS FOR STRUCTURAL OPTIMIZATION

Structural optimization problems have been solved with different objective functions and constraints to arrive at optimum shapes. The mathematical formulations and solution techniques greatly are influenced by the type of the structure studied and the nature of results required.

2.3 STOCHASTIC ALGORITHMS.

The stochastic algorithms are ,in nature, with probabilistic translation rules. These are gaining popularity due to certain properties which the deterministic algorithms do not have. The genetic algorithm (GA) and simulated annealing (SA) algorithm are two of the most popular stochastic optimization techniques (Fazil, 2007)

2.3.1Genetic Algorithms (GA)

This technique was developed by John Henry Holland in the late 1950s and in the beginning of the 1960s (Holland,1992).

Living organisms evolve the best ways to survive in critical situations. The fittest organisms survive and the others die. Similarly, Genetic algorithms explore ways to survive and move ahead trying to be the fittest every time. The evolution of the most optimum solution is attained through an iterative process, each iteration being called a generation. The fitness is usually the value of the objective function that needs to be minimised or maximised.

2.3.2 Simulated Annealing

Simulated annealing (SA) (Kirkpatrik, 1982) works on the analogy with the way molten metals cool and anneal. This method is a heuristic process to solve problems with several variables where there is a likelihood of a global optimum hidden between local optima. In this from an initial set of variables, small changes are made to find the new objective function. If the solution is not better, the variables move in random directions to find a better solution. Again, at that location, small changes are made to the variables to study the possibilities of a better solution, till the global optimum solution is obtained.

2.4 DETERMINISTIC ALGORITHMS.

These algorithms use specific rules for moving one solution to other. These algorithms are in use to suite sometimes and have been successfully applied for many engineering design problems

In the present work, deterministic algorithm is proposed using iteration procedure to improve quality of results. Some of the procedures and published materials in this category and the developments in the related area which have helped formulate the concepts of the present work are given here.

2.5 SHAPE, TOPOLOGY AND SIZE OPTIMIZATION

The structural design optimization generally start from a given shape and topology and then the search for a best solution under given conditions starts. Generally for continuum with complex boundaries Finite Element Method is used to determine the deformations and stresses at locations. In pure shape optimization, the boundary nodes are shifted to their changed position to give the best solution (Akira, 1992) keeping the thickness same.

Shape optimization attempts to integrate geometrical modeling, structural analysis, and optimization into one complete and automated computer-aided design process. (Hsu, 1994). There are many methods, classic and advanced that have been formulated, developed, experimented and established by engineers for optimization of structures. The never ending pursuit of optimization not only has interesting theoretical implications in mathematics, mechanics, multi-physics and computer science, but also has lead to important practical applications (Rozvany, 2007).

Waqas Saleem et. al (2010) described the strategy for optimal configuration design of existing structures by topology and shape design and how shape optimization can be achieved through an iterative procedure

2.6 EVOLUTIONARY STRUCTURAL OPTIMIZATION (ESO) AND RELATED METHODS

The optimization was recognised as a material distribution problem in the earlier stages of development of the shape and topology optimization (Bendsoe, 1989).Various methods for removal of material as density reduction were attempted by introducing a density function representing the volume of material at any location. A general "Layout Optimization" procedure was invented which integrated the Shape, Topology and Size optimization of Elastic structures (Diaz et al, 1993)

Evolutionary Structural Optimization (ESO) is based on the simple idea that the optimal structure for maximum stiffness or minimum weight can be produced by gradually removing the ineffectively used material from the design domain (Xie and Steven,1993).

The ESO method is simple to formulate through FEA packages and requires a relatively small amount of computer time. In gradual and calculated process, the method removes redundant material to evolve cavities, thus reducing the consumption. This method has been very popular and paved the way to further developments based on the performance of material at locations in the domain of the continuum.

There are two options in the classical ESO. At a locations not effective, it hard kills the elements, the methodology that need not work always. Instead of removing the non effective elements, "soft kill" methods came into existence where the capabilities of the material is nullified using the Young's modulus of such elements reduced to a small value in executing the next cycle of software solution (Papadrakakakis, et. al., 1996)

The use of ESO is to be employed with care, where its validity is not applicable in every situation. (Zhou et. al, 2001) Hence newer methods overcoming this weakness of ESO , came to existence. One of them has been to eliminate the element with stress which is lower than threshold using "deleted element algorithm" to restructure the remaining nodes and elements using the re-domain algorithm. Re-domain produces new elements with smaller size to obtain smooth boundary surface (Ismail,et al, 2004)

An optimization technique called Morphing Evolutionary Structural Optimization (MESO) was developed by Hatem et. al (2008) for stiffened plates. The method works on the principle of slowly removing the inefficient material changing the geometry every time, thus evolving a structure performing better.

2.7 PERFORMANCE BASED OPTIMIZATION (PBO)

The performance-based design is defined as the methodology in which structural design criteria are expressed in terms of achieving multiple performance objectives.The Performance can be measured in terms of functionality, strength, serviceability or cost.

A performance index for topology and shape optimization of plate bending problems with displacement constraints was presented by Qing Quan Liang (2001) in which method was proposed to keep track of the performance history when inefficient material is gradually removed from the design and to identify optimal topologies and shapes from the optimization process. The PBO method combines modern structural optimization theory with performance-based design concepts to produce a powerful technique for use in structural design. Performance levels for functionality, strength, serviceability and economy can be defined by the limiting values of measurable structural response parameters.

2.8 STRESS BASED OPTIMIZATION METHODS

Many mechanical parts are designed based methods which optimize the thickness of material used in constituting a component (Dulyachot , 2006). The thickness of material is adjusted inversely proportional to the von Mise's stress developed. During the trimming process , if very thin material is found at a location, it is removed changing the shape of the structure.

While attempting stress based optimization schemes for material reduction based on the force or stress values, at some locations, if the force becomes null, it tends to bring the stress values as zero, This is known as stress singularity, which leads to instability (Guilherme, 2007). asking for zero area requirement. of the material. Hence at many locations, within the design domain, a minimum density of material is assumed.

Another difficulty of stress-based topology optimization is due to the local nature of the stress constraint. In a continuum setting, stress constraints should be considered at every material point (Le Chau,2010).

Enforcing stress constraints in topology optimization presents some challenges. Topology optimization problems typically have a large number of elements, so satisfying the stress constraints at multiple points in each element would result in a large-scale optimization problems (Lee, 2012).

Extending this further, The research work has proceeded to develop new interpolation schemes like a separable stress interpolation (Seung, et. al ,2014) which allows stress-based topology optimization with multiple materials (STOMM).

2.9 SOME INTERESTING TECHNIQUES

An interesting methodology called Element Exchange Method was formulated by Mohamed Rouhi(2009) in an attempt to optimize a continuum. The algorithms initiates a random distribution of solid elements and void elements. Void elements are with a minimum density of negligible density value. Finite element analysis is performed to find the strain energy in the elements. A specific number of the solid elements with low strain energy are converted to void elements and equal volume of void elements with highest strain energy are converted to soild elements. In a sense, this works as transfer of material from a less efficient location to a better efficient location.

Zhan Kang (2005) solved Optimization problems based on mass moment of inertia. The topology optimization problem for minimization of structural compliance was formulated under a single constraint on the mass moment of inertia, rather than on the material volume. This can be considered as s step towards relating the disposition of material , rather than the total quantity of material to assess the capacity of structures in transferring loads.

Many of the optimization problems contain contaminated data showing uncertainity (Calafiore and Dabbene, 2007). These problems have been attempted as "in- the- worst- case" or Min-Max approach.

2.10 OPTIMIZATION OF 2D TRUSSES

During the development of Nodes-in Motions strategy conceptualized in the present work, the method has been first illustrated for 2D articulated structures. The capability of the procedure has been compared with the optimization solutions available in the literature. Hence a review has been carried out to know the developments in the field of optimization of trusses.

Tang et. al (2005) have developed an Improved Genetic Algorithm for design optimization of truss structures with sizing, shape and topology variables with mixed coding of integer and float types of variables.

A hybrid real-parameter genetic algorithm has been developed for trusses by Hwang et. al (2006) to solve optimization problems. The performance of the algorithm in discrete sizing variables and continuous configuration variables, both individually and combined has been studied.

Sizing, geometry and topology optimization of trusses via force method and genetic algorithm has been introduced by Rahami et. al (2008) which used a combination of energy and force methods in optimizing the weight of trusses.

An interesting Imperialistic Competitive Algorithm for truss structures has been presented by Hadi et. al. (2010) inspired from social human phenomenon , in which some empires with lowest cost are considered the best and the rest of the countries in the neighbourhood are considered colonies. The power of a country is inversely proportional to its cost. This has been used as a function to ultimately solve the optimization problem using Genetic Algorithm.

Kulkarni et. al (2012), have presented a mutation-based real-coded genetic algorithm (MBRCGA) for sizing and layout optimization of planar and spatial truss. The standard deviation of design variables has been used as a key factor in the adaptation of mutation operators. The reliability of the algorithm has been investigated in sizing and layout optimization, with both discrete and continuous design variables.

Pavel et. al. (2014) have presented a method for the simultaneous topology and size optimization of 2D and 3D trusses using evolutionary structural optimization with regard to commonly used topologies.

2.11 NUMERICAL INSTABILITIES

On the route to the optimum solutions, three types of numerical instabilities have been identified (Sigmund,1998 ,Fuji and Kikuchi 2000 ,Staffen Johnson ,2013). Fujii and Kikuchi have suggested a Sequential Linear Programming technique to avoid the occurrences of these instabilities.

2.11.1 The Checkerboard Problem :

If the material removal is tried based on the efficiencies, the procedure is likely to form alternating solid and void elements or create discontinuity in the continuum (Sigmund, 1998). Hence the full removal of the ineffective material through the creation of voids is to be avoided. The material density could be changed or thickness may be trimmed to a minimum suggesting the presence of the material

2.11.2 Mesh Dependent Solution:

The solution may differ based on the type and size of the elements. The same problem attempted with a different type and size of element may give a different solution. The initial discretization of the continuum plays a huge role as a starting point for optimization. The experience of the analyser should help selecting proper shape and size of the elements. Finer elements should be deployed at locations of expected steep gradient of stresses.

2.11.3 Local Minima:

The solution obtained may only a localised minimum. There can be a better or more optimum solution elsewhere. The methodology adopted should not lead to undulating solutions in convergence to a minimum.

2.12 SCATTERED DATA INTERPOLATION (SDI), METHOD OF MOVING LEAST SQUARES (MLS) AND STRESS SMOOTHING

In the present work, the analysis of 2D continuum structures has been carried out using Finite Element Method. Recognizing the limitations of FEM, like discontinuity of stress values along the element edges, a suitable smoothing technique has been formulated. A review has been made for the smoothing and interpolation methods applicable to 2D elastic problems.

Scattered Data Interpolation and Method of moving least squares (MLS) approximation was devised by mathematicians in data fitting and surface construction. Since then, this has been the foundation in the field of multi-dimensional scattered data interpolation.

The Moving Least Squares (MLS) approximation was devised by mathematicians in data fitting and surface construction (Lancaster and Salkausdas 1981). It can be categorized as a method of series representation of functions. The MLS approximation is now widely used for constructing shape functions.

Beckers ,et.al (1994) presented a review on error estimation and adaptivity of stress computation methods for finite element displacement models. Based on optimal stress extrapolation points , an original stress smoothing method called "averaging + extrapolation" was devised valid for general two dimensional meshes composed of iso-parametric elements of degree up to three.

Liyanapathirana ,et.al (2000) applied stress smoothing method to a finite element pile driving analysis considering the nodes of the finite element mesh as the most important locations for output stresses. The method shown that accurate nodal stresses can be obtained by approximating the stress distribution inside four-element patches by a polynomial with order equal to the order of the shape functions.

The analytical research in the field of application of MLS in structural analysis and development grew leap and bounds in the 21^{st} century, especially in solving problems in structural mechanics using meshless methods, to develop methods beyond the traditional FEM (Liu and Gu, 2003). The use of polynomials as basis functions, selection of polynomial terms using Pascal's triangle, solution of difficult simultaneous equations with singular

A method called Approximation Based on Smoothing (ABOS) has been devised by Miroslav Dressler (2009) for smoothing and scattered data interpolation. Depending upon the configuration of data points, the unwanted oscillations in the interpolations have been explained.

Silveira (2010) devised a smoothing technique for discontinuous linear and quadratic boundary elements (BEM) based on least square (LS) fit and obtained continuous solution where the traditional formulations showed discontinuities.

As an extension of meshless procedures, smoothed point interpolation S-PIM (Liu and Zhang, 2013) has been considered an alternative to the FEM in structural mechanics.

The MLS method is much more flexible than traditional LS based methods and is a promising method for measurement data processing (Huaiqing Zhang, 2015). The applicability and flexibility of the MLS procedure is evident from the amount of research work in various fields of engineering with variations in this approach.

Chapter 3

FINITE ELEMENT ANALYSIS OF 2D PLATES SUBJECTED TO IN-PLANE BENDING

3.1 GENERAL

Under the action of external forces, structures made of solids deform. Depending on the material property, this deformation can disappear and initial form can be recovered when the external forces are withdrawn. Elasticity is this property of a structural material to return to its initial un-deformed form completely after removal of external forces acting on it. In this deformation, the rate of displacement at a point in the solid is called strain. Internal forces are introduced in the solid, to balance the external forces. Magnitude of these forces is defined by their intensities. This intensity is called stress.

Determination of stresses in the material of construction is a very important step the analysis and design of any structure. With the advent of digital computers, structures of complex shapes and configurations have been attempted with the conceptualisation of approaches like Finite Element Method. In this chapter, the basic steps involved in the Finite Element Analysis of plate like structures the development of a generalised code for FEA are discussed.

3.2 PLANE ELASTICITY

Elasticity theory establishes mathematical model to determine the displacement, strain and stress distribution in elastic solids under the action of external forces. In many cases, materials display behaviour such that the stress and strain vary proportionally up to a limit. This behaviour is called linearly elastic behaviour and the limit is called proportionality limit.

Materials can be anisotropic or isotropic depending on the variation of material property with direction. Material displaying direction-dependent properties is called anisotropic. If the material properties are identical in all directions at a point, then the material is called isotropic.

It may not be an efficient way to solve all real problems using governing elasticity field equations developed for three-dimensional problems. Simplified formulations have been developed taking the advantages related to geometry, loading and boundary conditions.

Plane elasticity is a special case where a three dimensional problem is simplified to one involving two dimensions only. There are two basic cases of plane elasticity. One is the plane stress and the other is the plane strain.

3.2.1 Plane Stress Condition

In structural mechanics, a flat thin sheet of material is called a plate. The distance between the plate faces is the thickness, denoted by h. The midplane lies halfway between the two faces. The direction normal to the mid-plane is the transverse direction. Directions parallel to the mid-plane are called in-plane directions. The global axis z is oriented along the transverse direction. Axes x and y are placed in the mid-plane, forming a right-handed rectangular Cartesian coordinate system. Thus the equation of the mid-plane is z = 0. The +z axis conventionally defines the top surface of the plate as the one that it intersects, whereas the opposite surface is called the bottom surface.

If a plate whose thickness in z direction is very small in comparison to the dimensions in other directions is loaded by forces acting in the plane of the plate and uniformly distributed over the thickness , the stresses in z direction are all zero. The state of stress is then specified by σ_x, σ_y, τ_{xy} only and called the plane stress. These components are functions of **x** and **y** only.

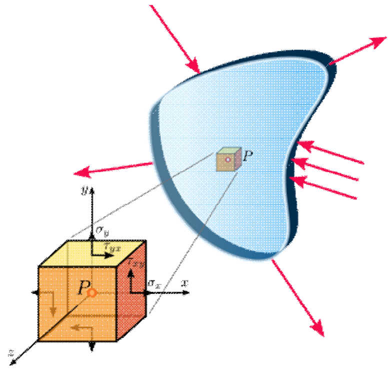


Fig 3.1 Plane stress case

3.2.2 Plane Strain Condition

A similar simplification can be obtained for the state of strain also. Consider an infinitely long prismatic body subjected to the load laterally. Assuming the load to be function of x and y only, all the sections experience the same deformation and therefore the strain components in the z direction are all zero. This deformation state is referred to as plain strain.

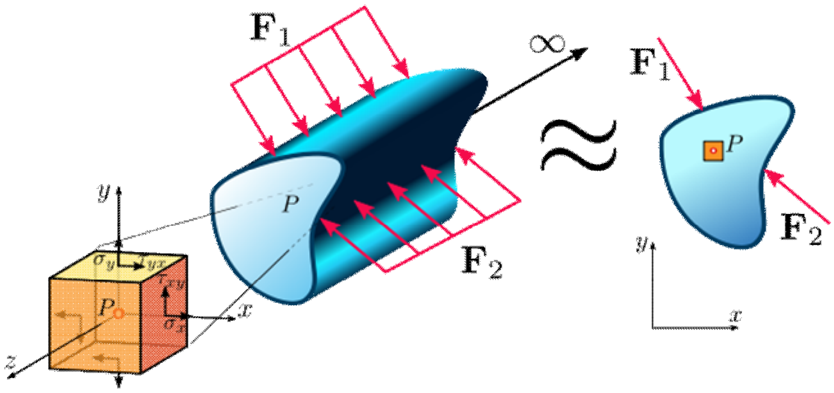


Fig 3.2 Idealization of Plain Strain Condition

The case of plane strain is not discussed in detail as the work carried out is related to plates where the thickness is assumed small in comparison with the dimensions in the other two directions.

3.3 PLATES SUBJECTED TO IN-PLANE STRESSES

Plates have large surface area, compared to the thickness. They are hence, considered as 2Dimensional structures. If they are subjected to in-plane forces, the condition leads to a Plane stress condition and the analysis can be done as plates undergoing in-plane bending.

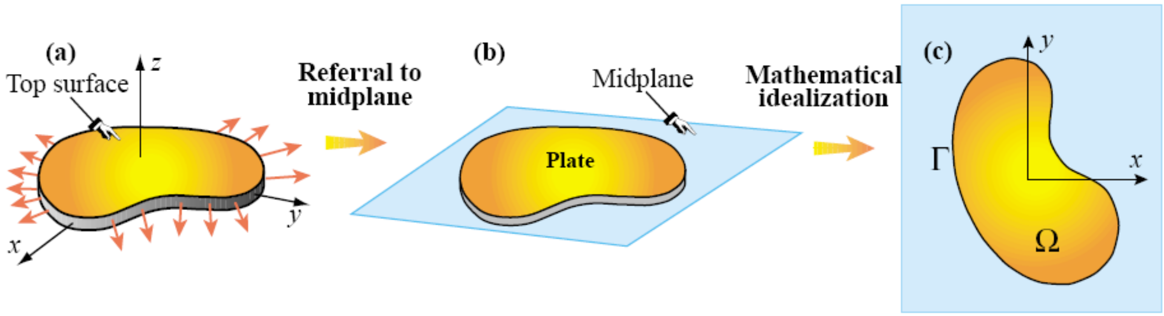


Fig 3.3 Idealization of Plane Stress Case

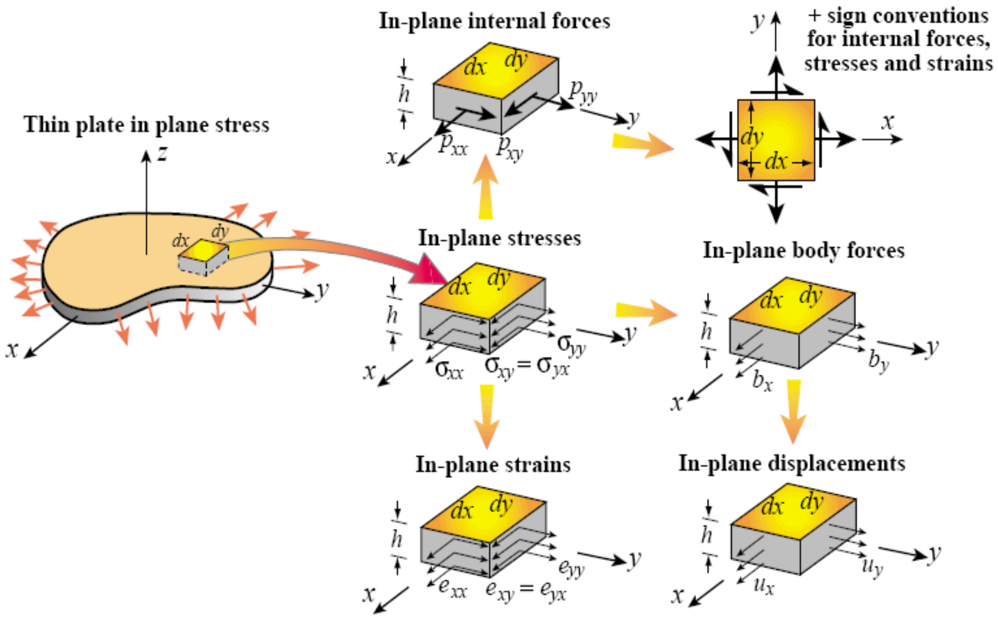


Fig 3.4 Displacements, Strains and Stresses in Plane Stress Condition

3.4 BEHAVIOURAL ASSUMPTIONS

The proximity of predicted behaviour of a structure to the actual one depends on the accuracy and genuineness of assumptions made at every stage of modelling and analysis. Following assumptions are made throughout this work.

1. The material is homogeneous, isotropic and linearly elastic.
2. All the loads applied on the plate are in its mid-plane. The normal and shear stress components in the third direction are zero or negligible
3. All support conditions are symmetric about the mid-plane.
4. In-plane displacements, strains and stresses can be taken to be uniform through the thickness

3.5 PROCEDURE INVOLVED IN FEA OF PLATES SUBJECTED TO IN-PLANE BENDING

FEA is a method of piecewise approximation by connecting the deformations to the stresses by simple functions, each valid for a small region through the process of discretization. One of the important facts of FEA lies in the fact that the accuracy of results depends on the size of elements that are created during discretization. Hence, FEA is used to obtain approximate solutions for values at discrete locations in a continuum with complex geometry, material properties, loading and boundary conditions. Common to all problems attempted using FEA, there are distinct steps involved from physical model to the solution.

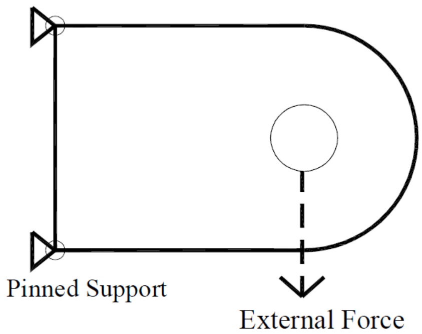


Fig 3.5 A Bracket

3.5.1 Discretization

The continuum is segmented into pieces called elements, the size of which depends on the accuracy of results sought. At locations where more accuracy is sought, the size of the element needs to be smaller compared to those at other locations. For example, the bracket shown in Fig. 3.5 is divided into smaller segments as shown in Fig.3.6. The shape of the element may be chosen as triangular, rectangular or polygonal for which the shape functions connecting the deformations anywhere in the region to those at known locations are derived. Detailed derivation of formulae and expressions related to triangular elements used for the FEA of Plates in plane stress , which are subsequently used in the development of code, are explained in **Appendix – A**. However, some terms, expressions and equations are mentioned here to highlight the developmental aspects of the source code.

Though the size of elements may vary in a given problem, the shape of the element remains the same throughout the structure being analysed.

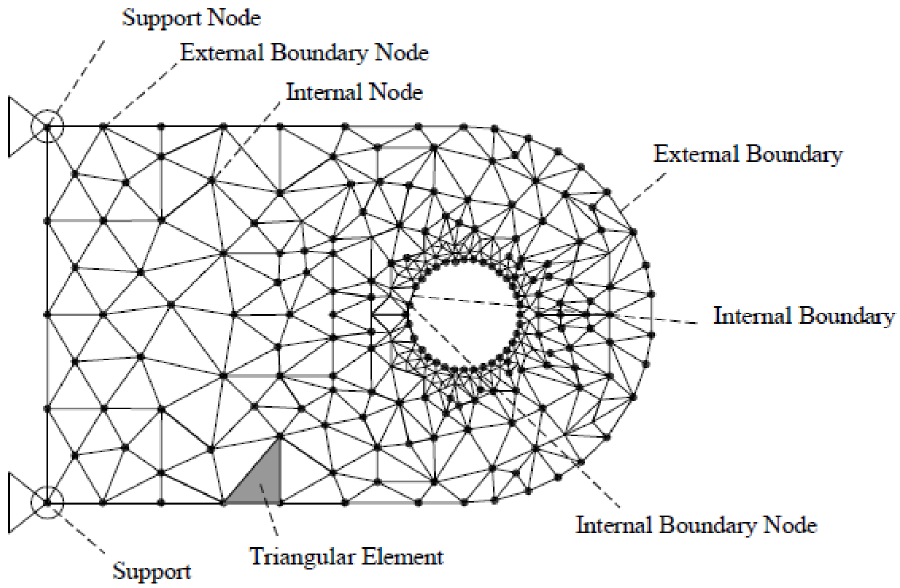


Fig 3.6 Discretization of the Bracket

3.5.2 Element Stiffness Matrices

These are derived for all elements individually considering the equilibrium of individual elements. They connect the stresses developed inside the element to the nodal deformations.

3.5.3 Global Stiffness Matrix.

Since the elements are connected at common nodes, considering the compatibility of connected elements at the node, a Global Stiffness Matrix is assembled which is used to relate the displacement at all nodes to the forces acting on the structure, in the form of simultaneous equations. The elements in the global stiffness matrix are modified to accommodate the boundary conditions.

3.5.4 Solution of Simultaneous Equations

The simultaneous equations are solved using any of the standard methods to obtain nodal displacements. The symmetric nature of the global stiffness matrix may be exploited for easy and quick solution

3.5.5 Recovery of Stresses in the Elements

Based on the requirement, the stresses in the elements at some locations are found out from the displacement values at the nodal locations.

3.6 TERMINOLOGY

The following summarizes the terms used while dealing with the plane stress problem.

External Boundary: This is the limit of attention within which the analysis is done, where the material is present. This is always well identified while attempting the analysis. The external boundary is defined as a closed polygon with the co-ordinates of apexes, as the equation of a closed region or as combinations of lines and arcs joined at the ends.

Internal Boundary: There could be voids, single or multiple, like punched holes, within the continuum where the material is not present. They are well defined as in the case of external boundary.

Thickness**:** Most plates used as structural components have constant thickness. If the thickness does vary, it should do so gradually to maintain the plane stress state. Sudden changes in thickness may lead to stress concentrations.

Material data**:** This is defined by the properties like elastic properties like Young’s Modulus, Poisson’s Ratio, yield stress and permissible stresses. The values of these properties may be different based on the nature of stress developed like tension, compression, bending or torsion. Here, it is assumed that the plate material is linearly elastic and the properties are same in all directions.

Specified Interior Forces**:** These are known forces that act in the interior of the plate. There are of two types. Body forces or volume forces are forces specified per unit of plate volume.. Face forces act tangentially to the plate faces and are transported to the mid-plane. For example, the friction or drag force on an airplane skin is of this type if the skin is modelled to be in plane stress.

***Specified Surface Forces*:** These are known forces that act on the boundary of the plate. In elasticity, they are called surface tractions. In actual applications it is important to know whether these forces are specified per unit of surface area or per unit length. The former may be converted to the latter by multiplying through the appropriate thickness value.

Displacement Boundary Conditions: These specify how the plate is supported. Points subject to support conditions may be fixed, allowed to move in one direction, or subject to multipoint constraints. If no displacement boundary conditions are imposed, the plate is said to be free or floating and will not be in static equilibrium.

Finite Elements: In Finite Element Analysis (FEA) of plates, the continuum is assumed to be made of so many pieces of smaller triangular, rectangular or polygonal units called elements. These elements are connected to one another along the edges such that no portion of the continuum is left-out in the analysis. In FEA, the shape of all the elements in an analysis remains the same. Each element is assumed to be of uniform thickness, while the thicknesses of all the elements need not be the same. However, all the elements are assumed to be connected through the mid-plane. Here, triangular (the basic shape in 2D FEA) elements, are used to divide the plate into finer parts.

Constant Strain Triangle (CST): Generally, an element is analysed assuming the strain in a direction remains same throughout its area, though the strain in another direction could be different. Such an assumption simplifies the procedure and is well justified when the triangular elements are finer.

Nodes*:* Similar to the edges of elements that are connected, the corners of elements are connected with those of other elements. These meeting points are called nodes.

***External Boundary Nodes*:** Those nodes along the external boundary are called external Nodes.

Internal Boundary Nodes: Those nodes falling along the internal boundaries are called internal boundary nodes.

Internal Nodes: All the other nodes which are inside the continuum are called internal nodes. At the internal nodes, material will be present all around.

Support Nodes**:** These are nodes at which the plate is supported or at which the displacement restrictions called boundary conditions are specified.

Displacements**:** These are in-plane deformations at nodes to be found out , due to the action of forces. They could be translations or rotation. In this analysis, the translational deformations are considered while the rotational movements at the nodes are neglected. The in-plane displacement field is defined by two components: **u (x,y)** and **v (x,y)** both in plane in x & y directions respectively

Degrees of Freedom**:** The basis of analysis depends of the number of movements expected per node. Here, each node is expected to be of 2 degrees of freedom, the translations in the direction of two global principal axes.

3.7 DEVELOPMENT OF GENERAL PURPOSE FEA CODE

A code has been developed for the FEA of plates in plane stress conditions , using triangular elements. Important steps used in the development of source code are stated here.

3.7.1 General Notations used

Following general notations are used in the derivations.

σ is the direct stress
τ is the shear stress
ε is the direct strain
γ is the shear strain
E is the Young's Modulus
υ is the Poisson's ratio
t is the thickness of an element
A is the area of an element
q is the local displacement vector
Q is the global displacement vector
ke is the element stiffness matrix
Kg is the global stiffness matrix
F is the global force vector

3.7.2 Sign Convensions Followed

Unless otherwise specifically mentioned, the following notations and sign conventions are followed through out the work . Stresses are considered as positive in the directions shown in Fig.3.7.

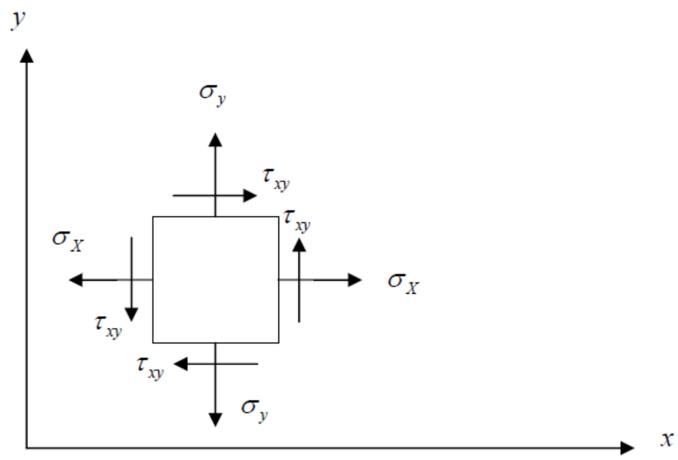


Fig 3.7 Notations and Sign Conventions

3.7.3 Relationship between Element Stress and Element Strain

The direct and shear stresses in an element are related to the constant strain through Poisson's ratio using the following relation.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad \text{----- (3.1)}$$

$$\sigma = D\varepsilon \qquad \text{----- (3.2)}$$

Where

$$D = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad \text{----- (3.3)}$$

3.7.4 Element Displacement Matrix.

For a triangular element, it is assumed that there are 6 independent displacements possible, with two translations at every corner.

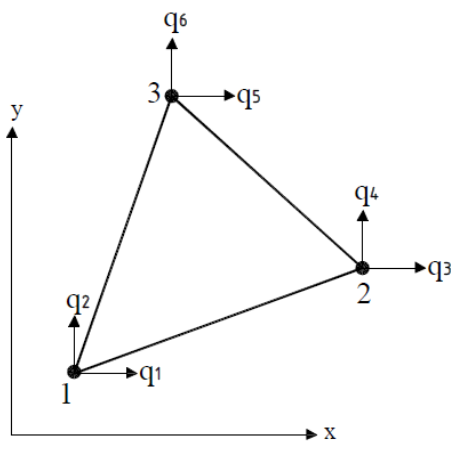


Fig 3.8 Degrees of Freedom for a Triangular Element

Thus, the element displacement vector q is defined as

$$q = \{q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6\}^T \qquad \text{----- (3.4)}$$

In a structure with many elements,the corner displacement values are common to all the elements connected at the node. Hence, it is necessary to develop generalised notations for the displacements for an element with the corners **i , j** and **k**

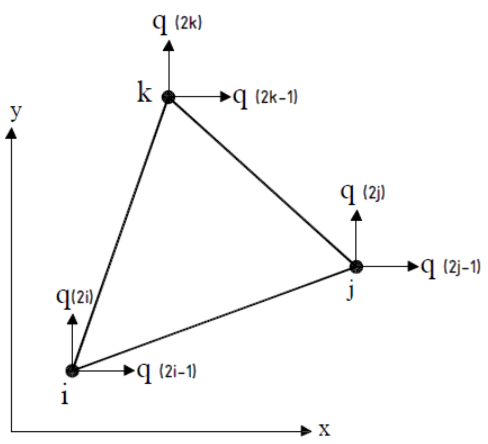


Fig 3.9 Displacements at Nodes of Generalised Element

3.7.5 Area of Triangular Element.

In the sequence of development of stiffness matrix for an element, the shape function mainly depends on the co-ordinates of the corners.

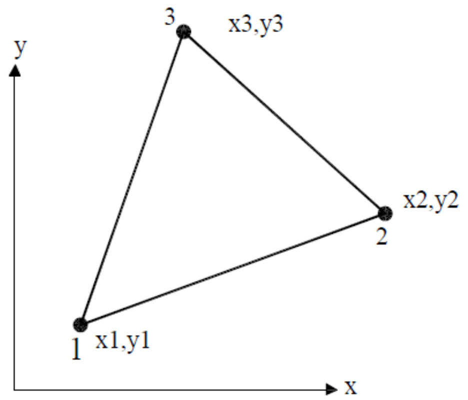


Fig 3.10 Nodal Co-ordinates of an Element

Area of the traingular element is given by

A_e= 0.5 x [(x2y3-x3y2)+(x3y1-x1y3)+x1y2-x2y1)] ----- (3.5)

3.7.6 Relationship between Element Strain and Corner displacement

The strain vector for an element is connected to the local displacement vector using the formula

$$\varepsilon = Bq$$ ----- (3.6)

$$B = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$ ----- (3.7)

Where, as a generalised rule,

$$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$$ ----- (3.8)

and

$$\mathbf{y}_{ij} = \mathbf{y}_i - \mathbf{y}_j$$ ----- (3.9)

3.7.7 Element Stiffness Matrix

The Element stiffness Matrix $\mathbf{k_e}$ of the order 6 X 6 is assembled as

$$k_e = t_e A_e B^T D B \qquad \text{----- (3.10)}$$

Where $\mathbf{t_e}$ is the thickness and $\mathbf{A_e}$ is the area of the element respectively.

3.7.8 Global Stiffness Matrix

The members of the element stiffness matrix are added into the global stiffness matrix **K** as decided by the node numbers of the triangular element.

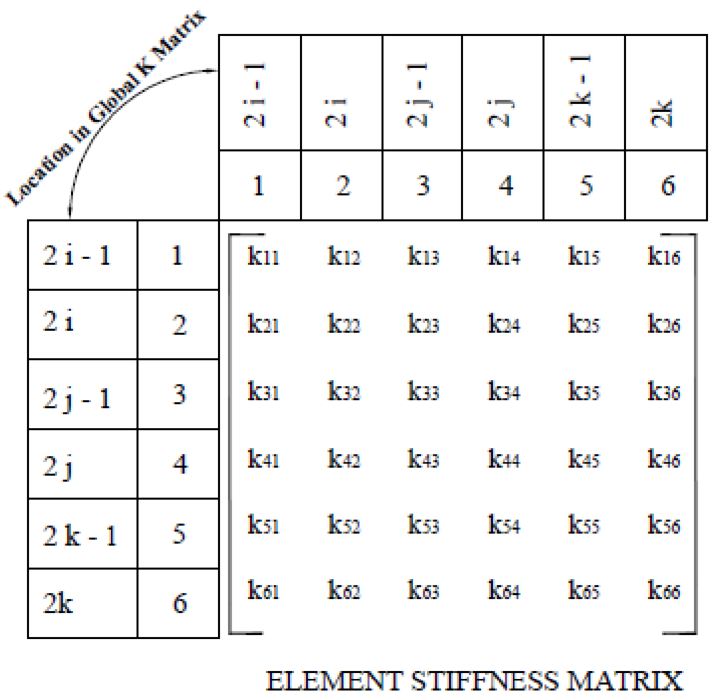


Fig 3.11 Assembly of Global Stiffness Matrix

3.7.9 Solution of Global Euilibrium Equation

The equilibrium equation for the entire continuum is formed as

$$KQ = F \qquad \text{----- (3.11)}$$

Where **K** is the global stiffness matrix, **Q** is the global displacement vextor and **F** is the global force vector.

The boundary conditions consisting of displacement restrictions are incorporated at this stage. There are many ways of incorporating these conditions. A popular method is to force the corresponding member in the global displacement vector to negligible quantity by multiplying corresponding diagonal member in the stiffness matrix by a very high value before the solution of simultaneous equations is attempted.

The next step is to solve the system of simultaneous equations by any popular method. The symmetric and sparce properties of the global stiffness matrix may be exploited to store it in banded format saving the storage space and the time required for solution. Solving equation (3.11) , we get the global displacement vector Q.

3.7.10 Element Stress Recovery

Extracting the displacement at the three nodes, the stresses in the individual elements can be obtained as

$$\sigma = \mathbf{DBq} \qquad \text{----- (3.12)}$$

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \frac{1}{\det J} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix}$$

----- (3.13)

3.8 CALCULATION OF PRINCIPAL STRESSES AND VON -MISE' S STRESSES

The FEA Code developed has been extended to calculate the principal stresses and von Mise's stresses in the elements.

The Principal Stresses are given by

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}} \qquad \text{----- (3.14)}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} \qquad \text{----- (3.15)}$$

The von Mise's Stress is calculated from the Principal Stresses as

$$\sigma_V = \sqrt{\sigma_1^{\ 2} - \sigma_1 \sigma_2 + \sigma_2^{\ 2}} \qquad \text{----- (3.16)}$$

3.9 FLOWCHART FOR A GENERALISED CODE FOR FEA OF PLATES

Fig 3.12 shows the flowchart used for development of the generalised code for the FEA of plates subjected to in-plane bending.

The input data consists of the description of geometry of the plate as a set of triangular elements connected at the nodes, material properties like Young's Modulus, Poisson's Ratio, yield stress. The magnitude of loads, positions and direction of their application, location of supports and movements allowed, initial thickness of elements and the permitted limits to the movement of individual nodes are specified.

The output data consists of the displacements at the nodes, the direct and shear stresses, Principal stresses, von Mise's stress developed in the elements, the utility ratio of material and the reactions at the support.

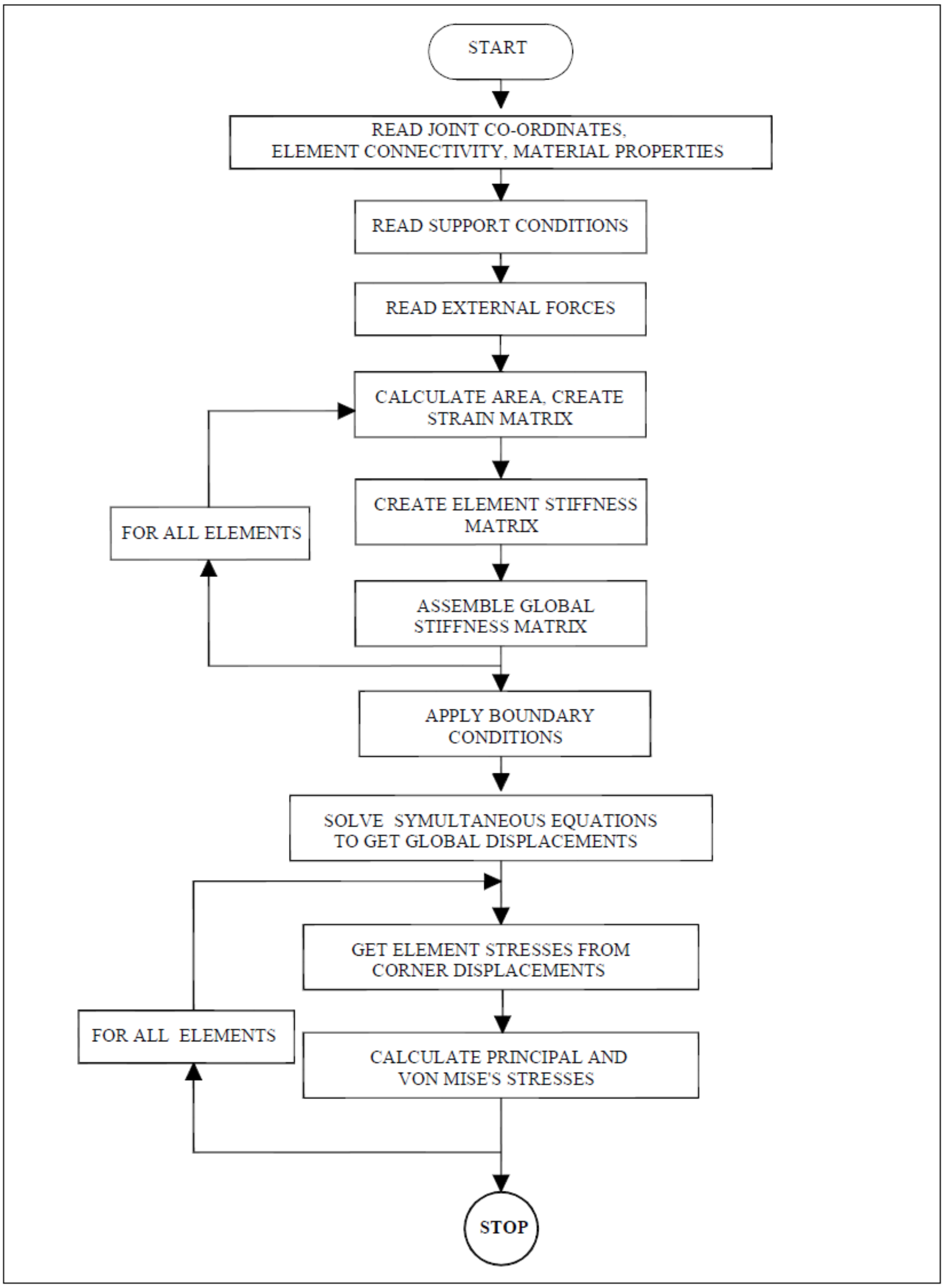
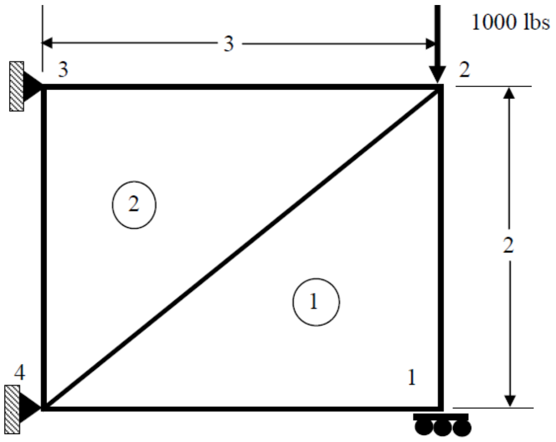


Fig 3.12 Flowchart for the FEA Code Development

3.10 VERIFICATION OF THE FEA PROGRAM DEVELOPED

A problem solved in the FEM Course material by Prof. Green Lee (2010) , University of New Mexico is solved to verify the source code developed for FEA of plates.

3.10.1 Illustrative Benchmark Problem :



$$E = 30 \times 10^6 \, psi$$
$$v = 0.25$$

Fig 3.13 The Benchmark Problem

To find the stresses in the plate **3 in X 2 in (75 mm X 50 mm)** subjected to a load of **1000 lbs (4448.22 N)** as given in the Fig. 3.13. The Values of Young's Modulus and Poisson's ratio have been taken as **30 x 10 ^{6} Psi (6.895 x 10 ^{6} Mpa)** and 0.25 respectively.

3.10.2 Solution Using FEA code Developed

NUMBER OF JOINTS = 4 NUMBER OF PLATES = 2

NUMBER OF MATERIALS = 1 NUMBER OF SUPPORT JOINTS = 3

NUMBER OF LOADED JOINTS = 1

MATERIAL	E ,	Poisson,	permissible stress,	uwt
1	3E+07	0.25	120	1000

JOINT AND CO-ORDINATES

1	3	0
2	3	2
3	0	2
4	0	0

MEMBER NO., node1 , node2 , node 3

1	1	2	4
2	2	3	4

Plate, Thk, Material

1	0.5	1
2	0.5	1

SUPPORTS, XRELEASE,YRELEASE

1	0	1
3	1	1
4	1	1

LOADED JOINT, XLOAD,YLOAD

2	0	-1000

Joint to ELEMENT Connection Details

Joint No.	No. Connected	ELEMENTS
1	1	1
2	2	1 2
3	1	2
4	2	1 2

Joint No.	Nos Connected	Joints
1	2	2 4
2	3	1 4 3
3	2	2 4
4	3	1 2 3

STRESS (psi) in ELEMENTS ------RESULTS.

Plate No.	N 1	N 2	N 3	AREA	Thk	Vol	Sigx	Sigy	Touxy
1	1	2	4	3.000	.500	1.500	-0093.12	-1135.59	-0062.08
2	2	3	4	3.000	.500	1.500	0093.12	0023.28	-0296.62

REACTIONS (lbs) AT THE SUPPORTS

NODE	Rx	Ry
1	0.0000	820.6510
3	-269.0235	165.7685
4	269.0235	13.5805

3.10.3 Stresses and Principal Stresses (psi) in Plates – Output

Plate No.	Sigx	Sigy	Txy	Sig1	Sig2	Tmax
1	-0093.12	-1135.59	-0062.08	-0084.87	-1143.84	0529.49
2	0093.12	0023.28	-0296.62	0157.66	-0041.25	0099.46

3.10.4 Joint Deformations (in)

NODE	DELTA X	DELTA Y
1	0.000019077	0.
2	0.00000873	-0.0000742
3	0.	0.
4	0.	0.

3.10.5 Solution Given In the Reference

$$q_1 = 1.908 \times 10^{-5}$$
$$q_3 = 8.73 \times 10^{-6}$$
$$q_4 = -7.415 \times 10^{-5}$$

STRESSES IN ELEMENTS psi

Element	σ_x	σ_y	τ_{xy}
1	-93	-1136	-62
2	93	23	-297

3.10.6 Inference

The output given by the FEA code developed for the analysis of plates is in agreement with the solution in the reference, verifying the correctness of the code developed.

3.11 APPLICATION OF THE FEA CODE

The FEA Code developed is a generalised one and can be used for the solution for the plates subjected to in-plane bending. This code has been used to determine stresses in plate material during the formulation of methodology related to optimization of plates, further discussed in Chapter 6.

Chapter 4

STRESS SMOOTHING

4.1 GENERAL

There are situations when a region of study is split into segments, for better accuracy of results. But when the results are coined together for all these segments to project the same for the whole region, there is a likelihood of discontinuities at the junctions in an expected continuous behaviour. Smoothing means removing sharp irregularities at boundaries of segments making them follow a regular pattern.

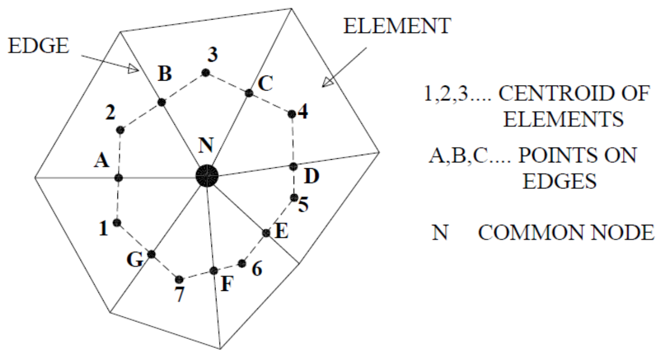


Fig. 4.1 Elements, Nodes and Edges in FEA

In FEA, the given region is split into a number of smaller segments called elements as shown in Fig 4.1. The joining point of several elements is called a 'node'. In a 2D continuum, with triangular or rectangular elements, the boundaries of the elements meet along lines called 'edges'. The main aim of FEA is to formulate a system of equilibrium equations to solve for the displacements at the nodes. The accuracy of displacement values is highly dependent on the size of the element. As the stresses in the elements are obtained from these nodal displacements, stress becomes derived quantity. The stress thus obtained is assumed to be uniform throughout the entire element. This gives rise to disparities along the edges of elements where the value of stress computed from two sides of the edge differ.

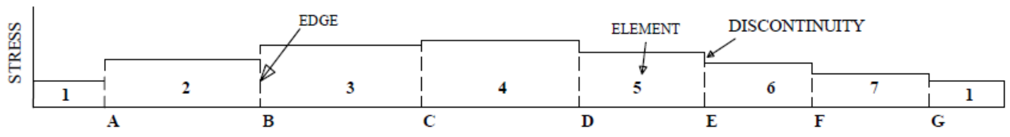


Fig. 4.2 Discontinuity of Stress values at the Boundaries of Elements

This leads to the stress being not single -valued across element interfaces as shown in Fig 4.2 , though the fact being the finer the elements- the less the disparity.

4.2 SMOOTHING

Removing such stress irregularities and ensuring a smooth variation along the element interfaces, at the same time honouring the stresses in the elements, is known as stress smoothing, depicted by a smooth curve shown in Fig. 4.3

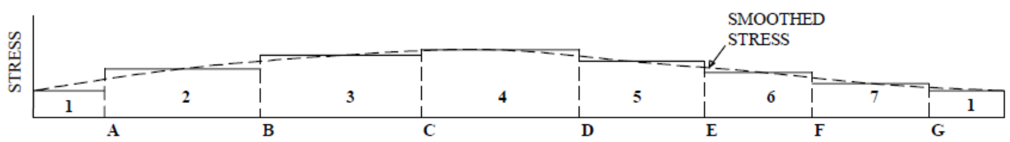


Fig. 4.3 Stress Smoothing

Similarly at nodes, where the corners of elements meet, the FEA stress values differ when computed from different directions.

In many problems, like in node based structural optimization, the investigator is interested in knowing the stress levels at a node, based on the stress values in elements connected to it. Hence smoothing is applied, usually, as a post processing technique at the end of FEA procedure to determine the value of field variable at a point of interest, in relation to those at a number of points in its neighbourhood.

4.3 CURVE AND SURFACE FITTING

In the mathematical field of numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points in the vicinity. Fitting a function satisfying all the data points can predict the variables within the range assuming a smooth variation.

In 1D problems, with a given set of values y1,y2, ...yn are known at locations x1,x2..., xn respectively, arriving at a relationship of the form

$$\mathbf{y = f(x)} \qquad \text{-----(4.1)}$$

is called curve fitting.

In 2D, finding a relationship of the form

$$\mathbf{z = f(x,y)} \qquad \text{-----(4.2)}$$

that can be used to used to obtain value of field variable z at any point, using a set of known values of **z** at known points (x1,y1), (x2,y2)....(xn,yn) in the vicinity, is known as surface fitting.

In both 1D and 2D, these functions may be polynomial, exponential, logarithmic or trigonometric.

4.4 MOVING LEAST SQUARES METHOD

Various methods are available to find or to predict the value of field variable at a point of interest in a domain from the obtained values in a sub-domain near the point. Collocation Method, Least square Method are some popular methods used for interpolation. Moving Least Square (MLS) Method is a modified version of Least Square (LS) Method in which continuous functions are reconstructed from a set of unorganized point samples via the calculation of a weighted least squares measure biased towards the region around the point at which the reconstructed value is requested.

In MLS, a field variable like deflection, can be approximated by

$$\mathbf{f\,(x)} = \sum_{i=1}^{m} p_i\,(\mathbf{x})\,a_i\,(\mathbf{x}) = \mathbf{p}^T\,(\mathbf{x})\,\mathbf{a}\,(\mathbf{x}) \qquad \text{-----(4.3)}$$

Where **f(x)** is the approximate value of the field variable at the location, $p_i(\mathbf{x}), i = 1, 2, \ldots, m,$ are called basis functions, m is the number of terms in basis functions, and $a_i(\mathbf{x})$ are the coefficients.

4.5 THE BASIS FUNCTIONS

$p_1(\mathbf{x}), p_2(\mathbf{x}), \ldots, p_m(\mathbf{x})$ are polynomials of variable x and use a vector consisting of monomials of the lowest orders to ensure minimum completeness.

$$\mathbf{p}^T(x) = \{ 1 \quad \mathbf{x} \quad \mathbf{x}^2 \quad \ldots\ldots \quad \mathbf{x}^m \} \qquad \text{-----(4.4)}$$

For example, the basis functions of one-dimensional polynomials have the following forms:

linear basis as

$$\mathbf{p}^T(x) = \{1, x\}, m = 2 \qquad \text{-----(4.5)}$$

and quadratic basis as

$$\mathbf{p}^T(x) = \{1, x, x^2\}, m = 3 \qquad \text{-----(4.6)}$$

In Moving Least Square Method, The coefficients given by

$$\{\mathbf{a}\} \quad \begin{Bmatrix} a_1 \\ \vdots \\ a_m \end{Bmatrix} \qquad \text{-----(4.7)}$$

vary with **x,** the location of the point of interest in the sub-domain. Assembling the basis function values for all the points in the vicinity , we get

$$\mathbf{P} = \begin{bmatrix} p_1(x_1) & p_2(x_1) & p_3(x_1) & \ldots\ldots. & p_m(x_1) \\ p_1(x_2) & p_2(x_2) & p_3(x_2) & \ldots\ldots. & p_m(x_2) \\ \ldots. & \ldots. & \ldots. & \ldots. & \ldots. \\ p_1(x_n) & p_2(x_n) & p_3(x_n) & \ldots\ldots. & p_m(x_n) \end{bmatrix} \qquad \text{-----(4.8)}$$

Here, **P** is known as the moment matrix.

The basis functions can be simple polynomials, Legrange polynomials, Trigonometric Functions, Radial Basis Functions, and so forth(Liu,2003).

The essence of MLS is the concept of a moving window (influence domain) inside the full domain, but concentrating only on the data visible through the window. The method uses a polynomial of certain degree, the value of which will be equal or very near to those at the data points. The main difference between the MLS and LS is in the nature of coefficients in the polynomials. The coefficients remain same for the entire domain in LS , where as they get a different set of values , based on the movement of the window, in MLS.

4.6 ILLUSTRATION OF MLS IN 1D

The Moving Least Square (MLS) Method may be explained considering a set of scattered data in 1D shown in Table 4.1.

Table 4.1

Given DATA	
X	Y
1.00	650.00
2.00	900.00
3.00	1600.00
4.00	1800.00
5.00	3000.00
6.00	3200.00
7.00	3800.00
8.00	4500.00
9.00	5800.00
10.00	6100.00

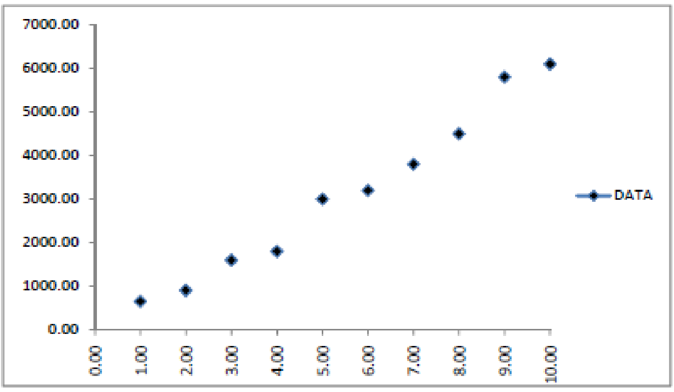


Fig. 4.4 Scattered Data Points for Illustration

The points in an x-y plot are shown in Fig. 4.4. We can fit a Least Square (LS) function in the form

$$y = a_0 + a_1 x + a_2 x^2 \qquad \text{-----(4.9)}$$

as

$$y = 172.3 + 374.2 x + 23.48 x^2 \qquad \text{-----(4.10)}$$

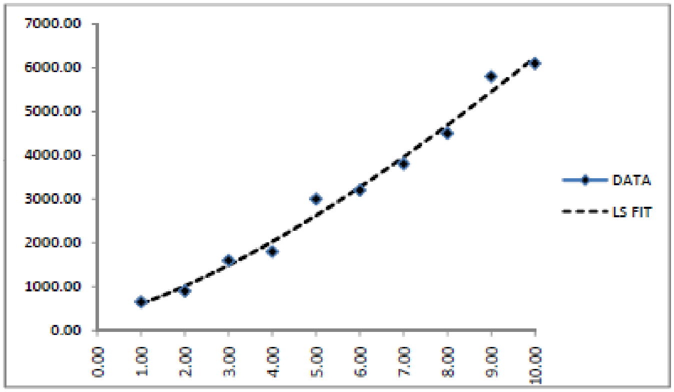


Fig.4.5 Curve Fitting Using Method of Least Squares

A plot for this trend line is shown in Fig 4.5. It may be observed that the fitted line does not pass perfectly through any of the data points though it is a best fit line with an overall minimum deviation.

If the whole range is split into 3 regions, $1 <= \mathbf{x} < 4$, $4 <= \mathbf{x} < 8$, and $8 <= \mathbf{x} <= 10$ trying to make a LS fit for each region, we can fit functions for individual regions as given in Table 4.2 and plotted as shown in Fig 4.6

Table 4.2 MLS Fit Functions

No.	Range of x	Function	
1	$1 <= \mathbf{x} < 4$	$y = 137.5 + 477.5\ \mathbf{x} - 12.5\ \mathbf{x}^2$	-----(4.11)
2	$4 <= \mathbf{x} < 8$	$y = - 1917.14 + 1134.29\ x - 42.86\ x^2$	-----(4.12)
3	$8 <= \mathbf{x} <= 10$	$y = - 41900 + 9800\ \mathbf{x} - 500\ \mathbf{x}^2$	-----(4.13)

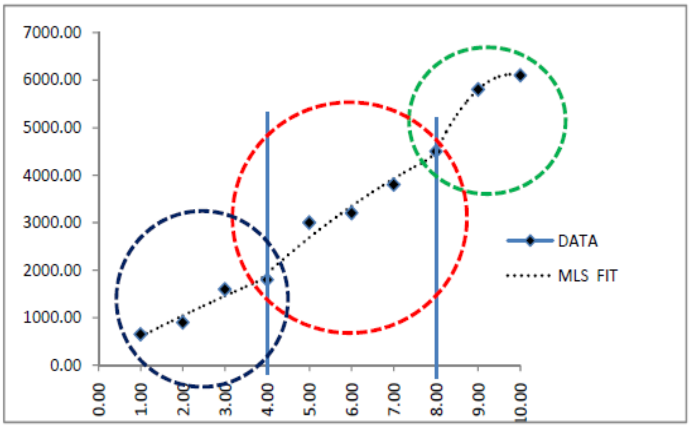


Fig. 4.6 Fitting Using Method of Moving Least Squares

Here too, it may be observed that the lines do not pass exactly through the data points, though the individual fitting lines show less deviations.

4.7 CONCEPT OF MOVING POLYNOMIAL MLS FOR FITTING DATA

The accuracy of fitting increases at the data points if higher degree polynomials are used for fitting. But the number of terms in the polynomial can be selected to the maximum of number of data points available in the vicinity. Thus selecting polynomials of 4 terms, 5terms and 3 terms for the regions 1,2 and 3 respectively, we get new fitting functions for the regions. These fitting functions are shown in Table 4.3,depicting , as we move from one region to the next, the number of terms in the polynomial may change. Hence this is called as Moving Polynomial MLS (MPMLS).

Table 4.3. Moving Polynomial MLS Fit Functions

Region No	Range of x	Function	
1	$1 <= \mathbf{x} < 4$	$y = 1800 - 2166.667\ \mathbf{x} + 1175\ \mathbf{x}^2 - 158.333\ \mathbf{x}^3$ **(4 Terms)**	-----(4.14)
2	$4 <= \mathbf{x} < 8$	$y = -100500 + 68158.33\ \mathbf{x} - 16679.16\ \mathbf{x}^2 + 1791.66\ \mathbf{x}^3 - 70.83\ \mathbf{x}^4$ **(5 Terms)**	-----(4.15)
3	$8 <= \mathbf{x} <= 10$	$y = -41900 + 9800\ \mathbf{x} - 500\ \mathbf{x}^2$ **(3 Terms)**	-----(4.16)

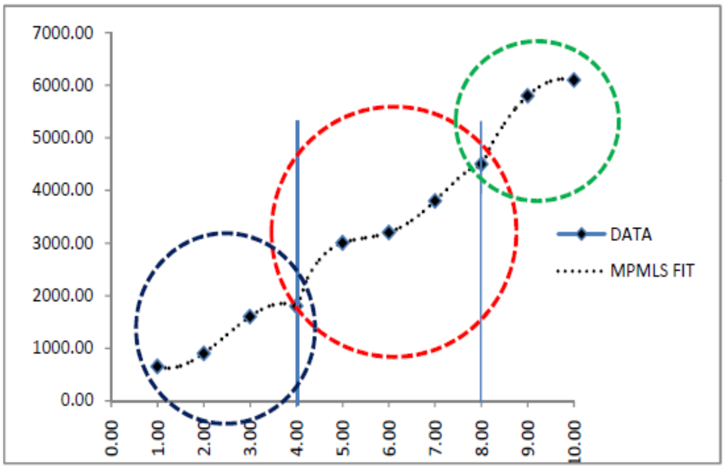


Fig. 4.7 Fitting Using Method of Moving Polynomial Moving Least Squares

It can now be seen that individual fitting functions almost exactly pass through the data points in the region of attention.

MPMLS gains its importance in situations where the fitting curve has to necessarily pass through the data points and at the same time be continuous in a given region while attention is focussed in the region.

4.8 MLS METHOD IN 2D PROBLEMS

4.8.1 Basis Functions

In 2D problems, The basis function **p**(**x**) is often built using monomials from the Pascal triangle given in Fig 4.8, to ensure minimum completeness . In the polynomial selected, equal participation is required from the **x** and **y** terms in their degrees to ensure a un-biased representation.

$$\mathbf{p}^T(\mathbf{x}) = \mathbf{p}^T(x, y) = \{1, x, y, xy, x^2, y^2, \ldots, x^n, y^n\} \quad \text{-----(4.17)}$$

In some special problems, enhancement functions can, however, be added to the basis to improve the performance of the MLS approximation.

Usually, the type of polynomial selected remains the same through out the domain, though values of the coefficients keep changing on the move. In finite element method or meshfree methods with a minimum of 3 scattered points guaranteed in the neighbourhood, a polynomial of the form

$$\mathbf{p} = \mathbf{a1}+\mathbf{a2}\, x + \mathbf{a3}\, y \quad \text{----- (4.18)}$$

is selected, which remains the same style, though set of values of coefficients **a1**, **a2** and **a3** keep changing.

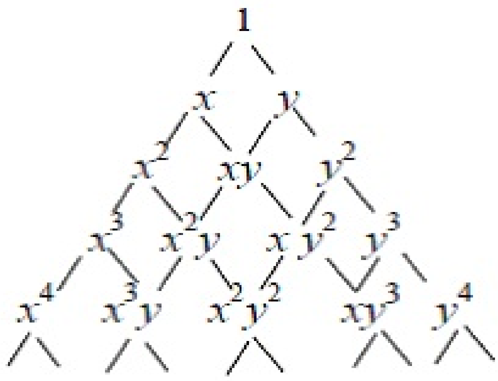


Fig. 4.8 Pascal Triangle used for the Formulation of Basis Function.

Assembling the basis polynomial function values for all the data points in the vicinity, the Moment matrix **P** is given by

$$\mathbf{P} = \begin{bmatrix} 1 & x_1 & y_1 & x_1y_1 & x_1^2 & y_1^2 & x_1^2y_1 & x_1y_1^2 & \ldots\ldots \\ 1 & x_2 & y_2 & x_2y_2 & x_2^2 & y_2^2 & x_2^2y_2 & x_2y_2^2 & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots \\ 1 & x_n & y_n & x_ny_n & x_n^2 & y_n^2 & x_n^2y_n & x_ny_n^2 & \ldots\ldots \end{bmatrix} \quad \text{-----(4.19)}$$

Or, generalising the terms as functions of x and y,

$$\mathbf{P} = \begin{bmatrix} p_1(x_1,y_1) & p_2(x_1,y_1) & p_3(x_1,y_1) & \ldots\ldots & p_m(x_1,y_1) \\ p_1(x_2,y_2) & p_2(x_2,y_2) & p_3(x_3,y_3) & \ldots\ldots & p_m(x_2,y_2) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ p_1(x_n,y_n) & p_2(x_n,y_n) & p_3(x_n,y_n) & \ldots\ldots & p_m(x_n,y_n) \end{bmatrix} \quad \text{-----(4.20)}$$

Where (x_1,y_1), (x_2,y_2), (x_3,y_3),..., (x_n,y_n) are the co –ordinates of n number of data points.

Thus,

$$[\mathbf{P}]\,\{\mathbf{a}\} = \{\mathbf{s}\} \quad \text{-----(4.21)}$$

Where **{a}** is a vector of unknown coefficients **of** ***'m'*** numbers of terms in the polynomial and **{s}** is the vector of known field variables at ***'n'*** number points in the vicinity.

4.8.2 Solution Techniques

In MLS method , a Polynomial Function containing ***'m'*** number of terms is fitted to pass through the function values at each of the given ***'n'*** number of scattered points. The polynomial may contain exactly the same number of terms as the number of scattered points.

If $m = n$, the solution would have been very simple in the form

$$\{a\} = [P]^{-1} \{s\} \qquad \text{-----(4.22)}$$

Though the solution looks simple, it suffers from an inherent property of a possible singularity of **[P]** which makes $[P]^{-1}$ non-existent. The existence of $[P]^{-1}$ depends on the distribution of the influencing points scattered in the vicinity. For a set of arbitrarily selected points, if x or y co-ordinates are the same, **[P]** may turn out to be singular and the method will not work straight away.

Moreover, if $m \neq n$ **,** the method fails **,** as the inverse of a non square matrix does not exist.

There are no chances that a polynomial is selected with terms more in number than the number of data points. Hence a case of $m > n$ does not exist.

When $m<n$, P is a non square matrix with an order $n \times m$. The solution needs to be obtained for n number of simultaneous equations to find the values of m number of coefficients represented by **{a}**.

Multiplying both the sides of equation (4.21) by $[P]^T$

$$[P]^T [P] \{a\} = [P]^T \{s\} \qquad \text{-----(4.23)}$$

$[P]^T [P]$ is a square matrix of order **m x m** and we can obtain inverse of this**.**

$$\{a\} = ([P]^T [P])^{-1} \; [P]^T \{s\} \qquad \text{-----(4.24)}$$

It can be proved that, the matrix will be singular, only if the columns in [P] are linearly dependent. The moment matrix is dependent only on the co-ordinates of the data points. Columns can be linearly dependent in cases where all the points are on global x axis or on global y axis or fall on a straight line or when two points merge. There are various methods suggested to avoid the singularity of the matrix. (Liu et al ,2003,2013). In these, rotation of local axes is a simple but very efficient method.

4.8.3 Local Co-ordinate and Rotation of Local Axes

In this methodology, singularity is avoided by rotating the local co-ordinates system of a sub-domain by an angle such that the data points possess new co-ordinate values and the moment matrix gets modified (Liu, 2003, 2013)

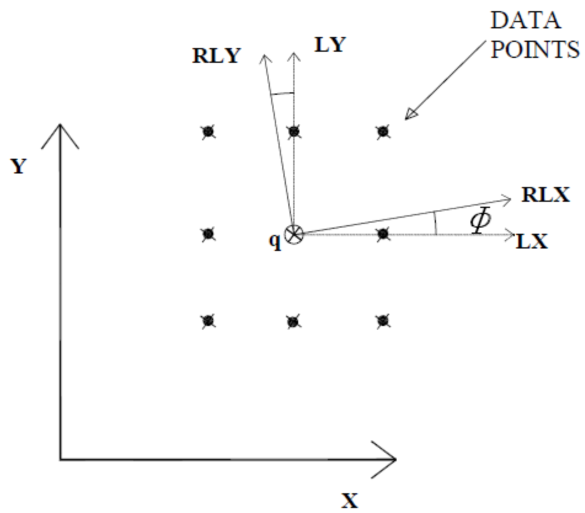


Fig. 4.9 Rotation of Local Axes

Shifting the co-ordinate system to the point of interest ***q (xq,yq)*** we get

$$lx = x - xq \qquad \text{-----(4.25)}$$

$$ly = y - yq \qquad \text{-----(4.26)}$$

where ***(lx,ly)*** is the new co-ordinate for a point with respect to the local axes.

Performing rotation of the local axes by an angle ϕ as shown in Fig. 4.9,

$$rlx = lx \cos\phi - ly \sin\phi \qquad \text{-----(4.27)}$$

$$rly = -lx \sin\phi + ly \cos\phi \qquad \text{-----(4.28)}$$

where ***(rlx,rly)*** is the co-ordinate of the data point with respect to the rotated local axes. Obtaining ***rlx*** and ***rly,*** corresponding global co-ordinates can be calculated as

$$xnew = xq + (rlx \cos \phi - rly \sin \phi) \qquad \text{-----(4.29)}$$

$$ynew = yq + (rlx \sin \phi + rly \cos \phi) \qquad \text{-----(4.30)}$$

where ***(xnew,ynew)*** is the new location of the data point with respect to the global co-ordinates. However, since the location of point of interest with respect to the data points is not changed, the interpolated value at ***q (xq,yq)*** and the function fitted for the region do not get affected .

Where ever the moment matrix turns out to be singular, such rotations by a small angle may be performed. If the rotated co-ordinates still give rise to moment matrix being singular, rotation in the reverse direction or by another angle needs to be tried.

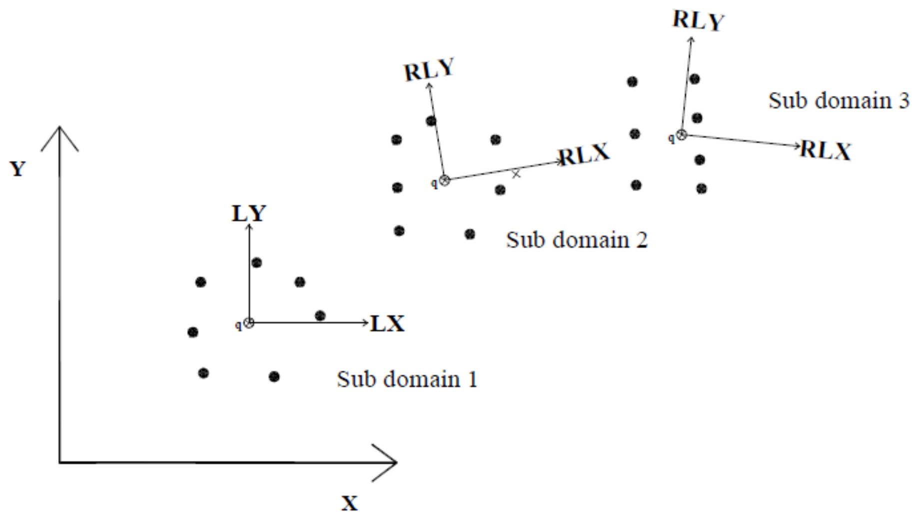


Fig. 4.10 Rotation of Local Axes in different Sub Domains

In the presentwork, this methodology has been adopted with a trial of rotaion by an angle one dgree , incremented everytime till singularity does not occur. Such rotations may be necessary at many locations where the data points form singular moment matrix, as shown in Fig. 4.10.

4.8.4 Weight Function

The measured values closest to the prediction location have more influence on the predicted value than those farther away. Hence while predicting a value based on a given set of data in the neighbourhood, more ‘weightage’ should be given to the data available near the point of prediction. One of the ways to implementation of weightages is through the use of weight functions. The value of weight functions decays with increasing distance to the data point.

A 'weight function' is a mathematical device used when performing a sum, integral, or average to give some elements more "weight" or influence on the result than other elements in the same set. They occur frequently in statistics and analysis, and are closely related to the concept of a measure. Weight functions can be employed in both discrete and continuous settings.

4.8.5 Types of Weight Functions

There are various types of weight functions used in weighted MLS Methods. The cubic spline weight function, the quartic spline weight function, the exponential weight function, new quartic spline weight function by Liu et. al. (2002) are some of them.

Basically, **W = f (x-xi)** -----(4.31)

and is represented as W(x-xi).

The Weight Function **W** is always chosen to have the following properties.

1. **W (x-xi)** > 0 within the support domain
2. **W (x-xi)** = 0 outside the support domain
3. **W (x-xi)** monotonically decreases from the point of interest at **x**
4. **W (x-xi)** is sufficient smooth, especially on the boundary of the domain

4.8.5.1 The Cubic Spline Weight Function

$$\widehat{W}(\mathbf{x}-\mathbf{x}_I) \equiv \widehat{W}(\bar{d}) = \begin{cases} \frac{2}{3} - 4\bar{d}^2 + 4\bar{d}^3 & \text{for } \bar{d} \le \frac{1}{2} \\ \frac{4}{3} - 4\bar{d} + 4\bar{d}^2 - \frac{4}{3}\bar{d}^3 & \text{for } \frac{1}{2} < \bar{d} \le 1 \\ 0 & \text{for } \bar{d} > 1 \end{cases} \quad \text{-----(4.32)}$$

4.8.5.2 The Quartic Spline Weight Function

$$\widehat{W}(\mathbf{x}-\mathbf{x}_I) \equiv \widehat{W}(\bar{d}) = \begin{cases} 1 - 6\bar{d}^2 + 8\bar{d}^3 - 3\bar{d}^4 & \text{for } \bar{d} \le 1 \\ 0 & \text{for } \bar{d} > 1 \end{cases} \quad \text{-----(4.33)}$$

4.8.5.3 The Exponential Weight Function

$$\widehat{W}(\mathbf{x}-\mathbf{x}_I) \equiv \widehat{W}(\bar{d}) = \begin{cases} e^{-(\bar{d}/\alpha)^2} & \bar{d} \le 1 \\ 0 & \bar{d} > 1 \end{cases} \qquad \text{-----(4.34)}$$

4.8.5.4 The New Quartic Spline Weight Function by Liu et al (2002)

$$\widehat{W}(\mathbf{x}-\mathbf{x}_I) \equiv \widehat{W}(\bar{d}) = \begin{cases} \frac{2}{3} - \frac{9}{32}\bar{d}^2 + \frac{19}{192}\bar{d}^3 - \frac{5}{512}\bar{d}^4 & \text{for } \bar{d} \le 1 \\ 0 & \text{for } \bar{d} > 1 \end{cases} \qquad \text{-----(4.35)}$$

Where

$$\bar{d} = \frac{|\mathbf{x}-\mathbf{x}_i|}{r_w} \qquad \text{-----(4.36)}$$

and r_w is the size of the support domain

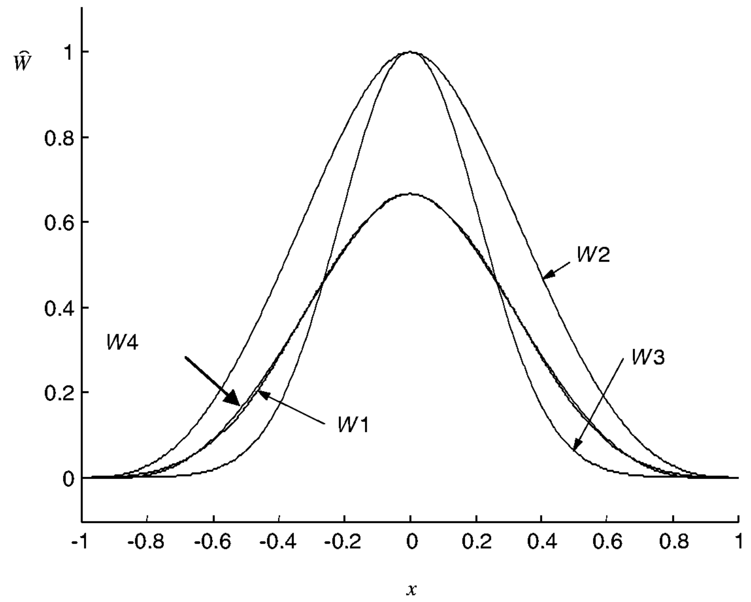


Fig. 4.11 Plots for Various Types of Weighting Functions

Fig 4.11 Shows the plot of weight functions where $\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$ and $\mathbf{W_4}$ are the function plots as per the equations (4.32) to (4.35) respectively.

The choice of the weight function is more or less arbitrary as long as the above requirements are met. The exponential function and spline functions are often used in practice. Among them, the most commonly used weight function is the cubic spline weightfunction $\mathbf{W_1}$. This has been used in the present work.

In 2D problems (Liu,2003),

$$\mathbf{W_x = W (x - x_i)} \qquad \text{-----(4.37)}$$

$$\mathbf{W_y = W (y - y_i)} \qquad \text{-----(4.38)}$$

Where $\mathbf{(x,y)}$ is the point of interest and $\mathbf{(x_i,y_i)}$ is $\mathbf{i}^{\mathbf{th}}$ the data point.

The net weightage to be given is

$$\mathbf{W = W_x \cdot W_y} \qquad \text{-----(4.39)}$$

The weighted MLS function for the sub-domain thus becomes

$$\mathbf{W\,P\,a = W\,s} \qquad \text{-----(4.40)}$$

where **W**(x) is a diagonal matrix of the form

$$W = \begin{bmatrix} w(x-x_1) & 0 & \cdots & 0 \\ 0 & w(x-x_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(x-x_N) \end{bmatrix}. \qquad \text{-----(4.41)}$$

4.9 SOLUTION FOR COEFFICIENTS

It is evident that **W, P** and **a** are dependent on the co-ordinates of the data points. In these expressions, W is of the order ***nxn*** , P is of the order ***nxm***, a is of the order ***mx1*** and **s** is of the order ***nx1***

$$\mathbf{P^T W\,P\,a = P^T W\,s} \qquad \text{-----(4.42)}$$

$$\mathbf{a = (P^T W\,P)^{-1}\,(P^T W)\,s} \qquad \text{-----(4.43)}$$

or

$$\mathbf{a = Q^{-1}\,R\,s} \qquad \text{-----(4.44)}$$

where

$$\mathbf{Q = (P^T W\,P)} \qquad \text{-----(4.45)}$$

and

$$\mathbf{R = (P^T W)} \qquad \text{-----(4.46)}$$

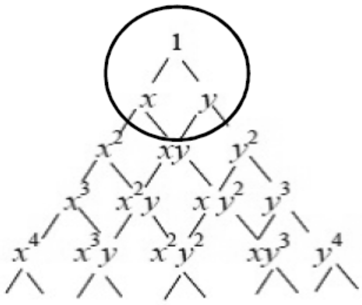
Solution of equation (4.44) yields the coefficient matrix **a** that can be used in the polynomial function to fit the given data points

4.10 MOVING POINT MLS IN 2D

In situations where the smooth fitting function has to necessarily pass through the data points, the accuracy of fitting will be better, if the number of terms and degree of polynomial are more. It is obvious that a relatively large support domain means more data points are involved in calculation. Similarly, the number of such data points vary from sub-domain to sub-domain. This means, there is a scope for using a different number of terms in the polynomial basis functions for every subdomain, every time selecting the number of terms in the polynomial equal to the number of data points available, ie ., $\boldsymbol{m} = \boldsymbol{n}$. This will involve reconstruction of polynomial , not only for a different set of coefficients, but also for number of terms and coefficients. This will exploit the advantages of MLS method leading to MPMLS in 2D.

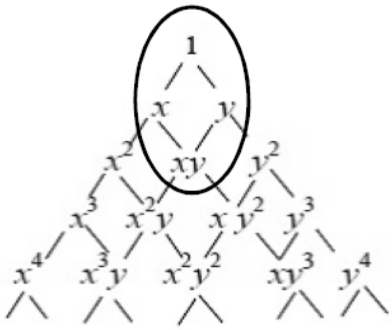
4.11 SELECTION OF BASIS FUNCTIONS IN MPMLS IN 2D

The process of selection of basis function is done as per the number of data points available in the vicinity, containing unbiased representation of x and y. Based on the principle that the equal representation from both the co-ordinate directions and un-biased selection of degrees of x and y, The Basis functions are selected as shown in Fig. 4.12 to 4.20, for number of terms varying from 3 to 11, respectively, for the sub-domains containing 3 to 11 data points. For the domains containing more data points, basis functions can be selected in similar lines.



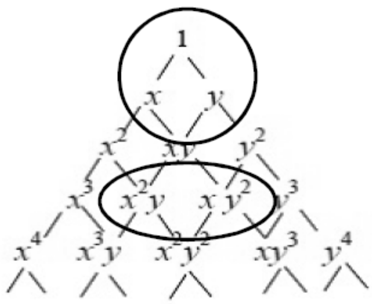
P3 = a1 + a2 x + a3 y

Fig. 4.12 Polynomial for 3 data points



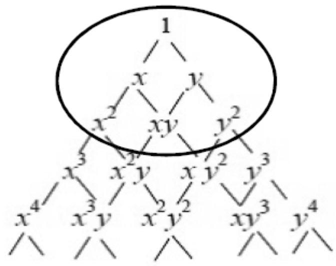
P4 = a1 + a2 x + a3 y + a4 xy

Fig. 4.13 Polynomial for 4 data points



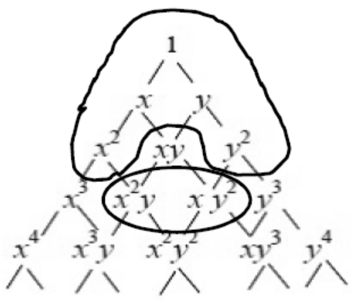
P5= a1 + a2 x + a3 y + a4 x^2y+a5xy^2

Fig. 4.14 Polynomial for 5 data points



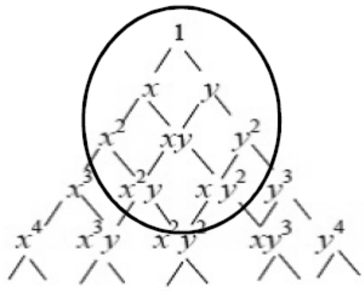
P6= a1 + a2 x + a3 y + a4 xy+a5x^2 +a6y2

Fig. 4.15 Polynomial for 6 data points



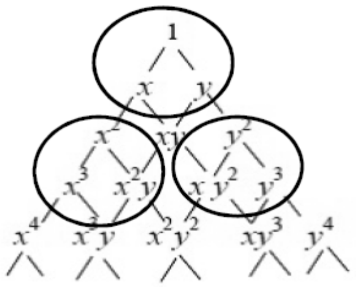
P7= a1 + a2 x + a3 y +a4x^2 +a5y2+a6 x^2y +a7xy^2

Fig. 4.16 Polynomial for 7 data points



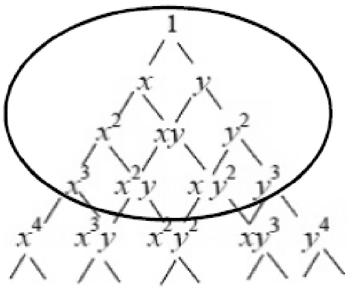
P8= a1 + a2 x + a3 y +a4x^2 +a5y^2++a6xy+a7 x^2y + a8xy^2

Fig. 4.17 Polynomial for 8 data points



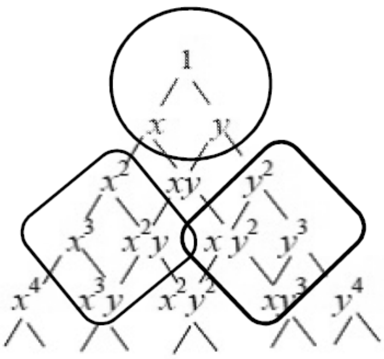
P9= a1 + a2 x + a3 y +a4x^2 +a5y^2+a6 x^2y + a7xy^2+a8x^3+a9y^3

Fig. 4.18 Polynomial for 9 data points



P10= a1 + a2 x + a3 y +a4x^2 +a5y^2+a6xy+a7 x^2y + a8xy^2+a9x^3+a10y^3

Fig. 4.19 Polynomial for 10 data points



P11= a1 + a2 x + a3 y +a4x^2 +a5y^2+a6 x^2y + a7xy^2+a8x^3+a9y^3+a10x^3y+a11xy^3

Fig. 4.20 Polynomial for 11 data points

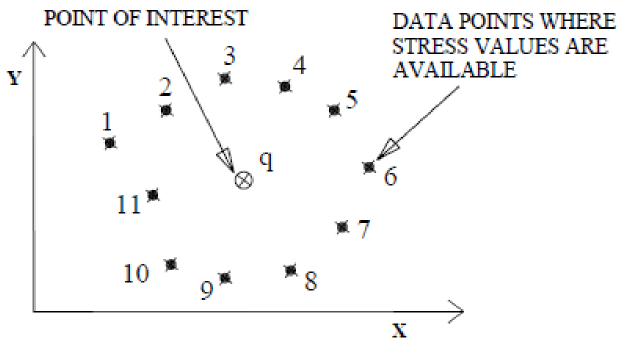
The usage of these basis functions and their relative advantages have been analysed through an illustration.

4.12 ILLUSTRATION ON STRESS SMOOTHING BY MPMLS METHOD

In this illustration, The use of polynomial basis function with different number of terms is studied by varying number of terms in weighted MLS solution.

Stress values, in MPa, $\mathbf{s_1, s_2, s_3, ... s_{11}}$ are given at 11 locations in a sub-domain containing 11 scattered points, shown in Fig.4.21. The stress value at a point '**q**' is needed.

In a typical FEA, a 3- term polynomial basis function is assumed for all sub domains, since, for all sub-domains, minimum 3 points will be available for interpolation in triangulated FEM Domain**.**



Point No	x	y	s
1	19.35	43.12	33.80
2	33.76	51.46	39.10
3	48.73	59.57	40.80
4	64.06	57.54	32.70
5	76.69	51.46	28.80
6	85.57	36.90	34.60
7	78.68	21.54	38.90
8	65.58	10.48	42.30
9	48.73	8.49	36.56
10	34.93	11.96	30.29
11	30.37	29.73	32.51
q	54.64	34.21	?

Fig. 4.21 Illustrative Stress Smoothing Problem

At this location, since there are 11 data points available and there is a possibility of using a polynomial basis function containing upto 11 terms. Thus polynomial fits are made with number of terms varying from 3 to 11, using weighted MLS, taking an intermediate point ***q*** (54.64 ,34.21) as the point interest.

Surface development of the boundary with the stress values as the ordinates is shown in Fig. 4.22. The aim is to fit a curve, as smooth as possible, which will represent the stress values along the boundary.

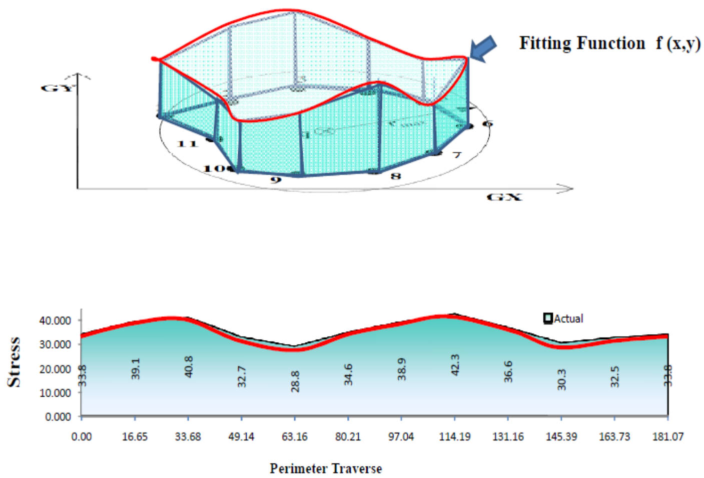


Fig. 4.22 Aim of Stress Smoothing Function

The solution is attempted with MLS method using a weighting function W1, given by equation (4.32) taking different number of terms in the polynomial . Here, since the data at 11 Points are available. Fitting functions are obtained as

With m = 3 $f(x,y) = + 36.27649579 + 0.01333471\ x - 0.03391594\ y$ -----(4.47)

With m = 4

$f(x,y) = + 9.69427425 + 0.50269835\ x + 0.73547957\ y - 0.01409307\ xy$ -----(4.48)

With m = 5

$f(x,y) = + 9.94349894 + 0.52431659\ x + 0.35839655\ y - 0.00014034\ x^2y + 0.00001402\ xy^2$ -----(4.49)

With m = 6

$f(x,y) = + 2.65382519 + 0.88630415\ x + 0.53115039\ y - 0.01456376\ xy - 0.00337254\ x^2 + 0.00337921\ y^2$ -----(4.50)

With m = 7

$f(x,y) = + 8.15389141 + 0.78449483\ x - 0.00066074\ y - 0.00385764\ x^2 + 0.00982873\ y^2 - 0.00003325\ x^2y - 0.00015986\ xy^2$ -----(4.51)

With m = 8

$f(x,y) = + 5.96838814 + 0.85847356\ x + 0.07994093\ y - 0.00233641\ xy - 0.00444588\ x^2 + 0.00929419\ y^2 - 0.00001779\ x^2y - 0.00015067\ xy^2$ -----(4.52)

With m = 9

$f(x,y) = + 15.06050144 + 1.54728324\ x - 0.81649742\ y - 0.03406718\ x^2 + 0.02957575\ y^2 + 0.00033269\ x^3 - 0.00038384\ y^3 - 0.00042707\ x^2y + 0.00044562\ xy^2$ -----(4.53)

With m = 10

$$f(x,y) = -86.54307967 + 5.40179595\,x + 4.07009625\,y - 0.08882328\,xy - 0.08537555\,x^2 - 0.06932155\,y^2 + 0.0005373\,x^3 + 0.00052654\,y^3 + 0.00023539\,x^2y + 0.00070261\,xy^2 \quad \text{-----(4.54)}$$

With m = 11

$$f(x,y) = -78.96361915 + 5.32852763\,x + 1.71967145\,y - 0.08770937\,x^2 + 0.01153016\,y^2 + 0.00055228\,x^3 - 0.0004538\,y^3 - 0.00098959\,x^2y - 0.00129275\,xy^2 + 0.00000897\,x^3y + 0.00002512\,xy^3 \quad \text{-----(4.55)}$$

The fitting function equations (4. 47) to (4.55) have been used to compute the stresses at the data points and these computed stresses values are compared with given stress values , in Table 4.4 The Variance is computed as given in equation (4.56) and tabulated. It is obvious that as the number of terms increases, the error decreases.

$$\text{Variance} = \frac{\sum\left(S_{actual} - S_{computed}\right)^2}{\text{No. of Data Points}} \quad \text{-----(4.56)}$$

Thus, it can be seen that , the errors are minimum when higher degree polynomials are used in the basis functions. Fig. 4.23 shows surface development of boundary stresses at data points for m=3, m=7, m=11 and that for given stress values. Fig 4.24 shows these plots for all the fitting functions.

It has been demonstrated that the accuracy of fitting will be better with a polymial function containing a number of terms as equal as the number of data points available in the viscinity.

Table 4.4 Comparison of Stress Values at Data Points

STRESS VALUES Mpa	MLS WITH VARIED NUMBER OF TERMS IN FUNCTION — STRESS VALUES OBTAINED USING FITTING FUNCTIONS Mpa								
	m = 3	m = 4	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10	m = 11
33.800	35.072	39.377	33.782	35.576	33.847	33.986	32.802	33.701	33.793
39.100	34.981	40.029	39.110	39.712	39.993	40.037	40.057	39.126	39.104
40.800	34.906	37.093	39.416	39.191	39.713	39.634	39.880	40.848	40.801
32.700	35.179	32.269	33.989	33.659	33.324	33.320	33.324	32.575	32.692
28.800	35.554	30.476	28.970	29.596	29.092	29.178	28.659	28.963	28.813
34.600	36.166	35.350	31.750	32.015	32.784	32.739	33.854	34.456	34.585
38.900	36.595	41.204	40.715	39.837	40.273	40.192	39.571	38.924	38.905
42.300	36.796	40.683	41.860	42.202	41.433	41.512	42.061	42.341	42.301
36.560	36.638	34.604	35.756	36.563	36.693	36.739	35.981	36.472	36.553
30.290	36.337	30.162	30.566	30.249	30.964	30.827	31.523	30.375	30.299
32.510	35.673	34.102	33.050	32.089	31.886	31.889	31.981	32.500	32.508
Variance =SUM OF THE SQUARES OF DEVIATIONS/n									
Variance	17.2	5.8	1.5	1.4	0.8	0.8	0.6	0.0	0.0

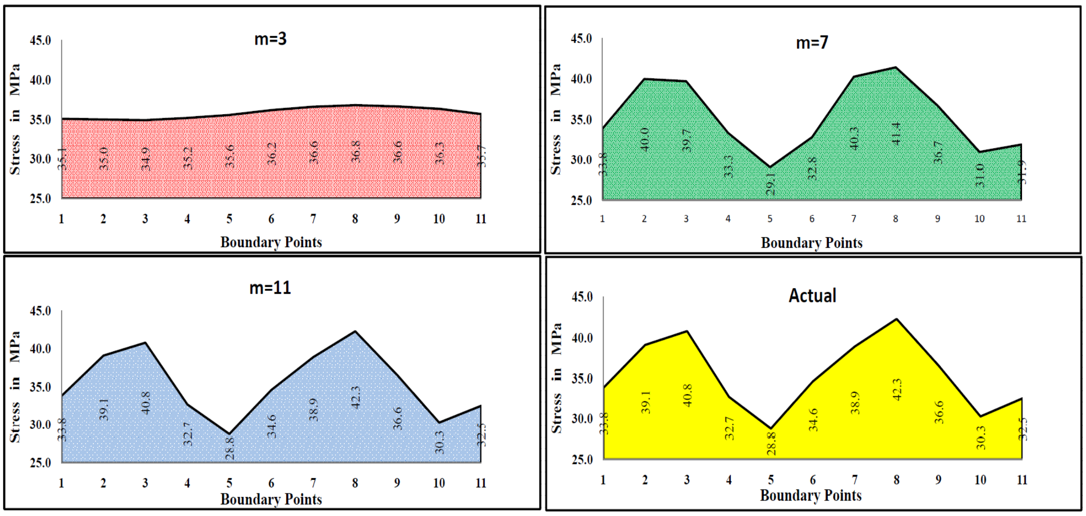


Fig. 4.23. Comparison of Stress Values

It may be seen that the plot for fitting function with m=3 has stress predictions much deviating from the given values at the data points. As the value of m increases, the function does justification of the fit by predicting accurate values at the data points, as shown in Fig. 4.24. The variance for fitted values of stress is represented in Fig.4.25 for increase in values of **m**=3 to **m**=11.

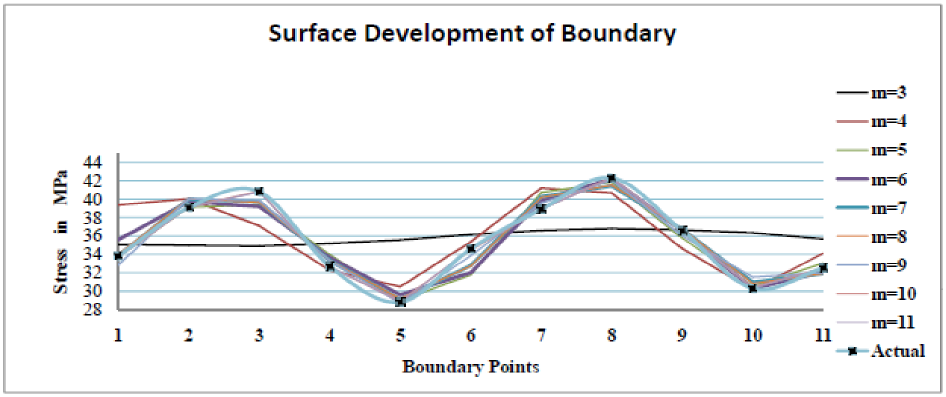


Fig. 4.24 Surface Development of Boundary Stress Values

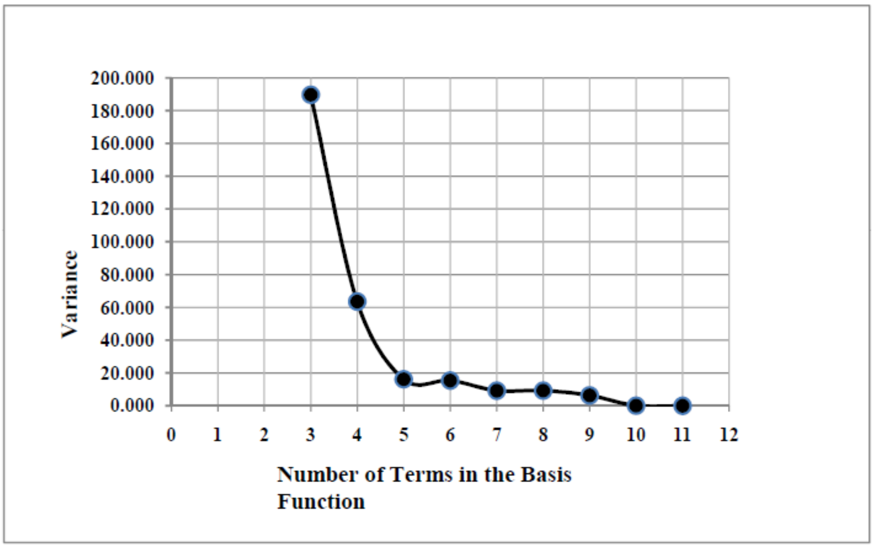


Fig. 4.25 Comparison of Variance

4.13 USE OF FITTING FUNCTION

The fitting functions of higher order truly 'honour' the values at the data points and represent smooth variation of stresses between the data points. Hence these can be used to smooth out the stresses across element edges and to predict the stress values all along the polygonal boundary, in a region containing the data points.

The surface plots corresponding to the fitting functions are given in Fig.4. 26.

The applicability of these functions for the local interpolation to determine the value of stress at an internal point depends not only on the number of sample points but also on the location of these sample points. If the location of samples points are not indicative of the surface trend , showing local peaks and valleys , scattered data interpolation by any method can lead to gross errors (O' Sullivan, et.al ,2002).

As seen in Table 4.5 the surface plots, the fitting function may not be directly used to evaluate the stress values at the intermediate point **q**, within the boundary, as the trend of variation of stress from outside the boundary to inside is not known.

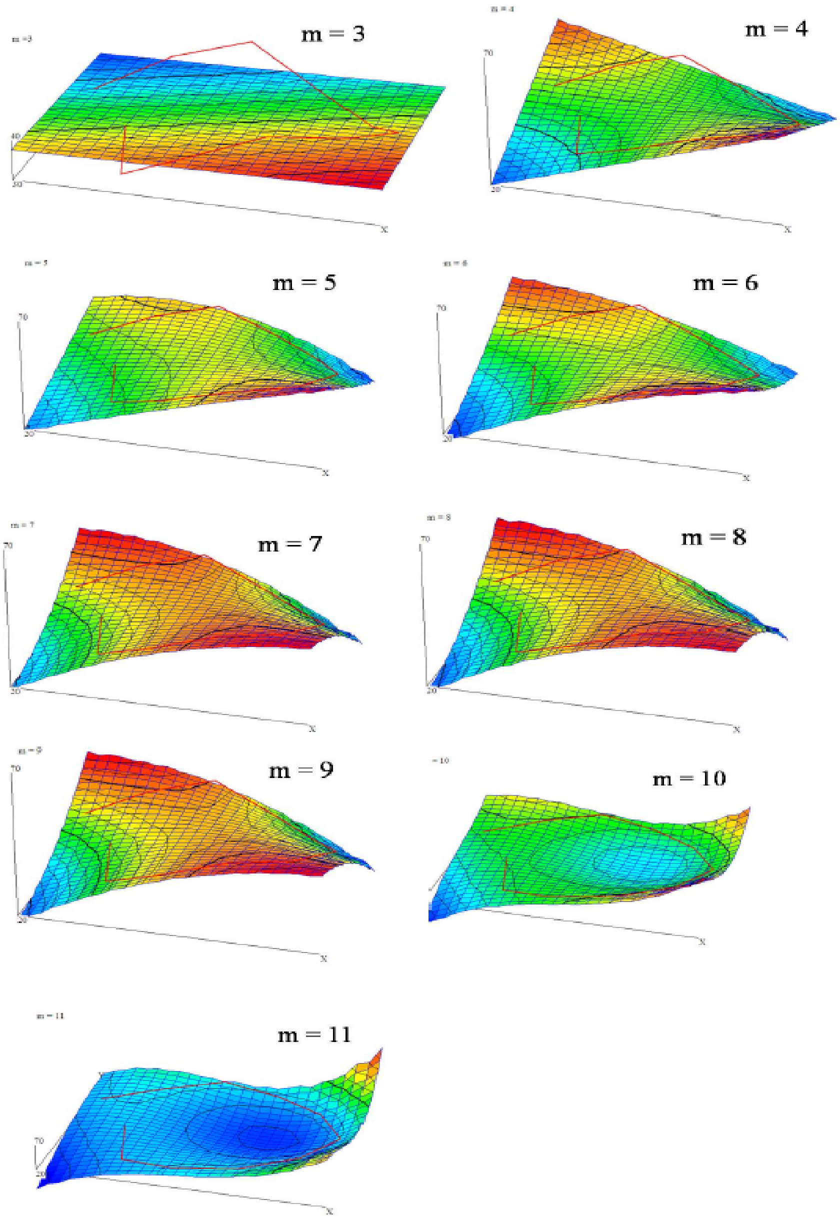


Fig. 4.26 Surface Plots for Fitting Functions

Table 4.5 Stress Values Computed at the Intermediate Point
Using Fitting Functions

STRESS AT (xq,yq) Mpa (USING FITTING FUNCTION)									
	m = 3	**m = 4**	**m = 5**	**m = 6**	**m = 7**	**m = 8**	**m = 9**	**m = 10**	**m = 11**
Func. Value	35.8448	35.9792	36.5455	35.915	37.3633	37.5677	37.395	23.499	15.8492

Table 4.5 shows that the fitting functions will have to be used with caution as an interpolation function, if a sample data point at a location indicative of trend of surface is not available. Under these circumstances, a suitable deterministic interpolation tool like Inverse Distance Weighting Method or Averaging Method may be employed for interpolation.

However, if any data point is available within such boundary, which indicates the trend of stress variation, the fitting function may be used for internal interpolation too. Similarly, for a small region with closely and irregularly spaced data points, the fitting function can predict the stress values valid throughout the region.

4.14 LOCAL INTERPOLATION USING INVERSE DISTANCE WEIGHTING METHOD

The concept behind interpolation by Inverse Distance Weighting (IDW) method is expressed in by Tobler in 1970 as the first law of geography which states "All places are related, but nearby places are more related than distant places" (O'Sullivan, 2002).

The influence of neighbours decreases with increase in distances , meaning farther neighbour has less influence.

Inverse distance weighting is a deterministic, nonlinear interpolation technique that uses a weighted average of the attribute (i.e., phenomenon) values from nearby sample points to estimate the magnitude of that attribute at interior non-sampled locations (Adrienne , 2014) .

$$Z = \frac{\sum (w_i z_i)}{\sum (w_i)} \qquad \text{-----(4.57)}$$

Where z_i is the attribute at the ith location. **z** is the attribute at the location of interest where interpolated value is required and w_i is the weighting added which decreases with the increase in distance of the i^{th} data point from the location of interest.

w_i is generally of the form

$$W_i = \frac{1}{d_i^{p}} \qquad \text{-----(4.58)}$$

Where d_i is the distance to the i^{th} point and **p** indicates the power to which the distance effect is enhanced. With p = 2, the above equation reduces to the form

$$z = \frac{\frac{1}{d_1^2} z_1 + \frac{1}{d_2^2} z_2 + \frac{1}{d_3^2} z_3 + \dots\dots\dots + \frac{1}{d_n^2} z_n}{\frac{1}{d_1^2} + \frac{1}{d_2^2} + \frac{1}{d_3^2} + \dots\dots\dots + \frac{1}{d_n^2}} \qquad \text{-----(4.59)}$$

Using the Given data points, if the interpolation is attempted using IDW, the interpolated value at **'q'** is found as 35.22 MPa. as shown in Table 4.6.

Table 4.6 Stress Values Computed based on Originally Available Data

INTERPOLATION FOR STRESS BY IDW WITH GIVEN DATA POINTS			
DATA POINT	**CO-ORDINATES**		**STRESS (GIVEN)**
GIVEN	**x**	**y**	**Mpa**
1	19.35	43.12	33.80
2	33.76	51.46	39.11
3	48.73	59.57	40.82
4	64.06	57.54	32.70
5	76.69	51.46	28.80
6	85.57	36.90	34.55
7	78.68	21.54	38.89
8	65.58	10.48	42.29
9	48.73	8.49	36.55
10	34.93	11.96	30.30
11	30.37	29.73	32.51
Point of Interest	**54.64**	**34.21**	
Stress Value Mpa	**(IDW)**		**35.22**

4.15 CREATION OF ADDITIONAL DATA POINTS USING SMOOTH FIT FUNCTIONS

It can be seen that a smooth surface which passes through the data points will be a good representative for the stress values along the boundary. Thus the fitting function can be used to create additional points along the boundary. These data points, in addition to the given ones, are useful in a suitable local interpolation method for the determination of stresses at an internal location as shown in Fig. 4.27.

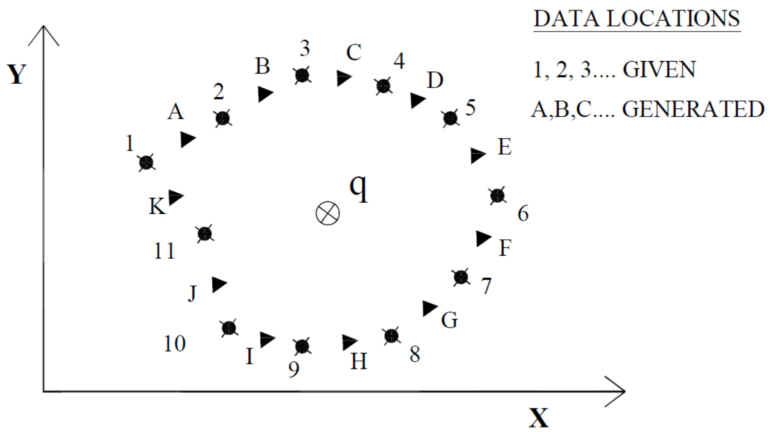


Fig. 4.27 Intermediate Data Points Generated Using Fitting Function

Additional data points A,B,C,...., K are generated using the fitting function corresponding to m=11. The stress values at the original and generated points are tabulated in Table 4.7. Using these 22 points, IDW improves the quality of interpolated stress value at q as 34.67 MPa as all stresses at these locations are estimated by a fitting function which is not only smooth, but also passing through the data points.

Table 4.7 Stress Values Computed based Data Generated Using Fitting Function

INTERPOLATION FOR STRESS BY IDW WITH DATA POINTS GENERATED USING FITTING FUNCTION m=11				
DATA POINT		**CO-ORDINATES**		**STRESS (FIT)**
ORIGINAL	**ADDED**	**x**	**y**	**Mpa**
1		19.35	43.12	33.80
	A	26.56	47.29	38.86
2		33.76	51.46	39.11
	B	41.25	55.52	38.14
3		48.73	59.57	40.82
	C	56.40	58.56	35.13
4		64.06	57.54	32.70
	D	70.38	54.50	27.87
5		76.69	51.46	28.80
	E	81.13	44.18	25.18
6		85.57	36.90	34.55
	F	82.13	29.22	32.74
7		78.68	21.54	38.89
	G	72.13	16.01	39.93
8		65.58	10.48	42.29
	H	57.16	9.49	38.95
9		48.73	8.49	36.55
	I	41.83	10.23	33.66
10		34.93	11.96	30.30
	J	32.65	20.85	31.09
11		30.37	29.73	32.51
	K	24.86	36.43	34.07
Point of Interest		**54.64**	**34.21**	
Stress Value Mpa		**(IDW)**		**34.67**

The effective combination of smoothing using the Moving Point Moving Least Square (MPMLS) method conceptualized and subsequent use of Inverse Distance Weighting (IDW) Method for interpolation has been used as the backbone for the stress recovery at nodal points, further extended to the formulation of Nodes- in -Motion -Strategy for the optimum material disposition in plates, detailed in Chapter 6.

Chapter 5

STRUCTURAL OPTIMIZATION BY NODES - IN - MOTION STRATEGY

5.1 GENERAL

Structural optimization, in its simplest sense, means the process of proportioning a structure for its best performance under given conditions. Depending upon the performance criterion, it could mean optimization for minimum material cost, for minimum manufacturing cost, for minimum overall cost, for minimum deflection, etc.

Based on the approach followed, structure optimized behaviour of the system and the objective, structural optimization has been further classified. A node based strategy is conceptualised and illustrated in the following sections.

5.2 TOPOLOGY OPTIMIZATION

The load transfer system, planned at the early phase, is of the paramount importance for the successful design of any structure. The conceptualisation of method of analysis and its mathematical implementation are dependent on this load transfer system and the initial configuration adopted for the structure. Explicit objectives like maximizing the stiffness or minimizing the compliance can be attained under a single or multiple load cases.

Topology optimization is some times referred to as layout optimization. Topological variables define the pattern of connected elements, regions of element removal or addition. In skeletal structures, topology optimization means to add some members where they are required, remove ones that are ineffective. This gives a new layout for the skeleton different from the old one. The member connectivity is revised and number of members changed. In this sense, 'topology optimization' is a term more used in skeletal structures.

5.3 SHAPE OPTIMIZATION

Shape is defined as the form of an object. The external configuration of any structure gives it a form.

Structures perform better if a proper shape is selected. The shape of a structure is defined by means of geometrical bounds containing the entire structure. These geometrical bounds are defined as co-ordinates in multi-dimensions and the change in the co-ordinates of bounds reflects are change in shape.

In shape optimization, boundaries of the component geometry are modified seeking better performance. Hence the co-ordinates of the bounds are treated as variables, while the connectivity among geometric sub domains remains constant. The basic concept of shape optimization is to achieve the minimum shape that satisfies all the necessary functional requirements such as stability, strength and stiffness.

An optimum shape for any structure depends on the location of supports externally applied loads, performance of materials in compression, tension or in shear.

5.4 SIZING OPTIMIZATION

In the case of skeletal structures, one may be forced to maintain the shape and topology but optimize the cross section of members, minimising the cost. The term 'Size Optimization', has more meaning in skeletal structures than in 2D and 3D continuum.

5.5 OPTIMUM MATERIAL DISPOSITION

Generally, the optimized solutions of plate like structures are attempted as a material disposition optimization. Optimum material disposition means the removing material from locations where it is not efficiently used and shifting/ increasing the material where it is best used. The methodology evolved in this research work for optimization of a continuum in covered in detail in subsequent chapters.

5.6 CLASSIFICATION BASED ON MODE OF BEHAVIOUR

In this classification structures are identified by the mode of behaviour under consideration.

5.6.1 Static Optimization

In this type of classification the structure is subjected to static external loads. Here the goal of structural optimization is to carry the applied loads safely and economically.

5.6.2 Dynamic Optimization

In this case, the structure is subjected to dynamic forces. The aim of optimizing structures not only in terms of strength, but also to avoid resonance. This can be achieved by increasing the difference between the forcing frequency and the natural frequencies of the structure. In certain cases the objective will be to match the frequencies with a predetermined set of values.

5.7 OPTIMIZATION PROBLEM DEFINITION

5.7.1 Mathematical Description of Optimization problem

Structural optimization problems are generally characterized by different objectives and constraints which are generally non linear functions of design variables. All optimization problems can be mathematically expressed as

Minimize (or maximize): $\mathbf{F(x)}$ -----(5.1)

Subject to constraints

$$\mathbf{gj(x) \leq 0} \qquad \mathbf{j=1,\ldots\ldots m} \qquad \text{-----(5.2)}$$

$$\mathbf{hk(x)=0} \qquad \mathbf{k=1,\ldots\ldots l} \qquad \text{-----(5.3)}$$

$$\mathbf{xi^{1} \leq xi \leq xi^{u}} \qquad \mathbf{i=1,\ldots\ldots,nd} \qquad \text{-----(5.4)}$$

in which, $\mathbf{x}$ is the vector of design variables, $\mathbf{F(x)}$ is the objective function to be minimized (or maximized), $\mathbf{gj(x)}$ and $\mathbf{hk(x)}$ are the behaviour constraints. $\mathbf{xi^{l}}$ and $\mathbf{xi^{u}}$ are lower and upper bounds on a typical design variable $\mathbf{xi}$.

5.7.2 Objective function

To select the best design, a criterion is to be chosen for comparing the different alternative acceptable designs. This is known as the objective function.

Mathematically, it is an expression that depends on design vector **x** which quantifies (in a mathematical sense) the worth of any design. The selection of the objective function depends on the nature of the problem. Depending upon the choice of the objective function, the optimization becomes either maximization or minimization. In structural optimization problems, the objective functions generally considered are minimizing the weight or volume of the structure, minimizing the strain energy, or minimizing the error in calculations.

5.7.3 Design Variables

The design variables largely depend on the problem at hand. Typical design variables can be coordinates of key points or thickness at key locations etc. Each individual design variable is denoted by **xi** and the set of design variables is grouped into the design vector **x**. In case of skeletal structures, the co-ordinates of joints, cross sectional areas of members are generally considered as design variables. In FEA based optimization problems of 2D or 3D continuum , the co-ordinates of all the nodal locations, thickness of individual elements are considered as design variables.

5.7.4 Constraints

In many practical design problems, the values of design variables are chosen so as to satisfy functional, geometrical and other requirements. The restrictions that must be satisfied in order to produce an acceptable design are called constraints. They can be further classified based on the situation or locations they are applied to.

5.7.4.1 Geometric Constraints

The constraints which represent physical limitations on the geometric design variables such as maintaining a shape, maintaining the loading points, limiting the movement of certain boundaries, maintaining minimum sizes at certain locations, locations of supports fall in this category..

5.7.4.2 Material Constraints

In optimization problems where selection is allowed from a variety of materials, the set of properties for a material needs to be maintained thorough out the solution. Similarly, from time to time, location to location, the availability of materials, ability to fabricate or assemble may also play as constraints. In this research work, the stress in any element is constrained to be within the maximum allowable stress.

5.7.4.3 Behavioural Constraints

The constraints which represent limitations on the behaviour or the performance of the system are called behaviour or functional constraints. It could be defined as maximum deflection permitted, limiting frequency of vibration, minimum stiffness constraints, etc.

5.7.4.4 Miscellaneous Constraints

There can be many other constraints imposed by the user while the process of optimization is on. It could be in the form of maintaining the locations of lower boundary nodes purely horizontal, allowing hole of constant size move within the shape, allowing a node to move relative to another node or nodes, maintaining one side of the plate vertical, etc. These conditions are checked at every stage of working towards betterment.

5.8 GENERAL NATURE OF PROCEDURE FOR OPTIMIZATION OF STRUCTURES

The procedure for optimization of any structure starts with an initial geometry of the structure based on the load transfer system suggested, loading & conditions assumed , material selected and objective of optimization defined.

The conceptualization and mathematical implementation of the analysis procedure follows and the internal forces developed in the structure are obtained to arrive at the stresses developed in the material of construction. Ensuring the stability, safety and serviceability conditions of the structure, a trial is made to modify the shape or thickness of the whole structure or size of its parts as an attempt to improve its performance towards betterment of objective function.

The remodelled structure is analysed again to determine its performance satisfying all the conditions. The whole procedure is repeated till the objective is attained to the level of satisfaction.

The procedure of optimization of any structure is repetitive in nature.

5.9 DEVELOPMENT OF A NODE BASED STRATEGY FOR 2D TRUSSES

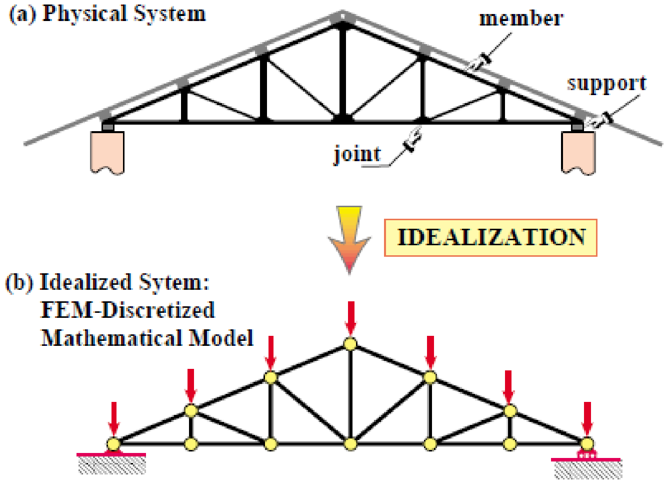


Fig.5.1 Idealization of Truss System

A truss is a structure composed of linear members assembled together at their end points. The interconnectivity of the joints and their relative disposition define the shape and topology of a truss. A truss forms an efficient structure capable of transferring loads expected on it to a supporting structure by virtue of its shape, topology and size of a group of straight members of varying lengths. Two dimensional trusses are basic and commonly used form of construction. They are treated as assemblages of 1D or Beam elements in FEM as idealized in Fig. 5.1.

5.9.1 Analysis and Design of Trusses

Analysis of a truss involves, sequentially, the initialization of the problem with nodal positions, member sizing, connectivity, support conditions and external loads. In a Standard Direct Stiffness Matrix Method, a global stiffness matrix is assembled from individual element stiffness matrices. support conditions are imposed by manipulation of corresponding elements in global stiffness matrix. With the force vector on the right hand side, simultaneous equations are solved to get displacements at nodes and forces in members. Stresses in the material are determined to check the adequacy of cross sectional areas provided for all the members. The Direct Stiffness Matrix Method used for the analysis of trusses which yields the forces in the members is outlined in Appendix B.

5.9.2 Objective Function

The Objective of the optimization problem related to the truss design is to reduce the whole weight, **WT,** of the whole truss to a minimum. For a truss with uniform material, it reduces to

$$WT = \sum_{i=1}^{m} (A_i \, l_i \, \rho_i) \qquad \text{-----(5.5)}$$

m is the total number of members, $\mathbf{A_i}$ is cross sectional area and $\mathbf{l_i}$ is the length , ρ_i is the unit weight of material of the **i** th member

5.9.3 Strength Constraints

These are the limits imposed on the stresses **S** developed in the materials of cross section of members.

$$S_{actual} <= S_{permissible} \qquad \text{-----(5.6)}$$

Permissible limits on the stresses would also depend on the type of force developed on the member. For the sake of development of strategy, here it is assumed that the permissible stresses are the same in tension as well as in compression.

5.9. 4 Design

Since the areas of cross-sections are initially assumed, the axial stresses in the material of section for every member can easily be computed.

$$\mathbf{S_i} = \mathbf{F_i / A_i} \qquad \text{-----(5.7)}$$

Where $\mathbf{F_i}$ is the force developed in the $\mathbf{i^{th}}$ member. The stress thus calculated is compared with the permissible stress in the material of every member.

The permissible values of stresses is generally dependent on the yield stress of material, limiting it well within the linear range of the stress –strain relationship. If the calculated actual stress exceed beyond the permitted values, the cross sections of such members are insufficient and need to be revised upwards and a final safe design to be ensured.

5.9.5 Re- analysis and Design

The change effected in the cross sections of the members will result in changes the element stiffness matrices earlier computed for analysis and design of the truss. Hence it is customary that we repeat the procedure of analysis and design to ensure a safe deign without any more requirement of area enhancement.

5.9.6 Utility Ratio

The efficiency of material in any member can be assessed by the percentage exploitation of the capacity of the material. The ratio of actual stress developed to the permissible stress in the material at a cross section of a member is called utility ratio, **U** of that member.

Thus, if **U**=1 the capacity is fully exploited, if **U** <1, the capacity is partially exploited and If **U** > 1 the stress exceeds the permissible limits indicating the member is unsafe.

Referring to Fig. 5.2, at a typical node, let there are ' **m**' number of members connected to it. Let the cross sectional areas be $\mathbf{A_1, A_2, A_3, \ldots\ldots A_m}$ and lengths be $\mathbf{l_1, l_2, l_3, \ldots. l_m}$.

Performing the analysis and getting a safe design, need not achieve an optimum design. Let the utility ratios of the members be $\mathbf{U_1, U_2, U_3, \ldots U_m}$, all of them being less than unity for a safe design.

Changing the areas of the members as 'just required' to make the utility ratio nearing unity will optimize the size of members for the given shape and topology. But this need not necessarily be optimum for the whole structure in terms of minimum weight to transfer the applied loads under specified support conditions. There could be a better shape or topology to serve the purpose with lesser weight. Hence, the effort is required to attempt shape, topology and size optimization simultaneously, in search of an optimum solution for the structure. This is accomplished through the following criteria.

5.9.7 Criteria Leading to Optimum Design of Trusses

Criterion-1 : For a given configuration, cross sectional properties, loading and support conditions, the members connected to a typical node, shown in Fig. 5.2, though all safe, need not have utility ratios equal to unity.

The shift of the node to another location, keeping the connectivity, and re-analysing the structure, modifies the utility ratios of members. The node may be moved such that the members with modified area are safe, at the same time, perform better. Successive modifying cross sectional areas, re-analysing, moving the connecting node for better performance of members will bring a situation where all the members attain utility ratios equal or very near to unity with a set of cross sectional areas, lengths. as shown in Fig. 5.3. The movement of nodes effect the shape and topology of the truss.

Criterion-2 : If a node can be moved to a new position such that all the members connected to it reach the utility ratios very close to unity simultaneously, then that position is deemed to be the perfect position for the node and hence a Nodes-in-Motion strategy would be formulated to find a perfect position for every node.

These conditions, when satisfied for all the nodes will result in all the members of the truss reaching utility ratios nearing unity.

In sizing optimization, the cross sectional areas modified, depending on the forces in members. Some of the members may end up with negligible cross sectional areas en-route the adaptive search for perfect nodal positions. This possibility is well addressed in Criterion -3 in the following section.

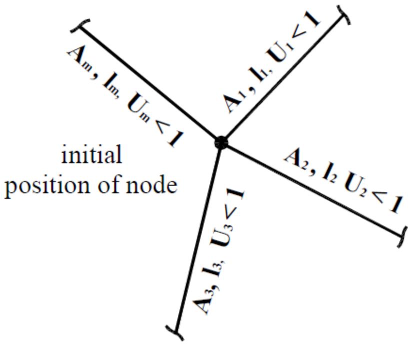


Fig. 5.2 Initial Position of Node

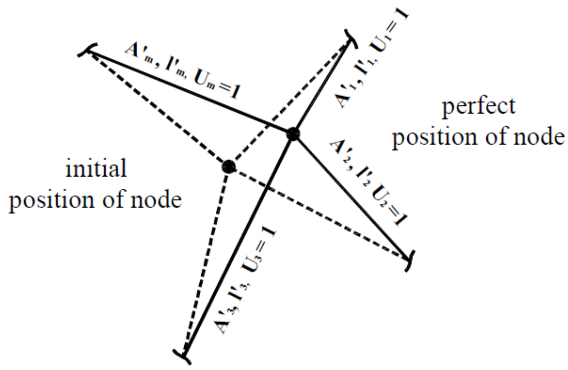


Fig. 5.3 Perfect position of Node

Criterion-3 : In the search process, if a member area demanded is miniscule (negligible cross sectional area), the indication is that such members are ineffective and the truss can perform without that member in question, as a part of the current configuration. Adaptive search may also encounter a situation where all the members at a node are ineffective. To tackle this situation, Criterion -4 is formulated.

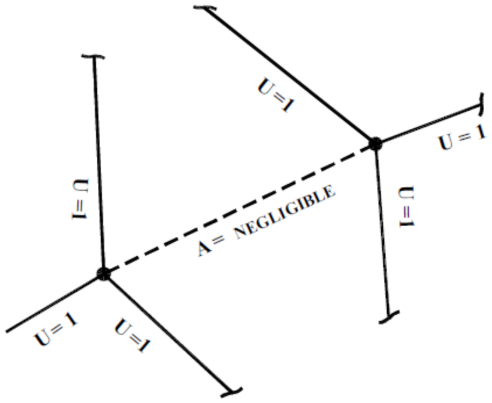


Fig. 5.4 Criterion for Removal a Member

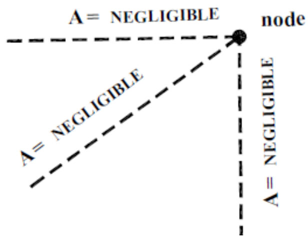


Fig. 5.5 Criterion for Removal of a Node

Criterion-4: If all the members at a node are ineffective, then the truss can survive without that node, indicating it can be collapsed to any other node it is linked to. This gives us an opportunity to remove such nodes from the truss and end up with a better topology.

5.9.8 Movement of Nodes

The multi objective optimization of trusses with same material can now be viewed as finding out the perfect nodal positions such that all the members of the truss, subjected to given set of constraints

a) have non negligible cross sectional areas
b) have utility ratios equal to unity (ideal situation)
c) have a set of cross sections and lengths such that total weight of the truss WT, is minimum

Movement of node changes the lengths of members connected to the node. The reverse is also true. The change in lengths of members connected to a node, moves the node.

To achieve the minimum, the problem is approached from three angles, simultaneously, for a member.

i. Reduce the length, if cross sectional area is to remain the same
ii. Reduce the cross sectional areas if the lengths is to remain the same.

iii. Increase the effectiveness

We can fix the maximum permitted change in any iteration, for the length of a member as ratio of its original length , in the iterative procedure to get a minimum weight. This ratio of change in length is termed as Modification Factor, **MF.**

$$MF = \frac{\text{CHANGE SUGGESTED FOR THE MEMBER}}{\text{ORIGINAL LENGTH OF THE MEMBER}} \quad \text{-----(5.8)}$$

Eg., If we can allow upto 10 % of change in length for a member, MF =0.10.

5.9.9 Weightage factors

The extend of movement of a node depends on the contribution of change in lengths of every member connected to that node. At the same time, an effective member should see that its effectiveness is maintained by not allowing the end nodes move much.

Similarly, a stronger member should have a relatively more say in controlling the movement of the node.

With these in mind, the lengths of members connected at a node are changed based on their relative qualifications. At a node , where **'m'** number of members are connected, for every member three factors are identified which are dependent on its weight, cross sectional area and utility ratio.

a) Factor for weight consideration (C_1)

$$C_{1i} = w_i \,/\, \text{sum of } (w_i) \quad \text{-----(5.9)}$$

i = 1 to m , where $\mathbf{w_i}$ is the weight of the **i** th member

b) ***Factor for area consideration (C_2)***

$$C_{2i} = A_i \,/\, \text{sum of } (A_i) \quad \text{-----(5.10)}$$

i = 1 to m, where $\mathbf{A_i}$ is cross sectional area of the **i** th member

c) ***Factor for inefficiency*** ***(C_3)***

$$C_{3i} = 1 - U_i \qquad \text{-----(5.11)}$$

where U_i is utility ratio of the **i** th member

Total forced change in length of ith member ,**dl** in the direction of member

$$dl_i = MF \cdot C_{1i} \cdot C_{2i} \cdot C_{3i} \cdot l_i \qquad \text{-----(5.12)}$$

where **MF** is the desired maximum modification desired per iteration

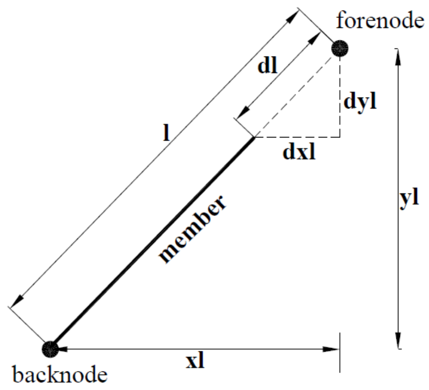


Fig. 5.6 Forced Change in Length of a Member

When the member length changes by **dl** , the node will be moved with respect to its original position by **dxl** in the global **X** axis and **dyl** in the Global **Y** axis.

$$X_{new} = X_{old} + \text{sum of (dxl)} \quad \text{for all the members} \qquad \text{-----(5.13)}$$

$$Y_{new} = Y_{old} + \text{sum of (dyl)} \quad \text{for all the members} \qquad \text{-----(5.14)}$$

The factor C2, defined for the area of a member becomes negligible if the member is ineffective. The factor C_3 , which is meant for the inefficiency of a member, reduces to zero, when utility ratio is unity. This means , the length of a member is not altered in both the cases. If all the members at a node are efficient, the node is not moved. This ensures the convergence of the solution for optimization.

On the other hand, if all of them are non-zero, the node will be forced to move relatively when every member is subjected to a forced change in length, every time checking the node position for a possible set of movement restrictions specified in the problem. This is repeated for all the nodes and the changed configuration of the truss is recorded.

5.9.10 Iterative Procedure

The optimum nodal position search method is an iterative procedure with distinct loops for sizing and shape optimization. Topology optimization is achieved during the course of shape optimization. Starting with the initial geometry, member properties, loading and support conditions and constraints, solution is obtained for the stresses. Cross-sectional areas are increased or decreased iteratively to obtain a safe sizing optimization for the shape and topology, till the utility ratios stabilized. This is named as the sizing loop. Member lengths are modified as per eq. (5.12) to effect change in nodal positions and the whole procedure is repeated to get another set of stabilized utility ratios.

The procedure is repeated till we get a stabilized set of utility ratios of all effective members equal to unity. While iterative loop in progress, some of the members of the truss are identified ineffective and Young's modulus values of such members are considered negligible for the consecutive loop. The final shape of the truss without these ineffective members is the optimum topology.

Fig. 5.7 shows the flowchart for the implementation of the nodes-in-motion strategy for 2D trusses.

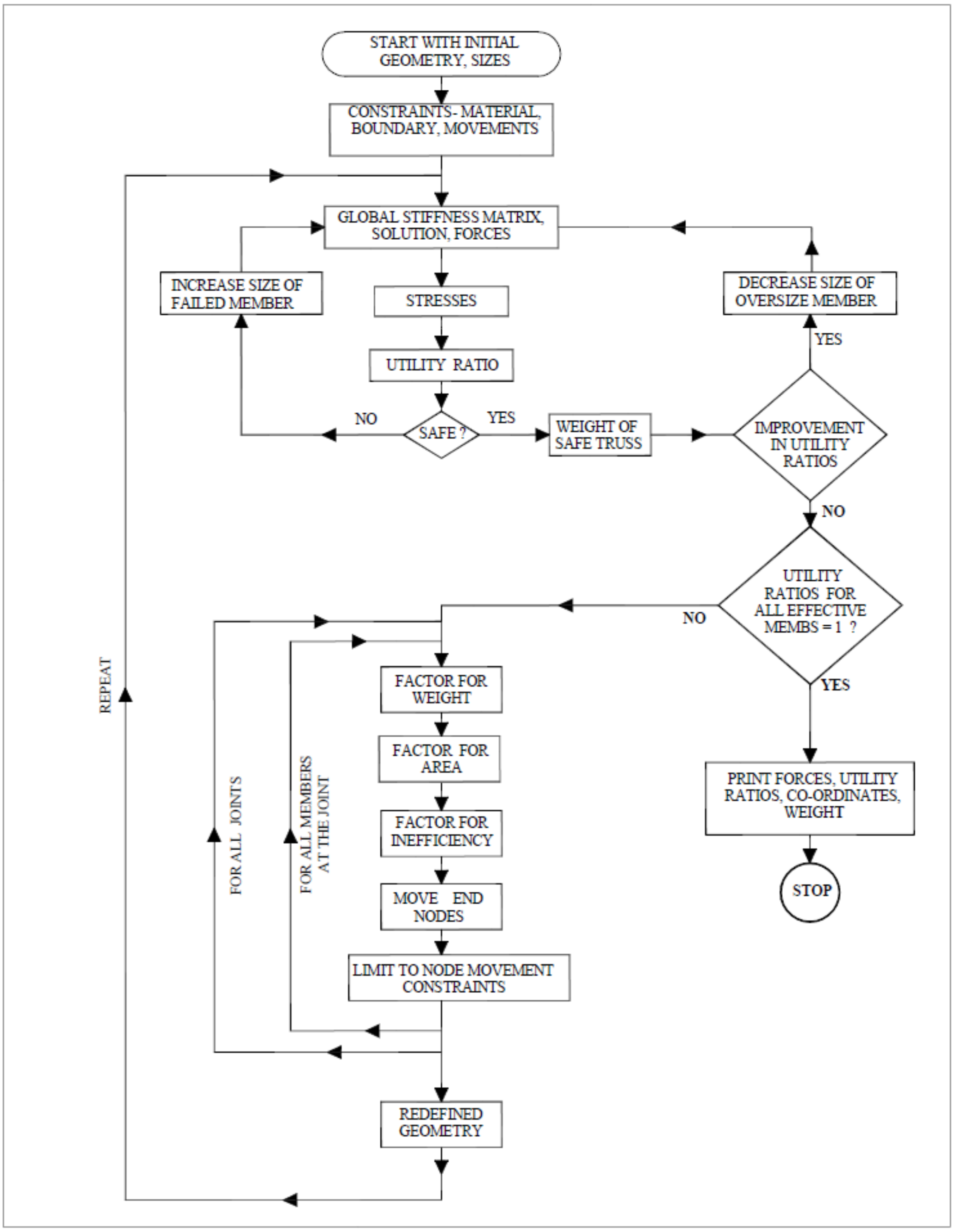
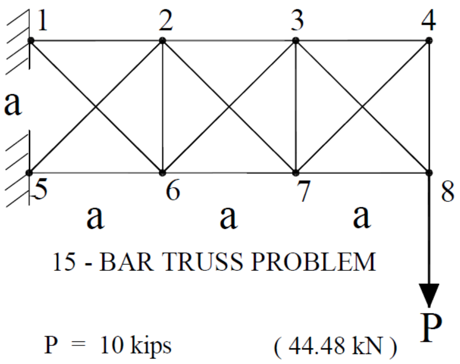


Fig. 5.7 Flowchart for Nodes -in -Motion Strategy for 2D Trusses

5.9.11 Illustrative Example

The 15 bar truss problem, shown in Fig.5.8 , solved by many researchers [Tang, et. al (2005), Rahami et al (2008), Gholisadeh (2012), Kulkarni et.al, 2012)] has been treated as benchmark to check the efficiency of the optimum nodal position search algorithm. The optimum design is to be achieved with the properties and movement restrictions stated in Table 5.1



P = 10 kips (44.48 kN)
a = 120 in (3048 mm)
Unit Weight = 0.1 lb/ cu. in (27.15 kN/ Cu. m)
E = 10000 ksi (68947.6 N/ Sq. mm)
Limiting Stress = 25 ksi (172.0 N / Sq. mm)

Fig. 5.8. Benchmark Problem

Table 5.1. Constraints for Nodal Movements

JOINT MOVEMENT CONSTRAINTS						
JOINT No.	**CO-ORDINATES**		**Permissible Freedom**			
	X	**Y**	**Min X**	**Max X**	**Min Y**	**Max Y**
1	0	120	0	0	120	120
2	120	120	100	140	100	140
3	240	120	220	260	100	140
4	360	120	360	360	50	90
5	0	0	0	0	0	0
6	120	0	100	140	-20	20
7	240	0	220	260	-20	20
8	360	0	360	360	20	60

Additional Conditions : $X_6 = X_2$, $X_7 = X_3$, $X_8 = X_4 = 360$

Table 5.2 shows the set of length, Area and utility ratio for every member at the instance of optimum design. It is noted that Nodes in Motion Strategy clearly identifies member numbers 3,7,8,9 and 15 as ineffective. The optimum topology evolved is shown in Fig. 5.9. The positional changes of the nodes for the optimum configuration have been affected only on the foundation of utility ratio wherein permissible stresses both in tension and compression remain the same, as stated in the benchmark problem

Table 5.2. Final Cross Sectional Areas and Utility Ratios

15 - MEMBER TRUSS - OPTIMUM DESIGN - FORCES ON MEMBERS								
	Length	AREA	WT	FORCE	Type	Stress	Allow. Stress	Uti. Ratio
Memb. No.	in	Sq. in	kips	kips		ksi	ksi	
1	120.0000	0.8671	0.0104	22.060	Tens	25.00	25.00	1.00
2	119.9030	0.7321	0.0088	17.750	Tens	25.00	25.00	1.00
3	123.8110	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
4	120.0000	1.1332	0.0136	-27.940	Comp	-25.00	25.00	1.00
5	119.9030	0.4670	0.0056	-11.910	Comp	-25.00	25.00	1.00
6	121.7860	0.4050	0.0049	-9.840	Comp	-25.00	25.00	1.00
7	120.0000	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
8	120.3080	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
9	70.0000	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
10	169.7060	0.4700	0.0080	11.230	Tens	25.00	25.00	1.00
11	169.7060	0.0951	0.0016	-2.910	Comp	-25.00	25.00	1.00
12	169.7860	0.0960	0.0016	3.280	Tens	25.00	25.00	1.00
13	169.7060	0.4710	0.0080	-11.660	Comp	-25.00	25.00	1.00
14	156.3420	0.5200	0.0081	12.580	Tens	25.00	25.00	1.00
15	150.2050	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
TOTAL WEIGHT OF TRUSS			0.07066	kips				

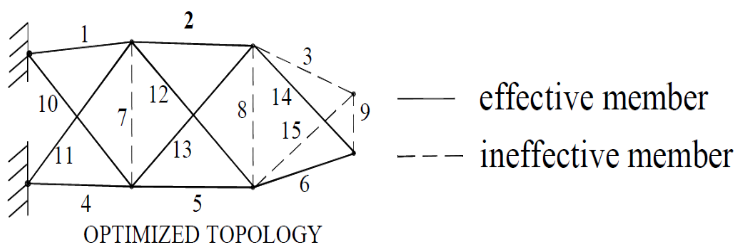


Fig 5.9. Configuration for Minimum Weight

5.9.12 Comparison

Table 5.3 shows the sizing and layout variables obtained by this method in comparison with the results given in references. It is seen that the results obtained are in agreement and are showing a comparable results in optimum design.

5.9.13 Convergence

Fig. 5.10 shows the weight reduction of the truss corresponding to iterations performed.

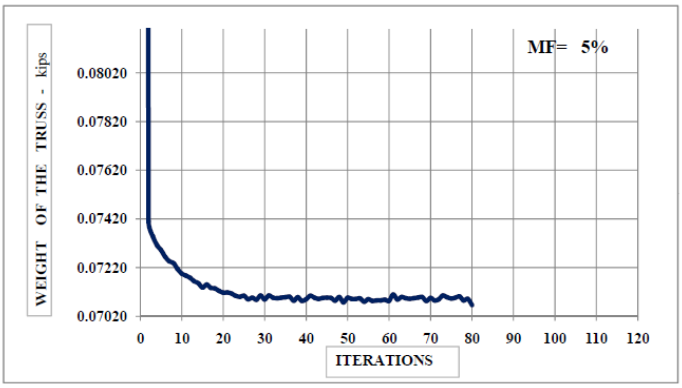


Fig 5.10 Graph Showing Weight reduction of Truss

Table 5.3. Comparison of Results for Benchmark Problem.

15 - MEMBER TRUSS - OPTIMUM DESIGN - COMPARISON OF RESULTS					
	EARLIER WORKS [Kulkarni et.al (2012)]				
MEMBER AREA	Gholizadeh et. al.(2012)	Tang et. al. (2005)	Rahami et. al. (2008)	Kulkarni et. al. (2012)	Present Work
Sizing variables (in.^{2})					
A1	0.954	1.081	1.081	0.954	0.8671
A2	0.539	0.539	0.539	0.539	0.7321
A3	0.27	0.287	0.287	0.111	0.0000
A4	1.081	0.954	0.954	0.954	1.1332
A5	0.539	0.954	0.539	0.539	0.4670
A6	0.174	0.220	0.141	0.347	0.4050
A7	0.111	0.111	0.111	0.111	0.0000
A8	0.111	0.111	0.111	0.111	0.0000
A9	0.44	0.287	0.539	0.111	0.0000
A10	0.44	0.220	0.440	0.440	0.4700
A11	0.347	0.440	0.539	0.44	0.0951
A12	0.22	0.440	0.270	0.174	0.0960
A13	0.22	0.111	0.220	0.174	0.4710
A14	0.174	0.220	0.141	0.347	0.5200
A15	0.27	0.347	0.287	0.111	0.0000
Layout variables (in.)					
X2	113.65	133.612	101.5775	105.7835	120.000
X3	254.47	234.752	227.9112	258.5965	239.903
Y2	128.97	100.449	134.7986	133.6284	120.000
Y3	115.73	104.738	128.2206	105.0023	120.098
Y4	59.364	73.762	54.8630	54.4546	90.000
Y6	-12.733	-10.067	-16.4484	-19.9290	0.000
Y7	3.5467	-1.339	-13.3007	3.6223	-0.211
Y8	59.29	50.402	54.8572	54.4474	20.000
Weight (lbs)	**73.93**	**79.820**	**76.6854**	**72.5152**	**70.660**

Optimum

5.9.14 The effectiveness of the method for 2D Trusses

The Nodes-in-Motion Strategy formulised for 2D trusses presented and demonstrated clearly has an edge over other techniques discussed. The method is appealing as convergence is faster and the search for optimum shape, topology and member sizing is accomplished simultaneously. Suitable modifications to element stiffness matrix, extends its usage potential to 3D trusses too.

5.10 USE OF NODES – IN - MOTION STRATEGY TO 2D ELASTIC CONTINUUM

A Plate is a continuum for which one dimension is very small compared to the other two dimensions. It is a surface with a small thickness. The Finite Element Analysis of Plates, has been dealt in depth in Chapter 3. Though both the 2D Truss and plate are dealt as 2 Dimensional, the analysis & design of plates varies distinctly from that of trusses in the following manner.

A truss is a frame work of linear members. The geometry of a truss is well defined with one dimensional members connected to nodes known as joints. The member connectivity is well defined in the problem itself. There is no need of 'discretization' of the structure further for analysis and the results of the analysis are accurate. There are distinct paths of transfer of forces from the loading points to the supports. The direction of forces acting at a joint are well defined and solution can directly obtained with equation of equilibrium applied at the joints.

A Plate is a 2-Dimensional continuum. It is defined by an external boundary with optional definition of holes punched in it. There is no well defined system of elements and node definitions before the analysis is attempted and hence discretization is a must. Such discretization will lead to two dimensional elements, the size (big, small or tiny) and configuration (triangular, rectangular or polygonal) are left to the discretion of the problem solver.

Though there is difference in concept, analysis and implementation , the Nodes- in -Motion Strategy has been identified as a fast converging and efficient technique with a great potential applicable to all types of structures. Thus, this node based methodology has been extended to 2D elastic continuum like plate, detailed in Chapter 6.

Chapter 6

STRUCTURAL OPTIMIZATION OF PLATES – COUPLING MPMLS METHOD AND NODES-IN-MOTION STRATEGY

6.1 GENERAL

Moving Polynomial Moving Least Square Method for Stress smoothing and Nodes – in - Motion strategy for optimization have been highlighted in the earlier Chapters 4 and 5 respectively. A synthesis of these two has been formulated and extended as an optimization tool for 2D elastic continuum, details of which are presented in the sections that follow.

6.2 PROBLEM FORMULATION

The objective of optimization procedure in plates is to minimize the weight of the plate for a given set of loading and boundary conditions, by allowing changes in shape and material disposition

6.2.1 Objective Function

The objective of optimization is to reduce the weight of the plate to a minimum, subjected to the given constraints. Search for a plate configuration of minimum volume, subject to stress constraints, is attempted.

6.2.2 Design Variables and Constraints

Thicknesses at various locations and nodal coordinates are treated as design variables

Constraints are :

a) The stresses in material at any location in the plate should be within the permitted limits

b)The movement of node,support ,boundary or load points is unrestricted or may be restricted to limits prescribed.

6.2.3 Mathematical Representation

Minimize

W , Total weight of the plate, subjected to

$$\sigma_{vms} <= \gamma \ \ x\ \sigma_y \qquad \text{------ (6.1)}$$

Where γ is the level permitted to be attained.

and **xmin ≤ x ≤ xmax, ymin ≤ y ≤ ymax,** for all points (x,y)

6.2.4 Implementation of Objectives

The objective are attained in step by step procedure through the development of

1. A generalised procedure for the analytical solution for stresses for a any structure with given geometry, loading and support conditions
2. An algorithm to assess the utility ratio of the material used, based on theories of failure
3. Trials for the reduction of material consumption based on established strategies
4. Re-defining the geometry of the system and repeating the procedure to refine the stresses.
5. A pre-defined criterion clearly dictating the termination of the repetition of the procedure.

6.3 STRUCTURAL ANALYSIS

6.3.1 Importance of Structural Analysis

Structural analysis is the vital part of the overall design optimization task because one needs to predict the structural behaviour for various trial designs in order to guide and improve the design process.

While the overall concept of design of a structure is a complex process related to creativity and vision, structural analysis is related to the knowledge of science and established mathematical formulations. The significant progress in structural optimization research is a parallel process in the field of structural analysis, mathematical programming, geometrical modelling and the ever increasing speed of computational capabilities of digital computations. Methods and formulations have been developed to assess and verify the capabilities of materials as a part of a structural system for their optimum use.

6.3.2 Structural Analysis of 2D Plates as a part of Optimization Process

Since the efficiency of a plate is measured in terms of the exploited capacity of material used, it is necessary to determine the actual stresses developed in the material and compare it with the maximum permitted as per theories of failure. The generalized FEA procedure to obtain the maximum stresses in triangular elements in plates subjected to in in-plane bending has been outlined in Chapter 3.

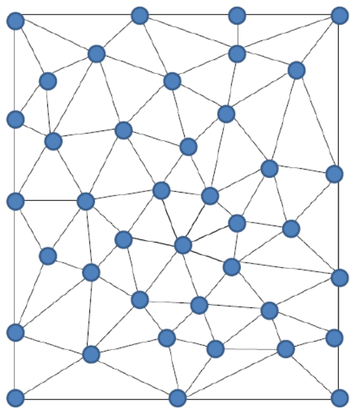


Fig. 6.1 The descritization of Plate into Elements and Nodes

6.3.3 Utility Ratio

To evaluate the utility of material, we define a factor called utility ratio defined as

$$\text{Utility ratio } \mathbf{U} = \sigma_{vms} / (\gamma \times \sigma_y) \qquad \text{------ (6.2)}$$

Thus utility ratio **U,** is the ratio of the actual von Mise's stress developed to the maximum permitted . The utility ratio acts as a measure of efficiency of material at a location and acts as the guiding factor is removing a part of material or depositing additional material at the location.

If the von Mise's stress developed in the material equals the permissible stress at a location, then the capacity of the material is said to be fully exploited. Hence utility ratio at a location acts as measure of utility of material.

Thus

Utility ratio < 1, the material is under exploited

Utility ratio = 1, the material is fully exploited

Utility ratio > 1, the material is stressed beyond the limits and it is unsafe.

6.4 CONCEPT OF OPTIMUM DISPOSITION OF MATERIAL

Generally, the optimized solutions of plate like structures are attempted as a material disposition optimization. Optimum material disposition means the removing material from locations where it is not efficiently used and shifting/ increasing the material where it is best used. The quantity of material at a location may be increased by expanding the boundaries or depositing the material. Similarly, it can be decreased by shrinking the boundaries or removing the material at that location.

Shrinking or expanding the element directly affects the shape of the plate depositing or removing the material affect the thickness at the location. Hence by aiming optimum disposition is a simultaneous shape and thickness optimization of plates. This method leads to the best geometry of the plate for perfect load path and best surface profile for the material utility. In this work, this double edged approach is used to optimize plates subjected to a given set of loads and support conditions.

The optimization procedure involves successive remodelling of the structure based on strategies decided. Nodes are moved towards relatively more stressed areas thus redefining the geometry giving rise to a more optimum shape in every iteration. This also gives rise to finer meshes at heavily stressed areas, for more accurate analysis in the successive iterations.

6.5 DETERMINATION OF STRESSES AT NODES

Determination of stress values at the nodes is an important step in the guiding movement of nodes. The stress at the nodes are obtained based on the stresses in the neighbouring elements using fitting and interpolating functions. The newly developed Moving Polynomial Moving Least Square (MPMLS) method , detailed in chapter 4 , is used here for stress smoothing along the boundary of influence area of the nodes and at nodes.

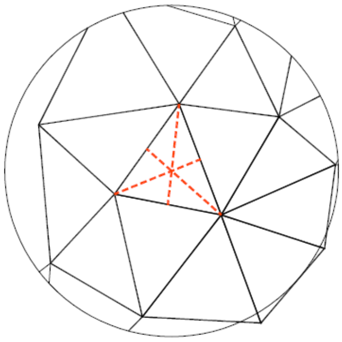


Fig. 6.2 Centroid of a Triangular Element

von Mises Stress values obtained for every element from FEA , is uniform throughout the element and is assumed to be prevailing at the centroid of the element. Centroids of all neighbouring elements of a node are the data points in 2D, with stress values available, to smoothen.

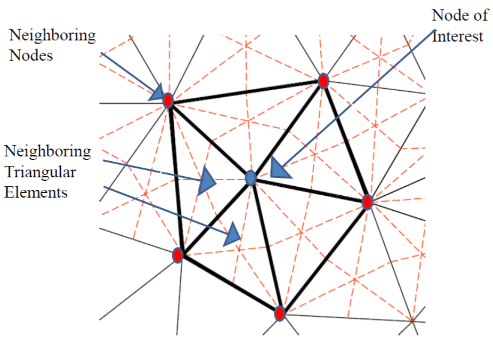


Fig. 6.3 Neighboring Elements

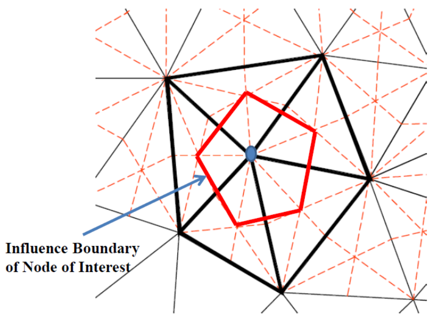


Fig. 6.4 Influence boundary of a Node

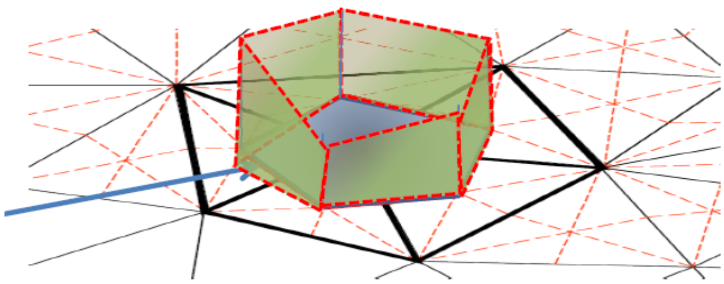


Fig. 6.5 Smoothing along Polygonal Boundary

A polynomial is fitted with number of terms equal to the number of neighbouring elements using weighted Moving Least Square method and compute the stress values along the boundary. Use weighted average to find the stress at the node.

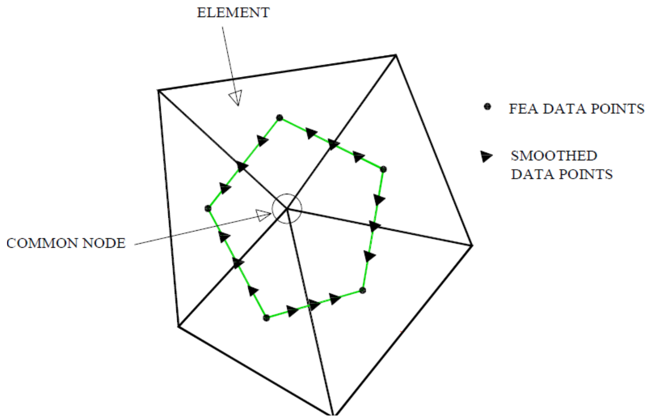


Fig. 6.6 Creation of additional data points for Interpolation

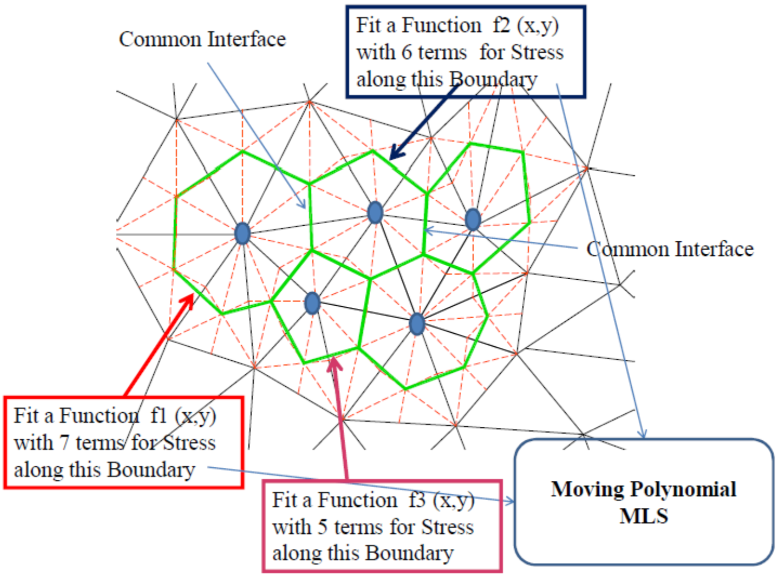


Fig. 6.7 Determination of von Mise’s Stress at Nodes

The process is repeated till the von Mise’s stresses are computed at all the nodes in the entire domain. These stresses are used to evaluate the worthiness of the material at the location.

Smoothing functions are generated only for the internal node locations. The external boundary and internal boundary nodes are likely have neighbouring elements less than 3 as polynomial in 2D MLS has minimum 3 terms and 3 coefficients. At these nodes, simple averaging is done to get the stresses .

6.6 RELOCATION OF NODES

Relative movements of nodes change the global co-ordinates of the nodes and affect the size and orientation of elements.

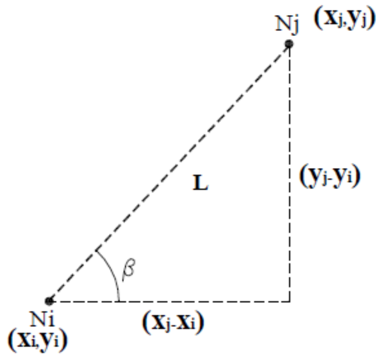


Fig. 6.8 Relative Locations of Nodes

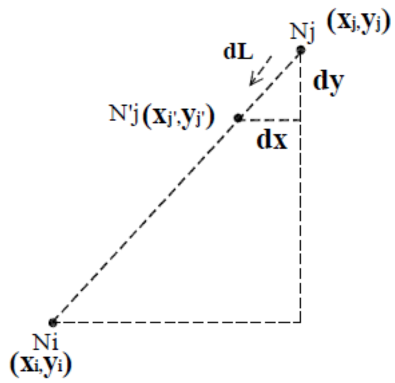


Fig. 6.9 Movement of Nodes

If a node **Nj** (x_j,y_j) is moved with respect to **Ni** (x_i,y_i) by an amount **dL**, a fraction of the total distance **L** between them,

$$dL = MF \times dL \qquad \text{------ (6.3)}$$

Where **MF** is the Modification Factor depicting the fraction of movement. . At a time, a relative movement upto 10 % of the distance between the nodes is considered for the problems attempted.

Referring to Fig. 6.8,

$$L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \qquad \text{------ (6.4)}$$

$$dx = MF \times (x_j - x_i) \qquad \text{------ (6.5)}$$

$$dy = MF \times (y_j - y_i) \qquad \text{------ (6.6)}$$

Similarly, The amount of push or pull is to be decided based on the difference between the utility ratios. The more the difference, the more should be the movement.

Hence the equations (6.5) and (6.6) are modified to accommodate the direction of moment and the amount of movement of node **Nj** as

$$dx = C1 \times MF \times (x_j - x_i) \qquad \text{------ (6.7)}$$

$$dy = C1 \times MF \times (y_j - y_i) \qquad \text{------ (6.8)}$$

Where

$$C1 = Ui - Uj \qquad \text{------ (6.9)}$$

The coefficient **C1** determines the direction as well as the amount of relative movement of **Nj** with respect to **Ni.**

The New Co-ordinates of **Nj** are now obtained as

$$x'_j = x_j - dx \qquad \text{------ (6.10)}$$

$$y'_j = y_j - dy \qquad \text{------ (6.11)}$$

The possibility of movement of the new location is checked with respect to the constraints dictated in the problem definition. If the location is feasible, the node **Nj** is moved to the new location.

6.7 MATERIAL RE-ALLOCATION

The Principle of optimum disposition of material operates on the premise of material re-allocation to zones where it is most effective. From initial configuration, based on utility ratios, the routine decides material shift direction for accomplishment of maximum utility ratios. Element size expands or contracts based utility ratios at nodes and thickness swells or shrinks element location. It is the location that matters and Utility ratio of the element decides its fate.

6.7.1 Location of Ineffective Zones Based on Utility Ratio of Elements.

The von Mise's stress determined for the element is assumed to be prevailing at the centroid of the element and the corner nodes are moved based on the utility ratio **Ue** at the centroid of element**.** The thickness of the elements in which the material is not exploited, is reduced based on **Ue**. The more the value of **Ue**, near to unity, the less is the reduction. However, it is worth to note that no element will develop a **Ue > 1** as the routine developed will always ensure a safe design before optimization is attempted.

Extending this concept further, if the area nearby too is less efficient, the element may be extended to this region too, before reducing the thickness, thus enhancing the efficiency of the formulation.

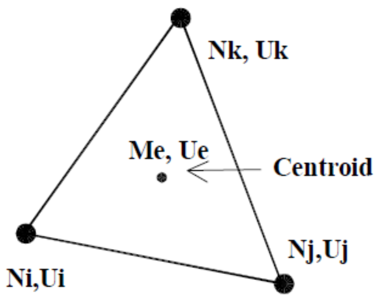


Fig. 6.10 Nodes of a Triangular Element

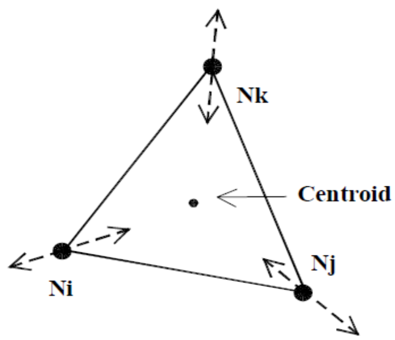


Fig. 6.11 Movement of Corner Nodes

If interpolated utility ratio at a corner of an element is relatively very less compared to the utility ratio at the centroid of the element, it is quite likely that the effectiveness of the zone is reducing in the direction of that corner. Hence the zone of removal of material may be extended in that direction.

This is accomplished by moving the corner nodes with respect to the centroid of the element, based on the relative values of the Utility ratio of the corners with respect to the utility ratio at the centroid as shown in Fig 6.11. Thus, if **Ucorner** < **Ue** then the corner is pushed out otherwise, the corner is pulled in.

6.7.2 Location of Ineffective Zones Based on Utility Ratio at Nodes.

The utility ratios at the nodes are indicators of effectiveness of zones. Increase and decrease of effective and ineffective zones can be accomplished by relative movement of neighboring nodes. Referring to Fig . 6.12, where Node **Ni** with a utility ratio **Ui** has 6 neighboring Nodes **N1,N2,....N6** with Utility Ratios **U1,U2,...U6** respectively, the neighboring nodes can be pulled or pushed with respect to Ni to reduce or increase the zones, based on the utility ratios.

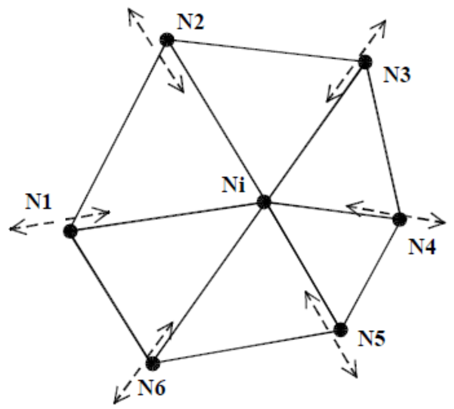


Fig. 6.12 Interior Node and Its Neighboring Nodes

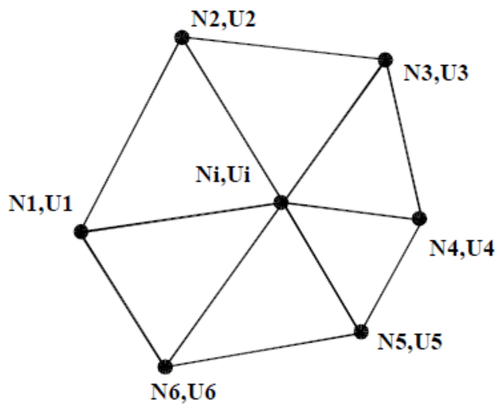


Fig. 6.13 Relative Movement of Neighboring Nodes

The push or pull is decided by the relative values of utility ratios **Ui** and **Uj** with **j** varying from 1 to 6. **Nj** is pulled in if **Ui < Uj** and pushed out if **Ui > Uj**.

6.7.3 Thickness Reduction at Ineffective Zones

Thickness of an element is revised on the following formula, if we try to achieve at least 90 % efficiency through thickness reduction.

If $1 >= U_e >= 0.9$ then $t_{new} = t_{old}$ ------ (6.12)

Else

$t_{new} = t_{old} - (0.9 - U_e) * t_{old}$ ------ (6.13)

eg., if $t_{told} = 6$ mm, $U_e = 0.6$,

$t_{new} = 6 - (0.9 - 0.6) * 6$

$t_{new} = 6 - (0.9 - 0.6) * 6 = 4.2$ mm

6.8 SOLUTION TECHNIQUE

The following step by step procedure is adopted to achieve the optimized shape and material disposition of material in the plate

Step 1: The geometry of the plate, Details of co-ordinates of nodes, triangular elements and their connectivity are given

Step2: The thickness of elements, loading , support conditions and material properties are given.

Step 3: The thickness given is treated as the intial thickness of elements to start with.

Step 4 : Do a Finite Element Analysis to compute the von Mise's stresses and the utility ratios of the elements as Stated in Chapter 3, moduled as in Fig. 6.14.

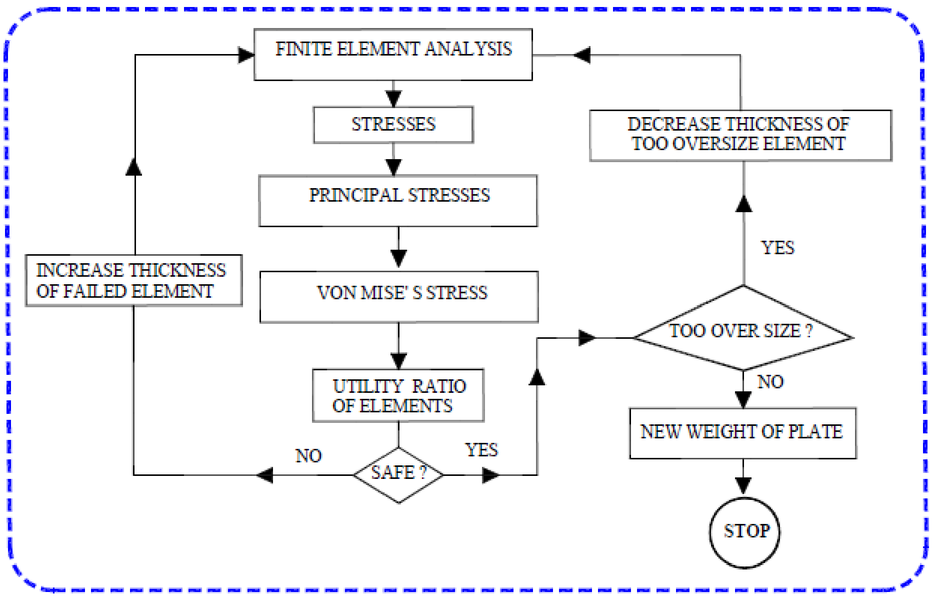


Fig. 6.14 Flowchart for Von Mise's Stress Module

Step 5: The thickness of elements are revised to safer limits if the stresses exceed the given values and procedure is repeated from step 4 till a safe design is achieved.

Step 6 : The weight of the plate is computed.

Step 7: Stress smoothing is carried out to compute the von Mise's stresses and utility ratios at all the nodal positions by Moving Polynomial Moving Least Square technique developed in Chapter 4. The module as shown in Fig. 6.15

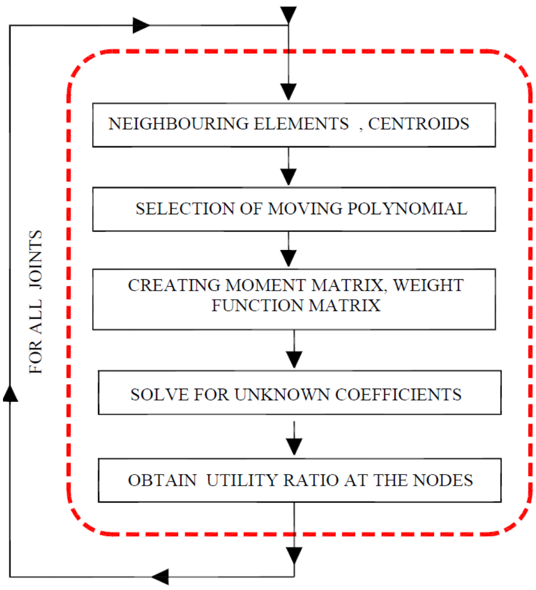


Fig. 6.15 Flow Chart for Stress Smoothing Using MPMLS

Step 8: Every node is relocated as per the relative values of utility ratios such that it attains a new position towards a better utility ratio as discussed in Chapter 5. This module is shown in Fig. 6.16.

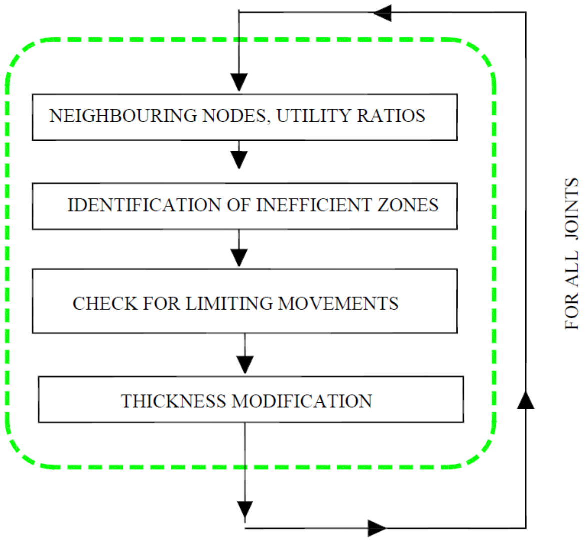


Fig. 6.16 Flowchart for Optimum Material Disposition

Step 9: Procedure is repeated from Step 3 ,till the decrease in weight is less than or equal to 2% of the earlier weight or the number of iterations specified by the user are completed. The overall flow chart, combining all the modules in given in Fig.6.17

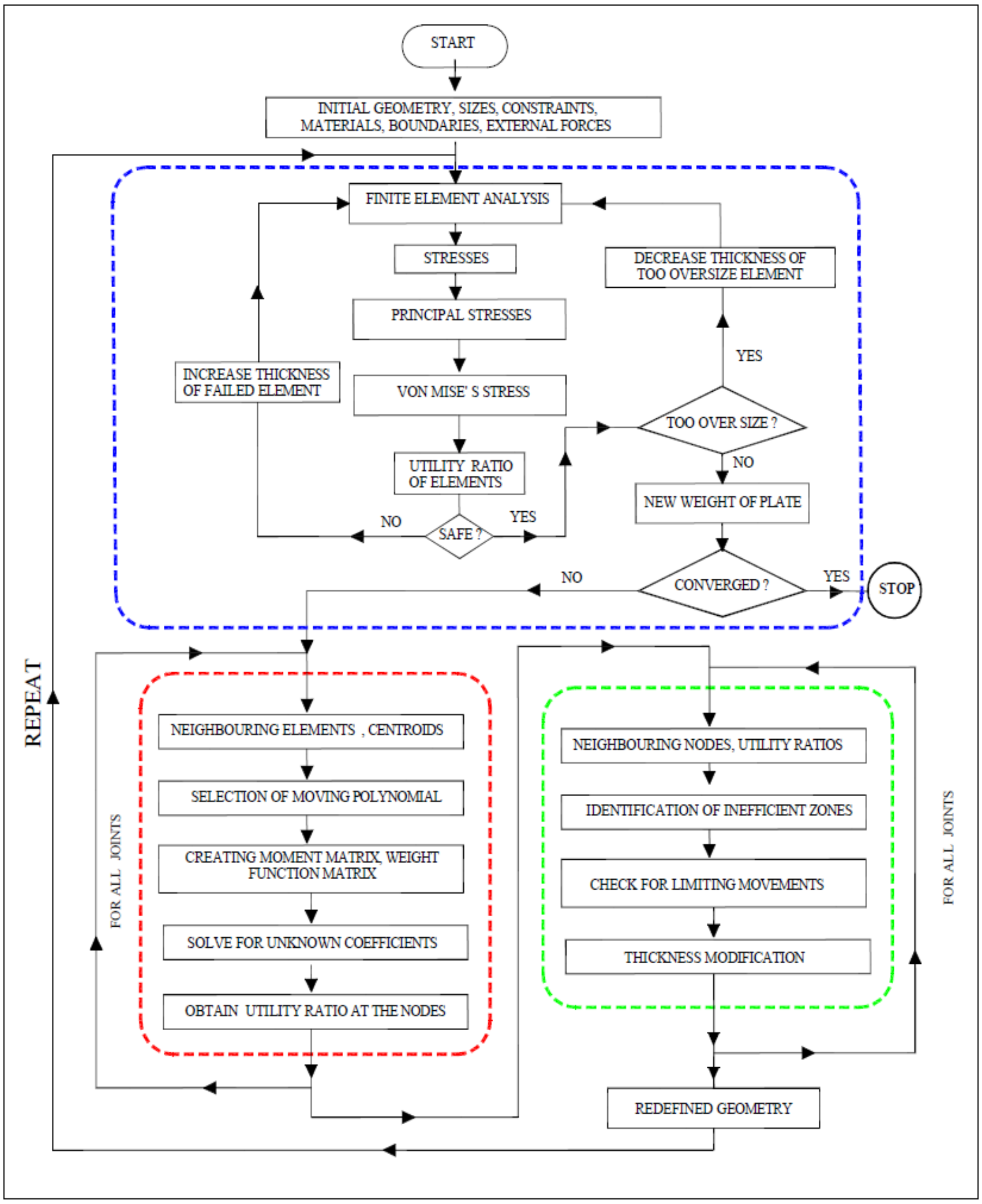


Fig. 6.17 Combined Flowchart for Optimum Material Disposition of Plates with MPMLS Method and Nodes- in Motion Strategy

6.9 ILLUSTRATIVE EXAMPLES

A few problems have been solved to verify the efficiency of the methodology developed.

6.9.1 Illustrative Example 1.: A plate, simply supported, subjected to a concentrated load at the centre. (Kim.H, 2000). In the reference, this problem has been solved using an Evolutionaly Structural Optimization Method (ESO) named Intelligent Cavity Creation (ICC). This problem has been solved with an intention to verify the evolution of an optimum shape called Michell Truss,(Fig. 6.19) considered to be the most optimum topology for the problem.

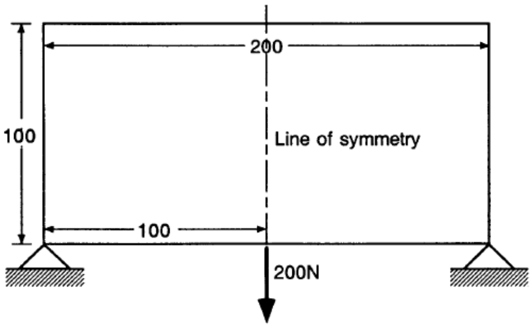


Fig.6.18 Simply Supported Plate with a Concentrated Load at Mid Span (Kim H, 2000)

Fig.6.19 Michell Truss Solution for Most Optimum topology for the Plate (Kim. H, 2000)

Solution:

The methodlogy developed in the current research work has been employed by segmenting the plate into 60 Finite Elements with 44 Nodes as shown in Fig. 6.20 .

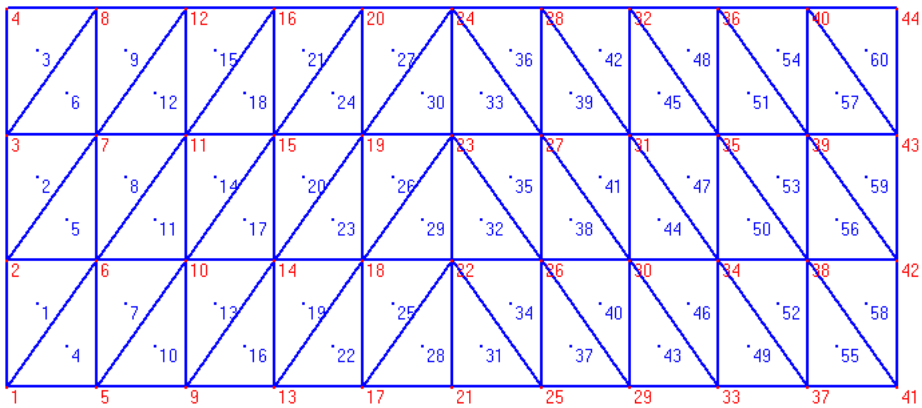


Fig.6.20 FEA Analysis of the Problem

The results obtained at various phases have been presented in Figures from 6.21 to 6.24 and they show the agreement of the shape refinement with that reported in Literature in Fig. 6.19 (Kim H, 2000).

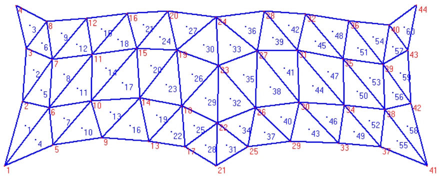


Fig.6.21 After 5 Iterations

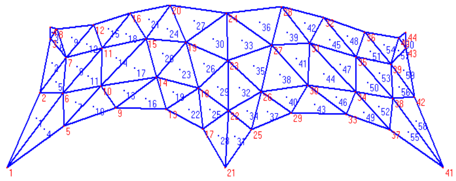


Fig.6.22 After 15 Iterations

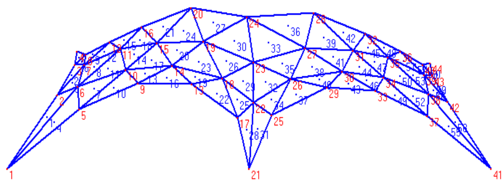


Fig.6.23 After 45 Iterations

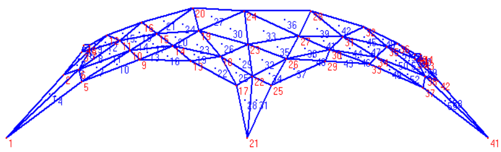


Fig.6.24 After 75 Iterations

6.9.2 Illustrative Example 2: A Cantilever beam made of plate is subjected to a concentrated load at the free end. (Helio,2014).

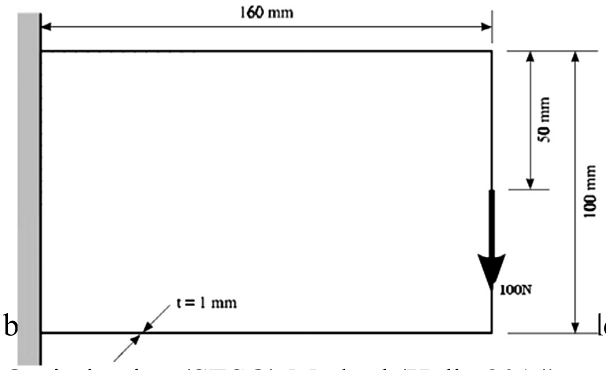


Fig.6.25 Cantilever Beam Subjected to Concentrated Load at the free end.

b led Smooth Evolutionary Structural Optimization (SESO) Method (Helio,2014).

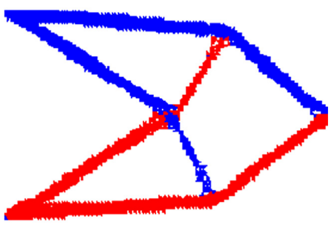


Fig. 6.26 Solution in the reference.

Solution:

The domain is divided into 32 triangular elements with 25 Nodes As shown in Fig. 6.27

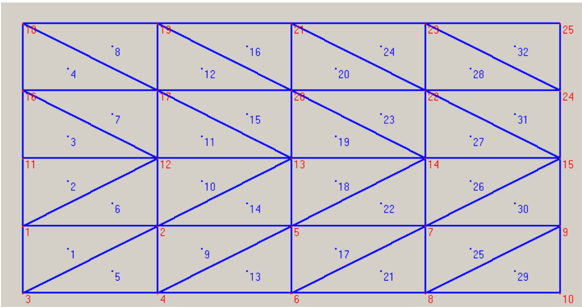
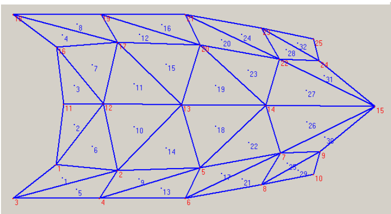
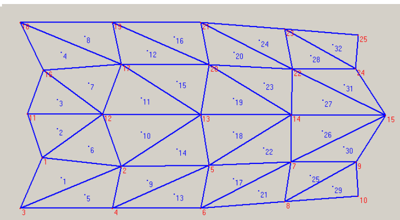


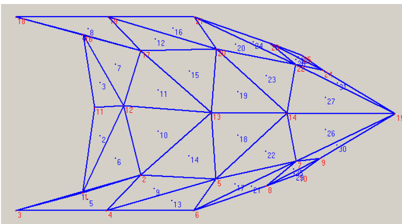
Fig.6.27 Discretization of the Cantilever Plate



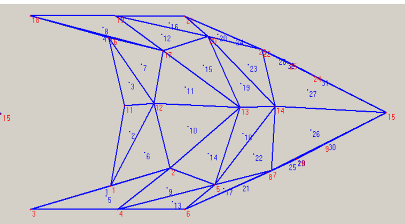
a) 5 Iterations



b) 30 Iterations



c) 50 Iterations



d) 90 Iterations

Fig. 6.28 Solution to Cantilever Problem

The Solution obtained adopting MPMLS and the Nodes-in- Motion strategy has been compared to the one obtained in reference (Heli, 2014) as in Fig. 6.26 The re-allocation of material to zones where it is most effective is clearly evident from the results at the end of various iterations presented in Fig. 6.28.

6.9.3 Illustrative Example 3.

Arriving at the best shape and material disposition for a spanner to apply 5000 N with an initial plate geometry assumed in Fig. 6.29 has been attempted and the results of shape optimization are presented in Fig.6.31 to 6.35.

Solution

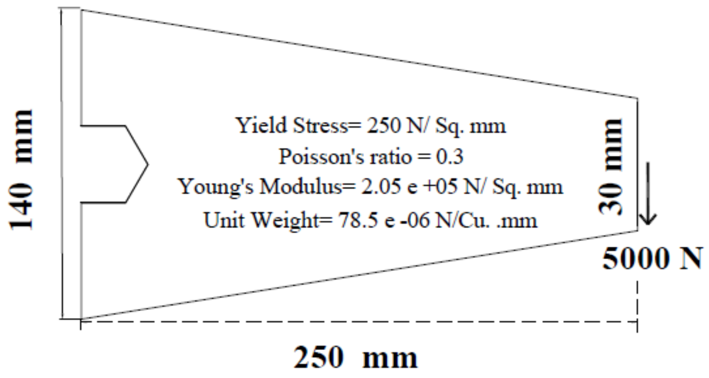


Fig. 6.29 Initial Geometry Assumed for the Design of Spanner

The permitted von Mise's stress is assumed as 0.6 times yield stress, as 150 MPa. The solution is attempted with an initial geometry as shown in Fig. 6.29. A uniform thickness of 8 mm is assumed to start with the procedure of optimization. The Initial weight of the spanner is 12.65 N

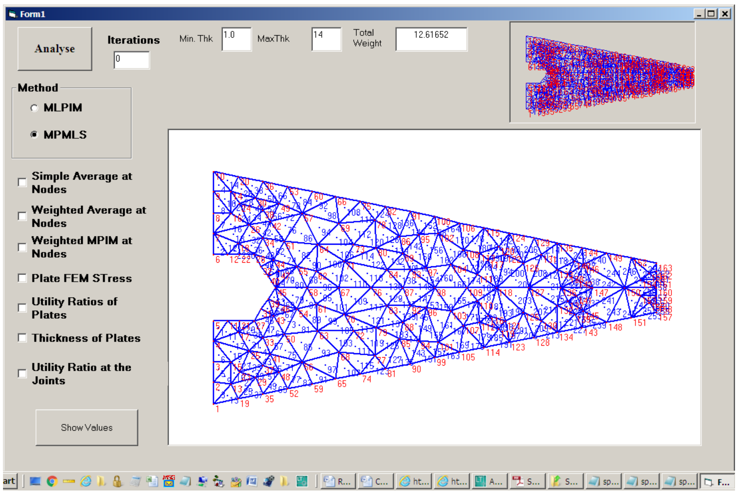


Fig. 6.30 Modelling of Spanner - Initial Weight 12.65 N

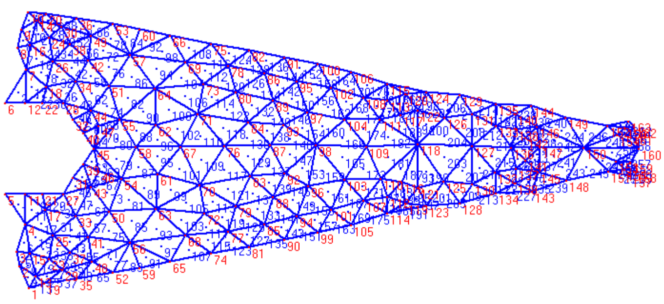


Fig. 6.31 5 Iterations - Weight 10.54 N

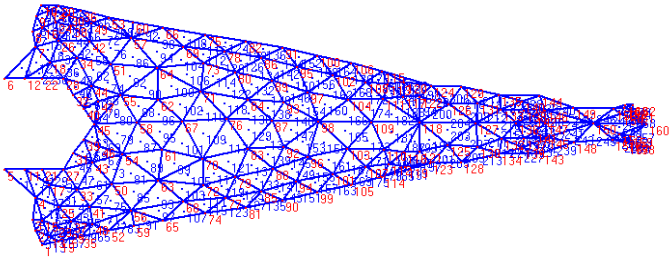


Fig. 6.32 15 Iterations - Weight 8.06 N

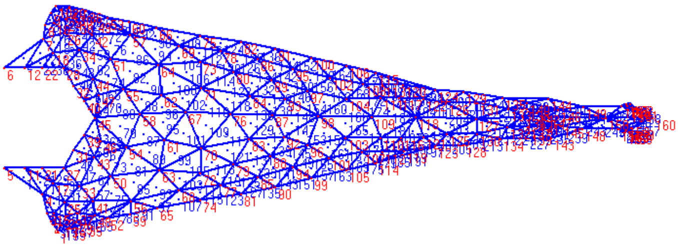


Fig. 6.33 30 Iterations - Weight 6.26 N

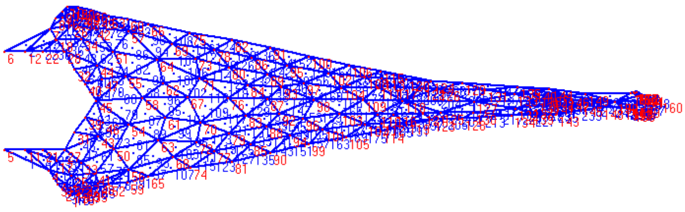


Fig. 6.34 50 Iterations - Weight 5.05 N

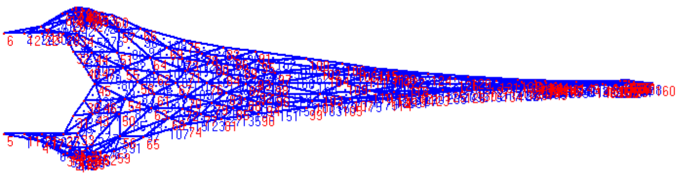


Fig. 6.35 102 Iterations - Weight 3.65 N

Convergence criteria with less than 2% variation in the weight compared to that of the earlier iteration has terminated the process of optimization in 102 iterations. Fig. 6.36 shows the details of weight reduction accomplished.

It is evident from the trend that drastic reduction is weight has happened in the earlier iterations as indicated by the steep slope of the graph and the drop in the steepness of the curve with increased number of iterations. The graph almost becomes flat as the termination criterion approaches indicating that the formulation is stable and converging.

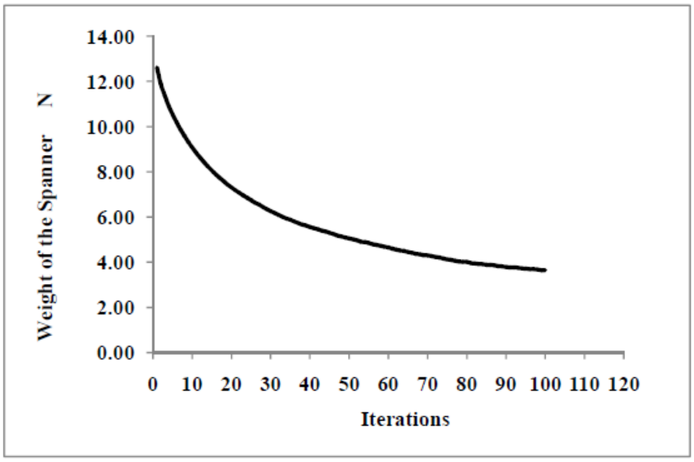


Fig. 6.36 Convergence to the Minimum Weight

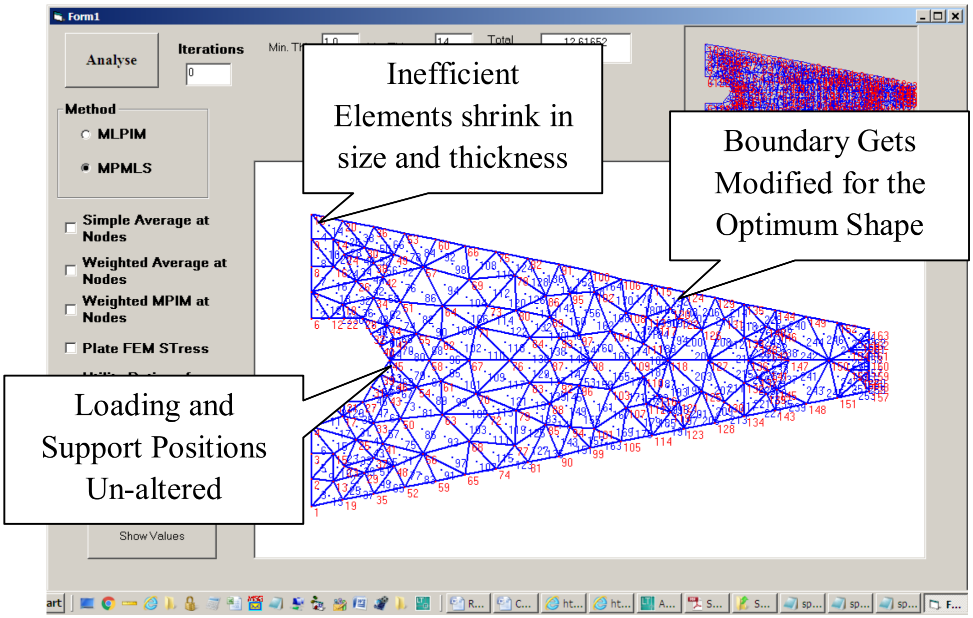


Fig. 6.37 Material Movement through Movement of Nodes

The iterative procedure carries out continuous refinement of shape of plate through shift of nodal positions, as shown in Fig.6.37, keeping the loading points and support points unmoved. The concept of relative utility ratios of elements moves the underutilised material.

In the process of optimization, the sizes and thickness of element No. 108 marked in the Fig 6.38 , as an example , automatically get refined as per the stress gradient. It also undergoes refinement in terms of location of corner nodes. As the solution reaches convergence, the element utility ratio converges to Unity, indicating the optimum material requirement at the location and its spread denoted by the location and area of the element.

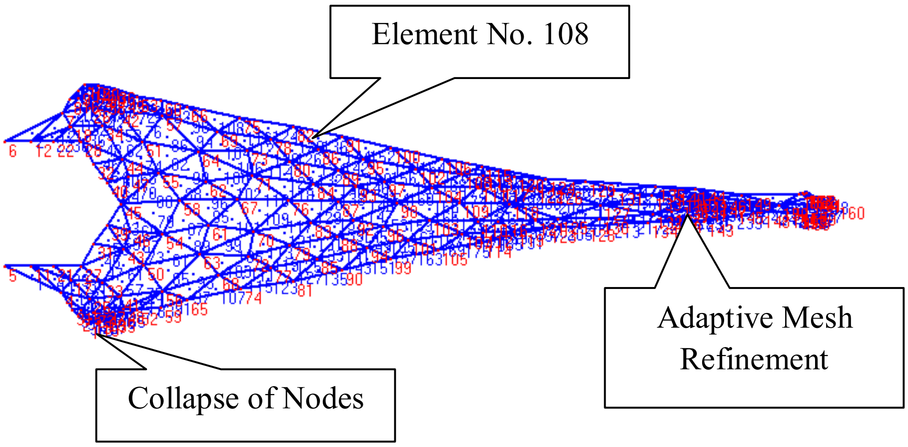


Fig. 6.38 Dynamic Mesh Refinement

Table 6.1 Refinement of Element 108

CONVERGENCE OF ELEMENT No. 108				
Iteration No.	Area Sq.mm	Stress Mpa	Utility Ratio	Thickness mm
0	109.00	53.40	0.36	8.00
5	76.44	66.32	0.44	7.98
10	59.44	76.51	0.51	7.96
15	50.22	85.02	0.57	7.94
20	44.61	92.97	0.62	7.90
25	40.90	100.56	0.67	7.84
30	38.56	107.97	0.72	7.77
35	37.21	114.74	0.76	7.71
40	36.30	121.10	0.81	7.66
45	35.76	127.05	0.85	7.61
50	35.56	132.91	0.89	7.53
55	35.66	138.05	0.92	7.48
60	35.97	142.97	0.95	7.43
65	36.45	146.69	0.98	7.43
70	36.97	149.17	0.99	7.70
75	36.94	149.23	0.99	8.25
80	36.84	149.77	0.99	8.85
85	36.86	148.04	0.99	9.82
90	36.87	149.26	0.99	10.17
95	36.84	149.06	0.99	10.90
100	36.81	148.66	0.99	11.68

The convergence of the properties of element 108 is tabulated in Table 6.1and Figure Nos 6.39, 6.40, 6.41 and 6.42 show the convergence graphs for the Area, Stress, Utility Ratio and Thickness, respectively of the element through the iterations. It may be noticed that the permitted stress value of 150 MPa is attained for the element in nearly 100 iterations. Likewise, the material moves to strategic locations based on the stress demand. As the element attains its best location, material is brought into this location, increasing its thickness.

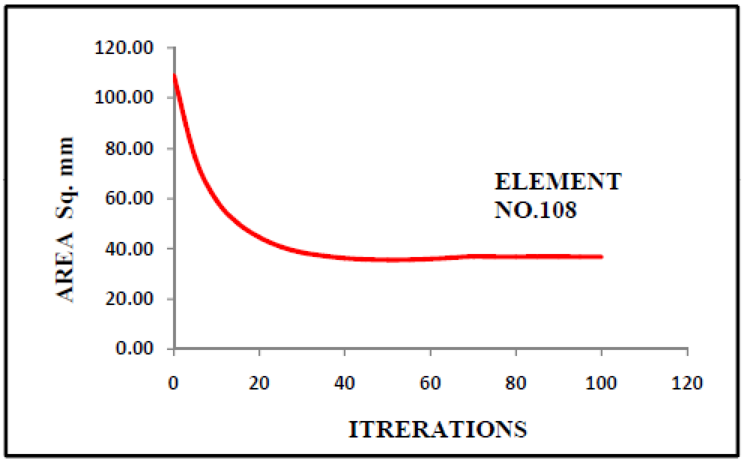


Fig. 6.39 Refinement of Area of Element 108

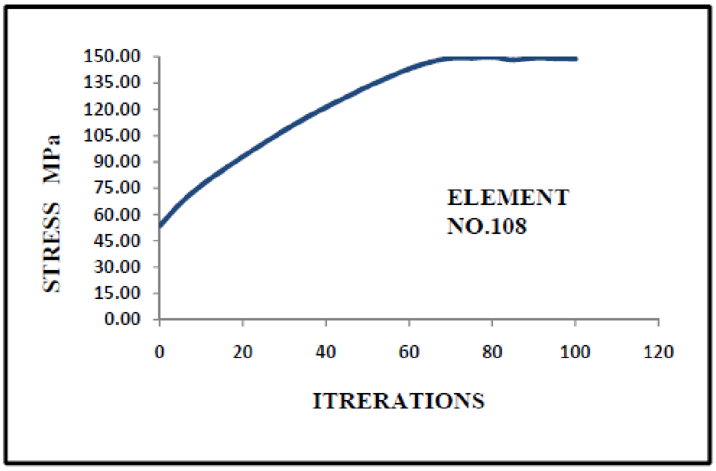


Fig. 6.40 Refinement of von Mise’s Stress in Element 108

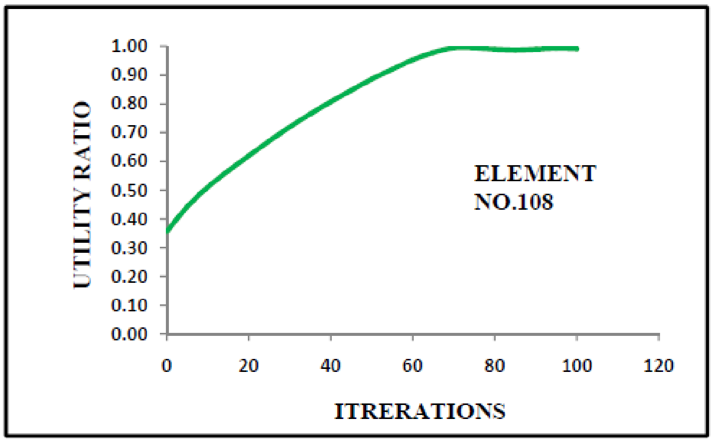


Fig. 6.41 Refinement of Utility Ratio of Element 108

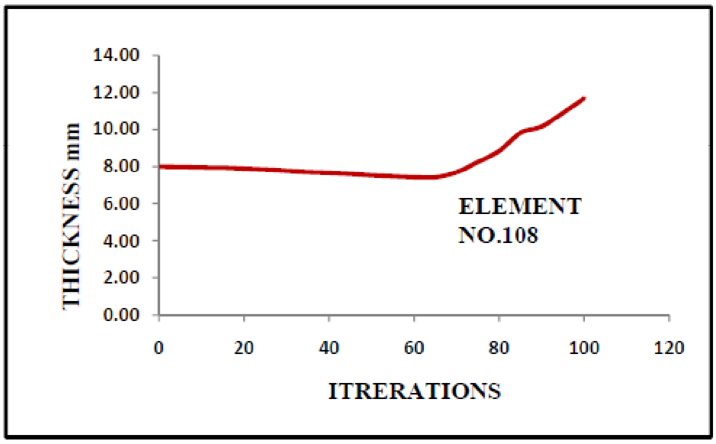


Fig. 6.42 Refinement of Thickness of Element 108

6.10 INFERENCE

The numerical stability, convergence and desired level of accuracy have been demonstrated through illustrations. The mile stones which led to the motivation, conceptualisation, formulation and successful implementation of the Nodes-in-Motion strategy for optimum material disposition in plates are discussed and the accomplishments are discussed in the succeeding Chapter 7.

Chapter 7

RESULTS , DISCUSSIONS AND CONCLUSIONS

7.1 GENERAL

The objective of the present research work has been to conceptualize, develop, implement and illustrate a comprehensive procedure for the optimum material distribution in plates subjected to in-plane bending.

The motivation behind taking up this area of research, the objective and scope of present work have been highlighted in the beginning of this report. A summary of conceptualisation of methodology , step by step achievement of milestones towards the objective, investigations carried out, results obtained, discussion on the results, accomplishments and scope for future developments in the area of research are presented in this Chapter.

7.2 METHODOLOGY

7.2.1 Structural Analysis

Structural Analysis gives an insight into the behaviour of a structure under given loading and support conditions. It is a tool to predict the performance of a structure before it is actually built. Hence any attempt to optimize a structure can not be fulfilled without an efficient and established Structural Analysis procedure.

The plane elasticity theory behind the Finite Element Analysis (FEA) of 2D elastic continuum structures in general and stress analysis of plates subjected to in-plane bending has been discussed in detail in Chapter 3. It has culminated in the development of an efficient code for the determination of stresses in plates. The code has been tested for its accuracy comparing the results given by it against established solutions. The problem of a plane structure given in Fig 3.13 (Green Lee,2010) has been solved and the results have been compared, shown in Table 7.1

Table 7.1 Verification of FEA Code

	Joint No	DISPLACEMENTS X- direction (in inches)	
		RESULTS FROM CODE DEVELOPED	SOLUTION IN REFERENCE
	1	1.9077	1.9080
	2	0.8730	0.8730
	3	0.0000	0.0000
	4	0	0
	Joint No	DISPLACEMENTS Y- direction (in inches)	
		RESULTS FROM CODE DEVELOPED	SOLUTION IN REFERENCE
	1	0.0000	0.0000
	2	-0.7420	-0.7415
	3	0.0000	0.0000
	4	0.0000	0.0000
STRESSES	Element No	DIRECT STRESSES X- direction (psi)	
		RESULTS FROM CODE DEVELOPED	SOLUTION IN REFERENCE
	1	-93.12	-93.00
	2	93.12	93.00
	Element No	DIRECT STRESSES Y- direction (psi)	
		RESULTS FROM CODE DEVELOPED	SOLUTION IN REFERENCE
	1	-1135.59	-1136.00
	2	23.28	-23.00
	Element No	SHEAR STRESSES X- direction (psi)	
		RESULTS FROM CODE DEVELOPED	SOLUTION IN REFERENCE
	1	-62.08	-62.00
	2	-296.62	-297.00

The close agreement of results of the current formulation with that reported in the literature , which has been utilized as the benchmark for comparison , suggests that the code developed works for addressing the objectives of the current investigation.

7.2.2 Stress Recovery at Locations of Interest

During the course towards attaining the objectives, , the importance of determination of stresses at points of interest has been identified. The correctness and accuracy of stress value at locations play a very important role in assessing the utility of material at that location.

The basic drawback of FEA in terms of lack of compatibility at the element interfaces has been overcome by a novel technique of smoothing called Moving Polynomial Moving Least Square (MPMLS) ,which guarantees accurate interpolation based on stress values at surrounding scattered data points. The conceptualization of this technique , the methodology of selection of polynomial terms and techniques to overcome singularity of moment Matrices have been covered in Chapter 4. The increase in accuracy of interpolation has been demonstrated using an illustration for interpolation for stress value using that available at 11 points scattered around it. The variance of smoothed fitting curve reproduced here as Fig. 7.1 has demonstrated the accuracy of predicted stresses along the polygonal boundaries and its use in creation of additional data points for better quality interpolated value at point of interest.

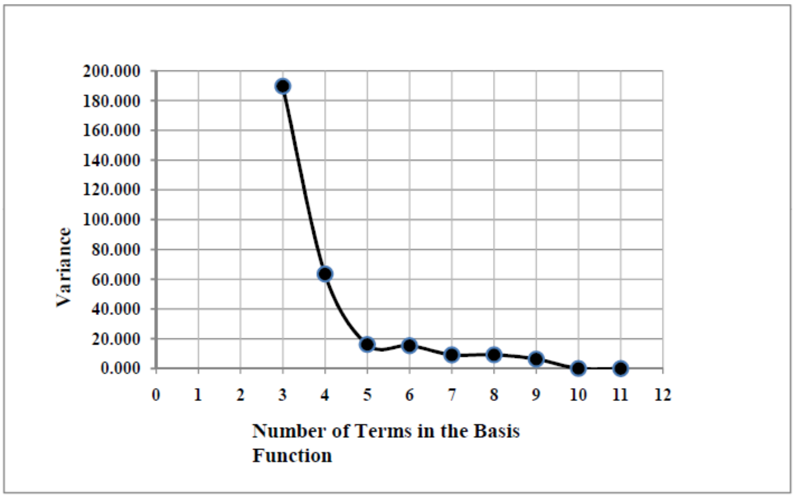


Fig. 7.1 Comparison of Variance

7.2.3 Nodes – in - Motion Strategy for Optimization

In order that an optimum configuration is achieved in node based structures, criteria have been conceived to obtain a perfect position of a node from the basic principles of structural mechanics. The mathematical formulations for the guided and controlled movement of nodes have been established in Chapter 5. The methodology has been used to solve a classic 15-bar truss problem attempted by many researchers. The results have been tabulated in Table 7.2, an extract from Table 5.3

Table 7.2 Verification of Nodes in Motion Strategy

15 - MEMBER TRUSS - OPTIMUM DESIGN - COMPARISON OF RESULTS					
	EARLIER WORKS (Kulkarni,2012)				
TRUSS	Gholizadeh (2012)	Tang (2005)	Rahami (2008)	Kulkarni (2012)	Present Work
Weight (lbs)	73.9300	79.8200	76.6854	72.5152	70.6600

The comparison shows that the current formulation of Nodes in Motion strategy is as efficient as those reported in the literature.

7.2.4 Synthesis of FEA, MPMLS and Nodes- in- Motion Strategies for Optimum Material Disposition in Plates

A combination of independently established facts related to the analysis, smoothing and Nodes- in-Motion strategies has culminated in an efficient formulation for the optimum disposition of material in plates. The relevance of earlier works in Chapters 3, 4 and 5 have been put into use in Chapter 6, as shown in Fig. 7.2 , to analyse for stresses, extract stress values at required locations and to move material for its best performance, respectively.

The combination of all the individual modules developed have been put together to act in unison as an end to end solution to the problems related the in - plane bending of plates. Results obtained for the optimum configuration of a simply supported plate with concentrated load at the centre, a cantilever plate with a concentrated load at the free end have been worked out in Chapter 6 and have been compared with those in the references (Kim H, 2000) and (Helio,2014). The solutions obtained for the configurations are in agreement .

The Code developed has been employed to arrive at the best shape and material disposition for a spanner. The results obtained very closely resembles the shape and material disposition of a spanner in regular use. Optimization has been attempted with the structural requirements and not functional.

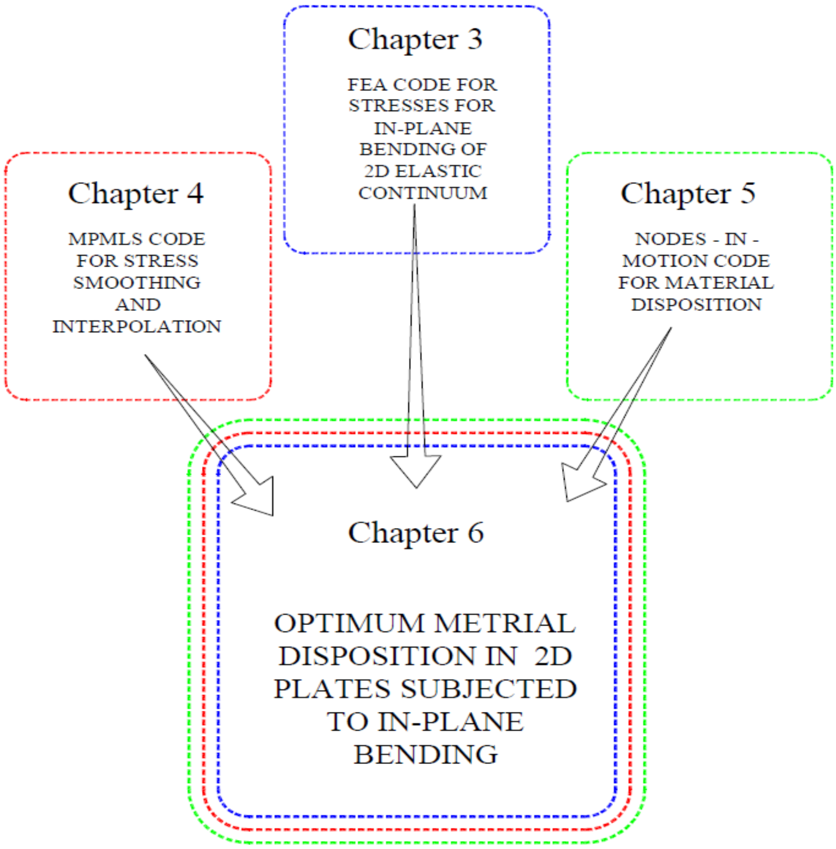


Fig. 7.2 Synthesis of Analysis, Smoothing and Nodes-in- Motion Strategy

7.3 ACCOMPLISHMENTS

A general purpose code for FEA of 2D elastic continuum structures has been developed, tested, verified and validated.

Concept of Moving Polynomial MLS Method has been introduced and its use in stress interpolation at any point of interest has been demonstrated. The analytical work conducted shows the adaptability of this method in any generalised smoothing.

The suggested Nodes- in-Motion strategy has proven to be a very efficient tool in formulation-solution of shape optimization problems.

Iterations leading to the desired level objective can either be in terms of number of iterations or based on the relative change in the weight in two consecutive iterations. In the problems solved, a 2% accuracy in the weight reduction is assumed as the termination criterion. It depends on the level of precision demanded by the designer. Similarly, the user can impose thickness constraints and minimum and maximum bounds for the shape of the plate. Irrespective of any starting thickness assigned, the program generates the same unique, safe and optimum configuration, satisfying the conditions of numerical stability, convergence and accuracy at the desired level.

The code developed terminates normally within expected accuracy level demanded by the industry. If the user demands unreasonable amount of accuracy levels, like 0.0001 mm thickness, the program may enter into an endless loop.

The Graphics User Interface developed gives a visual effect of morphing of original geometry into the optimized one. The interface optionally displays various parameters related to the optimization like, mesh sizes , stresses at nodes, utility ratios at locations and optimized thickness at various locations. It also generates a detailed output file showing results of each iteration.

7.4 POTENTIAL APPLICATIONS AND USE OF THE RESEARCH FINDINGS

7.4.1 In Construction Industry

In Construction Industry, the formulation can used in the design of plate like structures, connections, stiffeners, gussets, splices and in a variety of shapes and help cost cutting and savings.

7.4.2 In Component and Tool Manufacturing Industry

Components subjected to a known set of loads and support conditions can be designed for the best shape and thicknesses combination. This will be helpful in the Automobile, Mechanical Equipment and Tool manufacturing industry.

7.4.3 In Biomedical Implants

The work presented, thus, can be very useful in the field of bio-mechanics and in industries involved in the design and manufacture of orthopedic implants.

7.4.4 In Research Laboratories

In the research centres engaged in materials and manufacturing of plate like components, many samples are prepared for destructive testing purposes, the work presented here can make substantial savings in time and resources.

7.5 CONCLUSIONS

The objective of the research work has been accomplished through the conceptualisation, formulation, implementation and verification of the methodology of Moving Polynomial Moving Least Square method and Nodes-in- Motion Strategy for the optimum material disposition in plates subjected to in-plane bending.

7.6 SCOPE FOR FUTURE WORK

The Moving Polynomial Moving Least Square Technique for stress smoothing and Nodes-in- Motion strategy for optimum material disposition is worth extended to 3D structures.

The FEA module developed can be extended to address the stress concentration factors , crack formation, and stress intensity factors of at crack tips and propagation of cracks, which will find application in Fracture Mechanics and Health Monitoring.

Appendix – A

FEM FORMULATION USING TRIANGULAR ELEMENTS

Considering a small element of material which undergoes deformation u and v in the directions x and y respectively, equations for plane elasticity can be derived. Considering an infenitecimal element shown in Fig. A.1

σ	is the axial stress
ε	is the axial strain
τ	is the shear stress
γ	is the shear strain
E	is Young's modulus
ν	is Poisson's ratio

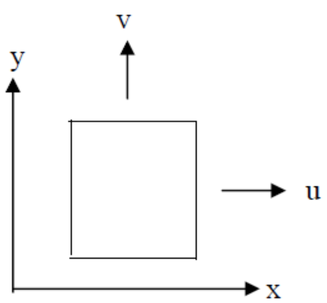


Fig. A.1 The deformations of a small element

Representing displacements in matrix form,

$$\mathbf{u} = \begin{bmatrix} u & v \end{bmatrix}^T \qquad \text{------ (A.1)}$$

The stress components in vector notation are expressed as

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \sigma_y & \tau_{xy} \end{bmatrix}^T \qquad \text{------(A.2)}$$

The strain components in vector notation expressed as

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_{xy} \end{bmatrix}^T \qquad \text{------ (A.3)}$$

The strain-displacement relationships can be written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \text{------ (A.4)}$$

or in matrix notation

$\boldsymbol{\varepsilon} = \mathbf{Lu}$ ------ (A.5)

where the differential operator **L** is expressed as

$$\mathbf{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$ ------ (A.6)

Thus the strain is expressed in terms of deformation.

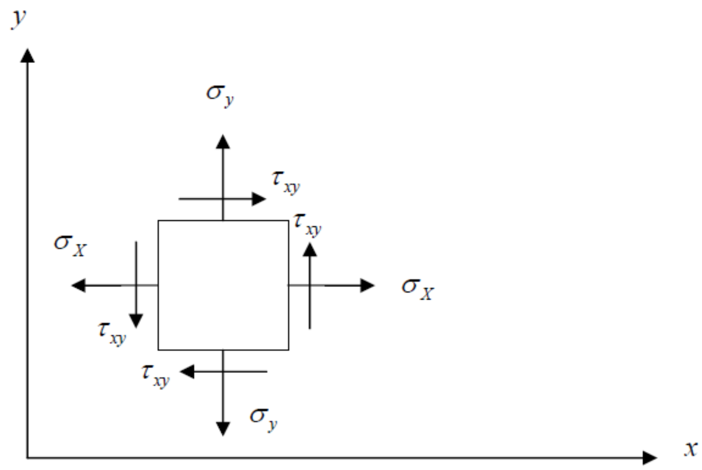


Fig. A.2 Notations for Stresses

Using the general Expression

$$\sigma = E\varepsilon$$ ------ (A.7)

For an elastic material subjected to stresses in perpandicular direction

$$\sigma_y = E\varepsilon_y + \nu\sigma_x$$ ------ (A.8)

The expressions for direct strains and shear strain can be obtained rearranging the above equation as

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$ ------ (A.9)

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$$ ------ (A.10)

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$ ------(A.11)

Substituting the value of σy from eq. A.   In eq.

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu(E\varepsilon_y + \nu\sigma_x)}{E}$$ ------(A.12)

$$E\varepsilon_x = \sigma_x - \nu E\varepsilon_y - \nu^2\sigma_x$$ ------(A.13)

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$$ ------(A.14)

Similarly,

$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$$ ------(A.15)

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$ ------(A.16)

In matrix form, this can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$ ------(A.17)

Thus, the linear stress-strain relationship in the matrix notation is expressed as

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$$ ------(A.18)

where matrix **D** for isotropic material with a Poisson's ratio υ in the plane stress case is given by

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$ ------(A.19)

We know the strain enery stored in a material is given by

$$U = \frac{1}{2}\int_A \varepsilon^T \sigma t dA \qquad \text{------(A.20)}$$

Substituting the stress in terms of strain,

$$U = \frac{1}{2}\int_A \varepsilon^T D\varepsilon t dA \qquad \text{------(A.21)}$$

For a triangular with two displacement components at each corner

The displacement vector q may be expressed as

$$q = \{q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6\}^T \qquad \text{------(A.22)}$$

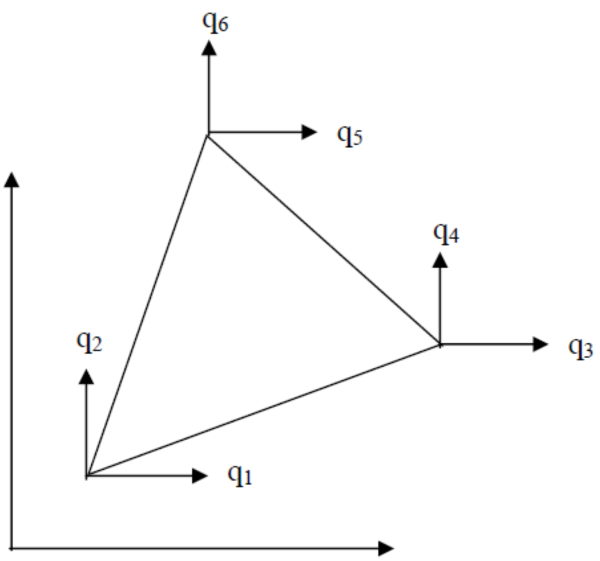


Fig. A.3 Displacements at the nodal Points of a Triangular Element

The next step is to derive general expression for the displacement components in the triangular portion in terms of the nodal displacements. Dividing the element into three parts, as shown in Fig. A.4,

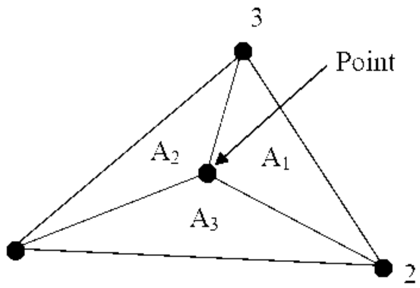


Fig. A.4 The division of Element Area by an Internal Point

The total area A is given by

$$A = A_1 + A_2 + A_3 \quad \text{------(A.23)}$$

Introducing ratios N_1,N_2,N_3 as

$$N_1 = \frac{A_1}{A} \quad \text{------(A.24)}$$

$$N_2 = \frac{A_2}{A} \quad \text{------(A.25)}$$

$$N_3 = \frac{A_3}{A} \quad \text{------(A.26)}$$

We know that the influence of individual components at the corners on u and v will depend on the proximity of the point to the corner. The proximity is can be assessed by the relative fraction of area of the traingle opposite that corner.

$$u = N_1 q_1 + N_2 q_3 + N_3 q_5 \quad \text{------(A.27)}$$

$$v = N_1 q_2 + N_2 q_4 + N_3 q_6 \quad \text{------(A.28)}$$

We also know that

$$N_1 + N_2 + N_3 = 1 \quad \text{------(A.29)}$$

expressing

$$N_1 = \xi \quad \text{------(A.30)}$$

$$N_2 = \eta \quad \text{------(A.31)}$$

$$N_3 = 1 - \xi - \eta \quad \text{------(A.32)}$$

$$u = (q_1 - q_5)\xi + (q_3 - q_5)\eta + q_5 \qquad \text{------(A.33)}$$

$$v = (q_2 - q_6)\xi + (q_4 - q_6)\eta + q_6 \qquad \text{------(A.34)}$$

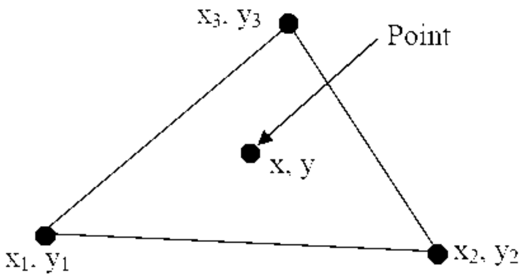


Fig. A.5 Co-ordinates of Corners of a Triangular Element

Similar to the deformation components, the cor-ordinates of the point may be expressed in terms of co-ordinates of the corners as

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 \qquad \text{------(A.35)}$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 \qquad \text{------(A.36)}$$

$$x = (x_1 - x_3)\xi + (x_2 - x_3)\eta + x_3 \qquad \text{------(A.37)}$$

$$y = (y_1 - y_3)\xi + (y_2 - y_3)\eta + y_3 \qquad \text{------(A.38)}$$

As per the Chain Rule in partial differentiation,

$$\frac{df}{dz} = \left(\frac{\partial f}{\partial x}\right)\frac{dx}{dz} + \left(\frac{\partial f}{\partial y}\right)\frac{dy}{dz} \qquad \text{------(A.39)}$$

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \xi} \qquad \text{------(A.40)}$$

$$\frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \eta} \qquad \text{------(A.41)}$$

$$\begin{Bmatrix} \dfrac{\partial u}{\partial \xi} \\ \dfrac{\partial u}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial u}{\partial y} \end{Bmatrix} \qquad \text{------(A.42)}$$

or

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} \qquad \text{------(A.43)}$$

But we know

$$\frac{\partial x}{\partial \xi} = x_1 - x_3 \qquad \text{------(A.44)}$$

$$\frac{\partial x}{\partial \eta} = x_2 - x_3 \qquad \text{------(A.45)}$$

$$\frac{\partial y}{\partial \xi} = y_1 - y_3 \qquad \text{------(A.46)}$$

$$\frac{\partial y}{\partial \xi} = y_2 - y_3 \qquad \text{------(A.47)}$$

$$x_{ij} = x_i - x_j \qquad \text{------(A.48)}$$

$$y_{ij} = y_i - y_j \qquad \text{------(A.49)}$$

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} \qquad \text{------(A.50)}$$

If

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \text{------(A.51)}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \qquad \text{------(A.52)}$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad \text{------(A.53)}$$

$$Area = \frac{1}{2}\left|\det J\right| \quad \text{------(A.54)}$$

$$2A = \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \quad \text{------(A.55)}$$

$$= (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3) + (x_1y_2 - x_2y_1). \quad \text{------(A.56)}$$

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} = \frac{1}{\det J} \begin{bmatrix} y_{23} & -y_{13} \\ -x_{23} & x_{13} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} \quad \text{------(A.57)}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\det J}\left(y_{23}\frac{\partial u}{\partial \xi} - y_{13}\frac{\partial u}{\partial \eta} \right) \quad \text{------(A.58)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\det J}\left(-x_{23}\frac{\partial u}{\partial \xi} + x_{13}\frac{\partial u}{\partial \eta} \right) \quad \text{------(A.59)}$$

$$u = (q_1 - q_5)\xi + (q_3 - q_5)\eta + q_5 \quad \text{------(A.60)}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\det J}\left(y_{23}(q_1 - q_5) - y_{13}(q_3 - q_5) \right) \quad \text{------(A.61)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\det J}\left(-x_{23}(q_1 - q_5) + x_{13}(q_3 - q_5) \right) \quad \text{------(A.62)}$$

$$\begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{Bmatrix} = \frac{1}{\det J} \begin{Bmatrix} y_{23}\frac{\partial v}{\partial \xi} - y_{13}\frac{\partial v}{\partial \eta} \\ -x_{23}\frac{\partial v}{\partial \xi} + x_{13}\frac{\partial v}{\partial \eta} \end{Bmatrix} \quad \text{------(A.63)}$$

$$v = (q_2 - q_6)\xi + (q_4 - q_6)\eta + q_6 \qquad \text{------(A.64)}$$

$$\frac{\partial v}{\partial x} = \frac{1}{\det J}\left(y_{23}(q_2 - q_6) - y_{13}(q_4 - q_6)\right) \qquad \text{------(A.65)}$$

$$\frac{\partial v}{\partial y} = \frac{1}{\det J}\left(-x_{23}(q_2 - q_6) + x_{13}(q_4 - q_6)\right) \qquad \text{------(A.66)}$$

$$\varepsilon = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} \qquad \text{------(A.67)}$$

$$\varepsilon = \frac{1}{\det J}\begin{Bmatrix} y_{23}(q_1 - q_5) - y_{13}(q_3 - q_5) \\ -x_{23}(q_2 - q_6) + x_{13}(q_4 - q_6) \\ -x_{23}(q_1 - q_5) + x_{13}(q_3 - q_5) + y_{23}(q_2 - q_6) - y_{13}(q_4 - q_6) \end{Bmatrix}$$

------(A.68)

$$y_{12} = y_1 - y_2 \qquad \text{------(A.69)}$$

------(A.70)

$$y_{12} = y_1 - y_3 - y_2 + y_3$$

------(A.71)

$$y_{12} = y_{13} - y_{23}$$

$$\varepsilon = \frac{1}{\det J}\begin{Bmatrix} y_{23}q_1 + y_{31}q_3 + y_{12}q_5 \\ x_{23}q_2 + x_{13}q_4 + x_{21}q_6 \\ x_{23}q_1 + y_{23}q_2 + x_{13}q_3 + y_{13}q_4 + x_{21}q_5 + y_{12}q_6 \end{Bmatrix}$$

------(A.72)

$$\varepsilon = Bq \qquad \text{------(A.73)}$$

$$B = \frac{1}{\det J}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$ ------(A.74)

$$U = \frac{1}{2}\int_A \varepsilon^T D\varepsilon t dA$$ ------ (A.75)

$$U = \frac{1}{2}\int_A q^T B^T DBqtdA$$ ------ (A.76)

Thus for the

$$U_e = \frac{1}{2}\int_e q^T B^T DBqt_e dA$$ ------(A.77)

$$U_e = \frac{1}{2}q^T B^T DBt_e\left(\int_e dA\right)q$$ ------(A.78)

$$U_e = \frac{1}{2}q^T t_e A_e B^T DBq$$ ------(A.79)

$$k_e = t_e A_e B^T DB$$ ------(A.80)

$$U_e = \frac{1}{2}q^T k_e q$$ ------(A.81)

The Total strain energy in the entire plate is given by summation of all energies of individual elements.

$$U = \sum_e \frac{1}{2}q^T k_e q$$ ------(A.82)

or

$$U = \frac{1}{2}Q^T KQ$$ ------(A.83)

Where K is the Global Stiffness Matrix, Q the Global displacement vector. With the Global Force vector F,

$$KQ = F$$ ------(A.84)

In this K matrix will be the order 2n X 2n, Q will be of the order 2n X 1 and F will be of the order 2n X 1

Solving this simultaneous equation, we get the Global displacement vector. From this, for every element, we can compute the element strain and stress using the equations

$$\varepsilon = Bq \qquad \text{------ (A.85)}$$

$$\sigma = DBq \qquad \text{------(A.86)}$$

or

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \frac{1}{\det J}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} \qquad \text{------(A.87)}$$

The Principal Stresses are given by

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{------(A.88)}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{------(A.89)}$$

The Von Mise's Stress is calculated from the Principal Stresses as

$$\sigma_V = \sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2} \qquad \text{------(A.90)}$$

The intermediate formulae derived in this Appendix - A are directly used in the code developed for the FEA of 2D plates subjected to in-plane bending, detailed in Chapter 3.

Appendix –B

DIRECT STIFFNESS MATRIX METHOD FOR ANALYSIS OF TRUSSES

B.1 GENERAL

Stiffness Matrix Method of Analysis of a truss is a systematic procedure involving basic principles of force displacement relationship and conditions of equilibrium and compatibility. Following procedure is adopted

B.2 TRUSS MEMBER CONSIDERED AS A BEAM ELEMENT

Consider a truss member with cross sectional area **A**, Length **L** made with a material having a Young's Modulus **E,** subjected to end forces **F1** and **F2,** undergoing a deformation **x1,x2** respectively at end nodes **N1** and **N2,** as given in Fig B.1

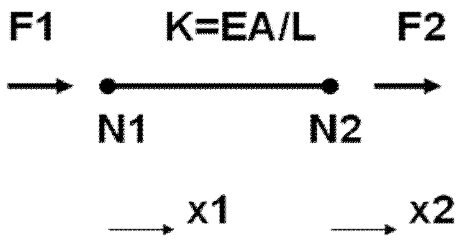


Fig. B.1 Force – Displacement Relationship for a truss member

K=EA/ L ----- (B.1)

Where K is known as the stiffness of the member.

The relationship between the end forces and displacement can be written as

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} +K & -K \\ -K & +K \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \quad \text{----- (B.2)}$$

or

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \quad \text{----- (B.3)}$$

B.3 LOCAL STIFFNESS MATRIX FOR AN INCLINED MEMBER

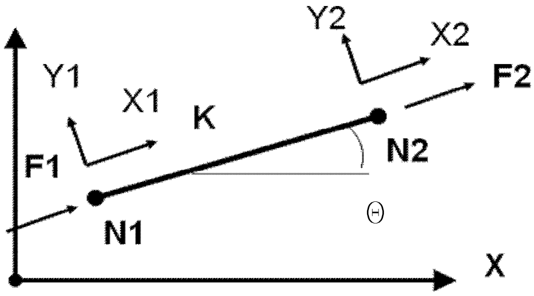


Fig. B.3 Force – Displacement Relationship in Local Co-ordinates

The relationship between the displacements in local directions and the forces in local axes is given by

local stiffness matrix

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{Bmatrix} \qquad \text{----- (B.4)}$$

B.4 ASSEMBLY OF GLOBAL STIFFNESS MATRIX

If the axis of the member is inclined with respect to the Global axes, with an inclination 'Θ' made with the X axis, the displacements in the global directions are related to those in the local directions as given in Eq. B.7 and Eq. B.8

$$s(\theta) = \sin(\theta) \qquad \text{----- (B.5)}$$

$$c(\theta) = \cos(\theta) \qquad \text{----- (B.6)}$$

$$\begin{Bmatrix} X_1 \\ Y_1 \end{Bmatrix} = \begin{bmatrix} c(\theta) & s(\theta) \\ -s(\theta) & c(\theta) \end{bmatrix} \begin{Bmatrix} {}^{g}X_1 \\ {}^{g}Y_1 \end{Bmatrix} \qquad \text{----- (B.7)}$$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{Bmatrix} = \begin{bmatrix} c(\theta) & s(\theta) & 0 & 0 \\ -s(\theta) & c(\theta) & 0 & 0 \\ 0 & 0 & c(\theta) & s(\theta) \\ 0 & 0 & -s(\theta) & c(\theta) \end{bmatrix} \begin{Bmatrix} {}^{g}X_1 \\ {}^{g}Y_1 \\ {}^{g}X_2 \\ {}^{g}Y_2 \end{Bmatrix} \qquad \text{----- (B.8)}$$

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \end{Bmatrix} = \begin{bmatrix} c(\theta) & s(\theta) & 0 & 0 \\ -s(\theta) & c(\theta) & 0 & 0 \\ 0 & 0 & c(\theta) & s(\theta) \\ 0 & 0 & -s(\theta) & c(\theta) \end{bmatrix} \begin{Bmatrix} {}^{g}F_{1X} \\ {}^{g}F_{1Y} \\ {}^{g}F_{2X} \\ {}^{g}F_{2Y} \end{Bmatrix} \qquad \text{----- (B.9)}$$

N1 N2

$$\left[{}^{g}K\right] = \frac{EA}{L} \begin{bmatrix} c^2(\theta) & c(\theta)s(\theta) & -c^2(\theta) & -c(\theta)s(\theta) \\ c(\theta)s(\theta) & s^2(\theta) & -c(\theta)s(\theta) & s^2(\theta) \\ -c^2(\theta) & -c(\theta)s(\theta) & c^2(\theta) & c(\theta)s(\theta) \\ -c(\theta)s(\theta) & -s^2(\theta) & c(\theta)s(\theta) & s^2(\theta) \end{bmatrix} \begin{matrix} \text{N1} \\ \\ \text{N2} \\ \\ \end{matrix}$$

----- (B.10)

Similarly, Force components in the global directions are related to those in the local directions and can be computed as given in Eq. B.9 and representing Global Force components in terms of global displacements, we get global stiffness matrix, given in equation (B.10)

The members of the stiffness matrix for every member is assembled at the required locations in the global stiffness matrix .

The displacement vector in the Global directions is connected to the global force vector through the relation

$$[\,^{g}\mathbf{K}]\,\{^{g}\mathbf{D}\} = \{\,^{g}\mathbf{F}\} \qquad \text{----- (B.11)}$$

B.5 SOLUTION OF SIMULTANEOUS EQUATIONS FOR NODAL DISPLACEMENTS

The group of simultaneous equations is solved for displacements using matrix inversion, decomposition or iteratively.

B.6 EXTRACTION OF MEMBER FORCES AND STRESSES

Once the displacements are obtained, axial forces in the individual members are computed from end displacements. Since the areas of cross-sections are initially available, the axial stresses in the material of section for every member can easily be computed.

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