TIME-DEPENDENT FAILURE POSSIBILITY ANALYSIS OF REINFORCED CONCRETE STRUCTURES

Thesis

Submitted in partial fulfilment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

by

Utino Worabo Woju

DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA
SURATHKAL, MANGALORE – 575 025

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DECLARATION

I hereby declare that the Research Thesis entitled **"Time-Dependent Failure Possibility Analysis of Reinforced Concrete Structures"** which is being submitted to the National Institute of Technology Karnataka, Surathkal in partial fulfilment of the requirements for the award of the Degree of **Doctor of Philosophy** in **Civil Engineering,** is a bonafide report of the research work carried out by me. The material contained in this Research Thesis has not been submitted to any University or Institution for the award of any degree.

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CERTIFICATE

This is to certify that the Research Thesis entitled "**Time-Dependent Failure Possibility Analysis of Reinforced Concrete Structures"** submitted by Mr. Utino Worabo Woju (Register Number: 187CV014) as the record of the research work carried out by him, is accepted as the Research Thesis submission in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy.

Dr. A. S. Balu
Research Guide

Prof. K. Swaminathan
Chairman - DRPC

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Utino Worabo Woju

ABSTRACT

Structural performance depends on the design, construction, environment, utilization, and reliability aspects. From these, other factors can be controlled by adopting proper design and construction techniques, but the environmental factors are difficult to control. Hence, the environmental factors in the analysis and design are mostly not considered sufficient in practice; however, they have significant effects on the performance of the structures in the design life. It is in this light that this study aimed at performing the time-dependent safety and serviceability performance analysis of reinforced concrete structures majorly considering environmental factors such as creep, shrinkage, and corrosion that possess uncertainty. To achieve the desired objective, a simply supported reinforced concrete beam was designed and detailed to Eurocode (EC2). Different design parameters such as corrosion parameters, creep and shrinkage, the time-dependent properties of the material have been identified and modeled through a thorough literature review. The empirical equations provided in design codes were modified to consider the time-variant parameters in time-dependent performance analysis.

In the presence of uncertainty of parameters, it is impossible to obtain the absolute reliability of the structure. The sources of uncertainties in reinforced concrete are the randomness of variables, mathematical models, physical models, environmental factors, and gross error. Uncertainties broadly classified as aleatory and epistemic uncertainties. This research mainly addressed the epistemic uncertainty of reinforced concrete structure to handle the imprecise data using fuzzy concepts. The fuzziness of variables identified and their membership functions were generated by MATLAB R2018a using the heuristic method. In addition to the identification of fuzziness of variables, the study further extended to design optimization and performance level evaluation of reinforced concrete structure using fuzzy relation and fuzzy composition to explore the application of fuzzy concepts. In the design of reinforced concrete structure using fuzzy relation and composition methods, the design is taken as optimum when the performance degree of membership tends to unity.

Failure possibility is a measure of safety when a structure encounters with fuzzy uncertainties. If uncertainties are time-dependent, the possibility of performance under zero results in time-dependent failure possibility, and it becomes more pronounced during improper consideration of environmental factors. Therefore, in this study, time-dependent parameters are taken into account for exploring the effects of environmental factors in reinforced concrete structures. Possible failure modes were identified and estimated using modified time-variant empirical equations to consider the propagation of input variables that are characterized by membership functions to output responses. Then, the time-dependent failure possibility is evaluated by the numerical optimization procedure. Real-time data has been collected from the city of Addis Ababa, Ethiopia for the case study to substantiate the methodology presented in this study. From the detailed modeling and analysis, considering the moderate corrosion rate with corresponding ambient temperature and relative humidity of the considered site, the structure safely performs for less than half of its design life.

Keywords: reinforced concrete, beam, environmental factors, time-dependent, flexure, shear, deflection, crack, performance, epistemic uncertainty, fuzzy sets, membership function, fuzzy operations, failure possibility

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ABBREVIATIONS

AASHTO	American Association of State Highway and Transportation Officials
ACI	American Concrete Institute
AFOSM	Advanced First Order Second Moment
ANN	Artificial Neural Networks
BS	British Standard
DLOM	Double-Loop Nested Optimization Method
EC	Eurocode
FORM	First-Order Reliability Methods
FOSM	First Order Second Moment
GOA	Global Optimization Approach
HDMR	High Dimensional Model Representation
IS	Importance Sampling Method
LRFD	Load and Resistance Design Factors
MCS	Monte Carlo simulation
MF	Membership Function
MVFOSM	Mean-value First-order Second-moment Method
PSIBDO	Possibilistic Safety Index-Based Design Optimization
RC	Reinforced Concrete
RSM	Response Surface Method
SLOM	Single-Loop Optimization Method
SORM	Second-Order Reliability Method
SQP	Sequential Quadratic Programming
TDFP	Time-Dependent Failure Possibility
TDP	Time-Dependent Performance
TIFP	Time-Independent Failure Possibility
TPBDA	Target Performance-Based Design Approach

SYMBOLS

A_c	Area of concrete cross-section
A_{st}	Calculated area of reinforcement
$A_{st,prov}$	Provided area of reinforcement
$A_{st,\min}$	The minimum area of steel required in the section
$A_{pit}\left(t\right)$	Net cross-sectional area
A_{sw}	Area of two-legged shear reinforcement
A	Fuzzy set of A
A_α	$\alpha - cut$ of the fuzzy number
b	Breadth of beam
C	Concrete cover thickness
C_0	Equilibrium chloride concentration at the concrete surface
C_{cr}	Critical chloride concentration
d	The effective depth of the beam
d_0	The thickness of the annular layer of concrete pores at the concrete-bar interface
d_a	Maximum aggregate size
D_c	Chloride diffusion coefficient
$d_c(t)$	The thickness of corrosion products along time
D_i	The initial diameter of the i^{th} bar before corrosion
$D_i\left(t\right)$	The diameter of the i^{th} bars after corrosion at the time t
$E_{c,ef}$	Effective modulus of elasticity of the concrete
$f(t)$	The internal stress of a thick-wall concrete cylinder
$f_1(t)$	Stress produced by corrosion products of the embedded reinforcement
f_2	Stress from applied load between the reinforcing bar and the concrete
f_{ck}	Compressive strength of concrete
f_{ctm}	Tensile strength of concrete
f_{yk}	Characteristic yield strength of steel
f_{yo}	Initial yield stress of non-corroded reinforcement steel
f_{ywd}	Design yield stress of the shear reinforcement
G_f	Fracture energy
i_{corr}	Rate of corrosion current density
k	Factor has taken in to account the distribution of bending moment
L	Effective span length of the beam
M_{cr}	Crack moment of the concrete section
M_E	The bending moment caused by permanent and transient loads

M_{Ed}	Design bending moment
M_G	The bending moment caused by permanent force
M_{Qp}	Moment of the quasi-permanent moment at the critical section
$\underset{t\in[t_s,t_e]}{Min}\, G(X,t)$	Minimum performance of the structure in time instant
r_0	The radius of the concrete cylinder
r_{corr}	Rate of corrosion
$r_{t,sh}$	Curvature induced by creep and shrinkage with time
$s_{\max}$	Maximum spacing of shear reinforcement
$s_{\min}$	Minimum spacing shear reinforcement
$s_{r,\max}$	Maximum crack spacing
t	Time
$[t_s, t_e]$	Time interval
T_i	Corrosion initiation time
u	Cross-section perimeter exposed to drying
$u(r)$	Radial displacement
V_{Ed}	Design shear force
$V_{Rd,s}$	Design shear resisted by stirrups
$V_{Rd,c}$	Shear resistance of the concrete section
$w_{k,ap}$	Maximum crack width from service load
$W_{rust}(t)$	Mass of corrosion products with time
$x^{-\alpha}$	Lower bound of fuzzy variable x
$x^{+\alpha}$	Upper bound of fuzzy variable x
$1/r_{QP}$	Flexural curvature due to quasi-permanent load
$\left(1/r(t)\right)_{sc}$	Time-variant curvature of the cracked section
$\left(1/r(t)\right)_{su}$	Time-variant curvature of the uncracked
^{o}C	Degree Celsius
α	Stiffness reduction factor
α_e	Modulus ratio
α_{rust}	Coefficient of corrosion products
β	Coefficient takes into account the duration of loading
$\delta_{\lim}$	Limit deflection
$\delta(t)$	Time-variant deflection
$\varepsilon_{cs}(\infty, t_s)$	Shrinkage strain at the infinite time
$(\varepsilon_{sm} - \varepsilon_{cm})$	Difference mean strain in the reinforcement concrete between cracks
$\varphi(\infty, t_0)$	Creep coefficient at the infinite time
$\mu_A(x)$	Degree of membership of x in A
v_c	Poisson's ratio of concrete

v_s	Poisson's ratio of steel
π_f	Possibility-safety index
$\pi_{ft\in[t_s,t_e]}$	Time-dependent failure possibility
ρ_{rust}	The density of corrosion products
ρ_{st}	The density of the steel
$\rho(t)$	Time-dependent reinforcement ratio
σ_r	Radial stress of a concrete cylinder
σ_s	Steel stress of quasi-permanent service load
σ_{sr}	Steel stress at first cracking
σ_θ	Tangential stress of a concrete cylinder
ξ	Coefficient allowing for tension stiffening

CHAPTER 1

INTRODUCTION

The performance of a structure for the intended purpose can be assessed by its safety, serviceability, and economy. The performance of the structure mainly depends on the input variables. The information of input variables is never certain, precise, and complete (Ranganathan 1999), i.e., uncertainty. The sources of uncertainties can be physical uncertainty, statistical uncertainty, model uncertainty, and gross errors. Due to the presence of these uncertainties of material properties, loads on the structure during its life, structural idealization model, limitation of numerical methods, and other unforeseen factors, the absolute safety of a structure is impossible (Biondini et al. 2004). Besides, the time-variant properties of the input variables lead the output performance to vary with the time (Fan et al. 2019).

Uncertainties are broadly classified into aleatory and epistemic uncertainties. Aleatory uncertainty arises from the inherent randomness in the physical properties and the system environment (Li et al. 2016; Kiureghian and Ditlevsen 2008) whereas, epistemic uncertainty originates from a lack of sufficient knowledge and imprecision of information about a system going to be studied. The type of uncertainty and the way of dealing with them has been addressed by many investigators (Marano and Quaranta 2008; Du et al. 2006; Brown et al. 1983; Pascal 1975) to solve problems in their respective areas of specialization. These studies concluded that, for aleatory uncertainty, the reliability estimation problem is usually carried out using probability theory, which requires a large number of samples and randomness of input variables; whereas epistemic uncertainty is usually modeled by possibility theory, which handles imprecision and fuzziness of input variables.

In structural engineering, the structure's response is governed by basic variables such as mechanical properties of materials, dimensions, unit weights, environmental loads, etc. Such essential variables possess fuzzy uncertainty due to degradation of quality, skill and workmanship experience, project environmental impacts, and condition of existing structures. Through the *extension principle* of fuzzy variables, the existence of fuzzy uncertainty of input variables propagates output results. A fuzzy uncertainty of a variable is expressed by a membership function. There are numerous types of membership functions, such as triangular, trapezoidal, Gaussian, singleton, etc., depending on the availability and characters of the variables. The triangular membership function is frequently used because of its simplicity.

To evaluate the safety degree of the structure in the presence of the fuzzy uncertainty, an area ratio index of the are membership function of output performance in the failure domain to the whole area of the membership function of output performance is proposed (Shrestha and Duckstein 1997). However, the area ratio index can reflect to some extent the safety degree of the structure it cannot exactly substitute the possibility of the output performance locating in the failure domain (Fan et al. 2019). Analogous to the reliability index to evaluate conventional reliability under random uncertainty, a failure possibility index measures the safety degree of the structure involving the fuzzy uncertainty by the possibility safety index of the performance locating in the failure domain. Moreover, the performance of the structure with time is nonlinear and deteriorate with time, consequently, the membership functions of the performance vary with time.

The time-dependent failure possibility index of the structure is estimated by solving the membership function of the nonlinear performance function (Shrestha and Duckstein 1997). The membership function of the fuzzy uncertain variable can be generated by different methods, such as heuristics, histograms, clustering, simulating annealing, genetic method, particle swarm optimization, and neural network, etc. To determine the membership function of the performance, it is necessary to identify all possible failure modes of the structure. The structural failure mode is generally divided into ultimate and serviceability failure modes (Hess III et al. 2000). The time-dependent failure possibility can be readily estimated by the *numerical optimization* from the membership function of performance whose value is less than zero.

Despite the expectation of long service of civil engineering structures, the resistances deteriorate with loads exceeding the expected design load and the various environmental changes that occur within the design life. The performance of reinforced concrete structures subjected to sustained load and corrosion and failure probability analysis, complementary to classical reliability analysis, are well-developed numerically and experimentally (Teplý et al. 1999; Afzal et al. 2016; Verma et al. 2014). On the contrary, the time-dependent performance and failure possibility analysis of concrete structures subjected to sustained load, corrosion, and creep and shrinkage lack sufficient research. Therefore, this study is significant to evaluate the time-dependent performance level, the fuzziness of input and output response parameters, and the time-dependent failure possibility of the reinforced concrete structure subjected to sustained load and environmental factors (i.e., corrosion, creep, and shrinkage).

1.1 Problem Statement

Nowadays, in urban and suburban areas, reinforced concrete is commonly used in the construction industry. Private buildings such as apartments, hotels, shopping malls, offices, and the like occupy large areas. Even though it emits greenhouse gasses and consumes more energy, material, and economy, its lifetime is limited to 50 years (ES EN 1992-1-1 2004) under normal conditions.

Design codes of reinforced concrete structures treat the exposure condition of reinforced concrete by only providing respective concrete cover. The concrete cover can delay the initiation of corrosion but does not completely control corrosion in the design life of the structure. Besides, creep and shrinkage are also other essential factors that reduce the bending stiffness and induce additional deflection of the structure. Therefore, environmental factors such as aggressive chemicals, temperature, and relative humidity in the phases of analysis, design, construction, and operation should be considered to reduce the uncertainty of the parameters.

To carry out this study, a particular reinforced concrete beam member has been designed and detailed as per Eurocode, and its time-dependent performance analysis is carried out to obtain the member's demand and capacity. These variations of cross-section, the strength of materials, the diameter of reinforcing steel, and load with time possess fuzzy uncertainty that leads to conduct the possibilistic analysis. To estimate the safety of a particular structure, the probability analysis is not worth to generalize the properties of a small sample size; thus, it is important to adopt a time-dependent possibility analysis to measure the particularity of the sample in order to recommend the appropriate maintenance strategy to avoid premature failure of the structure.

1.2 Research Objectives

The main objective of the study is to investigate the time-dependent failure possibility analysis of the reinforced concrete structures. To achieve this aim, the following specific objectives were set:

- To design an optimum reinforced concrete beam based on Eurocode
- To perform the time-dependent performance level of both ultimate and serviceability limit states
- To generate fuzzy membership functions of the input and output variables
- To analyze the time-dependent failure possibility of both ultimate and serviceability limit states

1.3 Scope and Limitations

A reinforced concrete beam that is subjected to a uniformly distributed load, which induces flexure and shear, and thus develops deflection and cracks. The considered beam is subject to sustained load, environmental loads, which have greater uncertainty, i.e., creep and shrinkage, and corrosion. However, the corrosion initiation period of flexural and shear reinforcement is different, the same corrosion rate was used in this study for both mild and high yield stress steel. There is no regard for the torsional constraint (i.e. torsion, flexure-torsion, and shear-torsion). However, the findings obtained from empirical expressions from different experimental and analytical researches, from relevant literature, can lead to possible errors due to the real problem's environmental factors.

Fuzzy uncertainty is employed to handle the ambiguity of variables. The degree of the uncertainty of materials performance, load intensity, action responses, and performance level is expressed by membership functions. Since membership function is established through the available data and expert's experience which inhold uncertainty. Additionally, there is no clear guideline and consensus for choosing the type of membership function for a particular fuzzy variable. This uncertainty of membership function can be refined by considering $n-type$ fuzzy, whose fuzzy variable is $m-1$ fuzzy membership function, but not considered in this research.

1.4 Significance of the Study

The resistance of the structure decreases with time due to the degradation of material strength and sectional dimension, creep and shrinkage, and residual stresses with time. The time-dependent failure possibility analysis is used to estimate failure possibility, complementary of reliability, from the initial function of the system for the desired purpose throughout its design life. The performance of time-dependent failure possibility analysis is important to plan maintenance strategies to ensure the performance of the structure, resolve premature failure, and the consequence of failure. This research is used to familiarize the possibility analysis of sample particularity instead of probability analysis because civil engineering structures are not identical in shape, size, material properties, analysis, and design methods, detailing, and construction techniques. Furthermore, the possibility safety index π_f is used to employ the design optimization and evaluate the reliability of the structures in which the less π_f more the structure reliable. Besides, the possibility-analysis can be extended to all civil engineering structures.

1.5 Overview of the Thesis

In this study, time-dependent failure possibility analysis of the reinforced concrete structures. Analystical study has been carried out for evaluating time-variant performance of the structure and finally time-dependent failure possibility analysis.

The first chapter introduces the background, problem statement, objectives, scope and limitation of the study, and its significance. The second chapter presents a review of literature on reinforced concrete, design optimization, factors accelerating the failure possibility of concrete structures, uncertainties of design parameters, fuzziness and fuzzy set theory, fuzzy sets and its membership function, standard operation fuzzy set, reliability of structure and methods used for reliability analysis.

The third chapter demonstrates the design optimization of the RC beam, time-dependent performance analysis, deterioration of input variables due to corrosion, identification and estimation of possible failure modes, generation of membership functions, and failure possibility analysis. The fouth chapter discusses the optimal design of the RC beam used for the case study, general performance of structures, evaluation of time-variant performance input variables, and the time-dependent safety and serviceability performance of the reinforced concrete structure.

In fifth chapter, uncertainty of parameters, types of uncertainties, generation of membership function of fuzzy variables, uncertainty propagation, and a numerical approach for fuzzy uncertainty propagation, and application of fuzzy concepts in reinforced concrete structures were presented. The sixth chapter portrays the basic concepts of TDFP analysis, failure possibility index, and estimation of time-dependent failure possibility of the structure. The seventh chapter presents a summary, conclusion of the study, practical implication related to the study area ant the recommendation for further studies of the area.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents a review of literature on reinforced concrete, factors that accelerate the failure of concrete structures, and its design optimization. The review here mainly focuses on the reliability of structure and several methods employed to estimate the reliability of the structure. Besides, the sources and types of uncertainties in the structure that define the methods of reliability analysis are presented here.

2.2 Reinforced Concrete

The main objective of reinforced concrete design is to achieve a structural system or part of the structure that will result in a safe, serviceable, and economical solution. To achieve safety, serviceability, and economy criteria of structures, three design philosophies, working stress design method, ultimate strength design method, and limit state design method, had been adopted to consider uncertainty in loading pattern and materials strength. Among these limit state design method, in which both load and material uncertainties, and serviceability criteria are considered, is recommended to adopt in design codes (ES EN 1992-1-1 2004; IS 456 2000). Despite the availability of the constituents, economy, and easily molded to the desired shape, the properties of the concrete are complex compared to other construction materials, i.e., steel and timber. As a result, the uncertainty of the concrete structure requires more attention in analysis and design to ensure the reliability of the structure.

2.3 Design optimization

Design optimization is a design methodology using a mathematical formulation of a design problem to support the selection of the optimal design among many alternatives. Design optimization involves (Guerra and Kiousis 2006; Rahmanian et al. 2014; Ferreira et al. 2003; Lin and Frangopol 2002): *variables* that describe the design alternatives; *objective(s) implies* the selected functional combination of variables to be maximized or minimized; *constraints* that imply a combination of variables expressed as equalities or inequalities that must be satisfied for any acceptable design alternative, and *feasibility, i.e.,* values for the set of variables that satisfies all constraints and minimizes/maximizes objective.

The commonly adopted practice in reinforced concrete structure design includes determining cross-sectional dimensions, materials property, and area of reinforcement that would meet the requirements proscribed by a given code of practice considering primarily strength, serviceability, and economy, as well as other imposed demands that result from the environment, and architectural requirements.

The design optimization of the structure in the presence of the probabilistic uncertainty, probabilistic safety analysis methods are well developed and adopted by many investigators. Lin (1996) presented the reliability-based design optimization approach to the design of reinforced concrete girders for highway bridges. In the study, two optimization formulations were presented, i.e., the load and resistance design factors (LRFD) and the reliability approaches, in which both are based on their corresponding AASHTO format. Ferreira et al. (2003) carried out the optimization of the steel area and the steel localization in a T-beam under bending, analytically by applying the non-linear behavior of steel and concrete. Also developed alternative analytical expressions and delivered illustrative examples of design optimization. Ceranic (2018) investigated new approaches in the use of cost-efficient optimization and applied these to the multi-level design of skeletal systems. The study had shown that optimizing structural problems with a single load case does not give a realistic minimum cost of a structure and that frames consisting of multiple beam and column groups in general produce a more cost efficient design.

Furthermore, the validity of structural optimization had been established, dependent directly on the balance between the mathematical model of the objective function and the design constraints, the algorithm that is applied, and the physical reality of the structural problem and its practical application. Tliouine and Fedghouche (2010) developed an analytical approach for design optimization of reinforced concrete T-beams under ultimate loads based on a minimum cost criterion and a reduced number of design variables. Milajić et al. (2013) reviewed the investigations being conducted 20 years back and summarized as a great number of optimization methods in civil engineering have not found implementation in practice, mainly because the problems were treated only from a mathematical point of view, disregarding applicability of obtained solutions in reality.

Other investigators developed and applied computer programming for structural design optimization. Guerra and Kiousis (2006) presented optimal sizing and reinforcing for beam and column members in multi-bay and multi-story reinforced concrete structures incorporate optimal stiffness correlation among all structural members and results in cost savings over typical-practice design solutions using sequential quadratic programming algorithm (SQPA).

Merta et al. (2010) formulated optimization as finding the minimum self-manufacturing cost of the beam subjected to the structural analysis constraints based on Eurocode provisions for the conditions of both the ultimate and the serviceability limit states by the nonlinear programming approach. Babiker et al. (2012) presented the cost optimization of RC beams designed according to the requirements of the ACI 318-08 code using the artificial neural networks (ANN) model. They compared the results obtained from the proposed model with the results obtained by using the classical optimization model, developed in the Excel software spreadsheet, and concluded that the two modes are in good agreement. Kulkarni and Bhusare (2016) presented the design optimization of multi-story reinforced concrete structures using structural analysis and design software and optimization algorithm.

2.4 Factors accelerating the failure possibility of concrete structures

Concrete is one of the composite materials normally used at every stage of construction and it may suffer damages (ES EN 1992-1-1 2004; Bakri and Mydin 2014; MacGregor et al. 1997) during its service life due to several reasons, i.e., poor workmanship, inappropriate design, structural overloading, chemical reaction, creep and shrinkage, the permeability of concrete, corrosion of reinforcement, poor maintenance, and foundation settlement. These factors are generally classified as (a) *design aspects* include the design criteria, accuracy of analytical and design equations, and design errors, (b) *construction aspects* include the natural variation of strength parameters and construction errors, (c) *environment aspects* include aggressive chemicals which lead to corrosion and ambient humidity and temperature which cause creep and shrinkage, (d) *utilization aspects* include the natural variations of service loads, utilization errors and the man-made hazards, and (e) *reliability aspects* include the statistical and model uncertainties (Arafah 2000).

However, it may not be possible to eliminate whole defects (Kumar et al. 2001; Wilmot 2006; Ho and Rahman 2004; Bremner et al. 2001; Bakri and Mydin 2014; Liubin et al. 2011; Mohamed et al. 2018; Qiao et al. 2014) the remedial measures for minimizing the defects of concrete structures can be used. These are adopting appropriate design, construction, utilization, and maintenance; improving quality of concrete by adopting appropriate constituents selection, mix design, compaction, and curing; preventing corrosion of reinforcement by increasing the depth of concrete cover, coating rebars, chloride extraction, waterproofing and patch repair, and upgrading supporting ground to control foundation settlement.

2.4.1 Creep and Shrinkage

Creep of concrete is defined as plastic deformation under sustained load or stress whereas, *shrinkage of concrete* is the property of diminishing in the volume of concrete during the process of hardening. The effect of creep and shrinkage of the concrete depends on the ambient temperature and humidity, the dimensions of the element, and the composition of the concrete. Creep is also influenced by the maturity of the concrete when the load is first applied and depends on the duration and magnitude of the loading (ES EN 1992-1-1 2004). Creep and shrinkage affect the bending stiffness and significantly increase the deflection of the reinforced concrete structures (Lluka et al. 2015; Haldar et al. 2010). The effects of creep and shrinkage increase with the decrease of relative humidity and increase of temperature and vice versa (Li and He 2018; Lluka et al. 2015). To prevent the effect of the creep and shrinkage, it must account for sustained loading for long-term effects while designing (Haldar et al. 2010; Madsen and Bazant 1983) and cyclic wetting and drying is recommended during construction (Li and He 2018).

2.4.2 Corrosion

Many researchers investigated the effects of aggressive environments on reinforced concrete structures. Some of them are: Otieno et al. (2010 and 2016) investigated the corrosion rate prediction model by incorporating the effect of crack width, concrete cover depth, and concrete quality. Besides, Yalciner et al. (2012) investigated the effects of corrosion on a 25-year-old reinforced concrete building's performance and found that corrosion reduces the bond strength and the service life, affects the safety and economy of the built structure. As per Almusallam (2001) investigation, corrosion of embedded reinforcement and the consequent cracking of concrete due to the diffusion of chloride ions to the reinforcement bar surface is significantly predominant than that due to carbonation. The effect of reinforcement steel corrosion has been investigated in many studies and founded that: reduce load carrying capacity; loss of diameter or effective cross-sectional area; significantly reduce bond strength; increase crack width, strongly reduced elongation of reinforcement steel (Almusallam 2001; Stanish 1997; Xia et al. 2013; Zandi Hanjari et al. 2008; Loreto et al. 2011; Adukpo et al. 2013; Zhou et al. 2014; Baskaran and Gopinath 2011), and induces internal pressure (hoop tension), which is due to increasing corrosion products with time, that easily exceeds the limited tensile strength of concrete (Allam et al. 1994; François et al. 2013; Cabrera 1996; Li 2004; Hagino et al. 2013) that leads to cracking and spalling of concrete cover, and hence reduce the service life of the structure.

Before corrosion takes place, the concrete structure is only subjected to the applied load, creep, and shrinkage. To prevent the effect of an aggressive environment, design codes (ES EN 1992-1-1 2004; IS 456 2000) provided respective concrete cover based on the exposure condition of the structural element. The concrete cover may delay the corrosion initiation time but does not fully control the corrosion. To consider the effect of corrosion on the serviceability of the structure the corrosion initiation time is a critical factor. The corrosion initiation time depends on the concrete cover, chloride diffusion coefficient, chloride concentration percentage to the weight of concrete, concrete strength, and the expression to determine the corrosion initiation time was derived by (Thoft-Christensen et al. 1996) using Fick's law of diffusion. The corrosion initiation time T_i can be obtained from the determining parameters C, C_0 and D_c given in expression (Thoft-Christensen et al. 1996) as:

$$T_i = \frac{C^2}{4D_c}\left[erf^{-1}\left(\frac{C_0 - C_{cr}}{C_0}\right)\right]^{-2} \tag{2.1}$$

where C is the concrete cover thickness in (cm); D_c is the chloride diffusion coefficient in (cm^2/year); C_0 *(% weight of concrete*) is the equilibrium chloride concentration at the concrete surface, and C_{cr} *(% weight of concrete*) is the critical chloride concentration

The time-variant resistance of the concrete section is then determined by considering the deterioration of the reinforcing steel diameter about the corrosion initiation time. The reduction of the steel diameter is determined from the expression (Val et al. 1998):

$$D_i(t) = \begin{cases} D_i & \text{for } t \le T_i \\ D_i - r_{corr}(t - T_i) & \text{for } T_i \le t \le T_i + \dfrac{D_i}{r_{corr}} \\ 0 & \text{for } t \ge T_i + \dfrac{D_i}{r_{corr}} \end{cases} \tag{2.2}$$

where, $D_i(t)$ is the i^{th} diameter of the reinforcing bars at a time t; D_i is the initial diameter of the i^{th} bar before corrosion at the time t; *n* is the number of the bars, and r_{corr} is the rate of corrosion that is given by $0.0232 i_{corr}$ in which 1 μA/cm^2 is equal to 11.6 μm/year

Table 2.1: Ranges of corrosion rate and its significance level (Val et al. 1998)

Current density i_{corr} (μA/cm^2)	Corrosion rate r_{corr} (μm/year)	Corrosion level
< 0.1	< 0.026912	Negligible
0.1-0.5	0.0269-0.13456	Low
0.5-1.0	0.13456-0.26912	Moderate
> 1.0	> 0.26912	High

Corrosion of embedded reinforcement not only reduces the bar diameter but also the strength of concrete and yield stress of steel with time. The reduction of concrete strength with time is given by the expression (Kliukas et al. 2015):

$$f_{cc}(t) = \alpha_{cc} k_2(t) f_{ck} \tag{2.3}$$

in which $\alpha_{cc} = 1 - 0.1 M_G / M_E$; $k_2(t) = 0.85 - 1.7\rho(t)$ and $\rho(t) = \dfrac{A_s(t)}{A_c}$, *where* M_G is the bending moment caused by permanent force; M_E is bending moment caused by permanent and transient loads, and $\rho(t)$ is time-dependent reinforcement ratio.

Similarly, the yield stress of the embedded reinforcement bars reduces according to the law proposed by (Du et al. 2005).

$$f_y(t) = f_{yo}\left(1 - \alpha_y \frac{A_{pit}(t)}{\pi D_o^2 / 4}\right) \tag{2.4}$$

where f_{yo} is the initial yield stress of non-corroded reinforcement steel; α_y is the coefficient and its value is taken as 0.005 which is recommended by (Du et al. 2005) and adopted by (Marano et al. 2010); $A_{pit}(t)$ is the net cross-sectional area of a corroded bar at a time $t > T_i$

2.5 Uncertainties of Design Parameters

Civil engineering structures like buildings, bridges, transmission towers, and the like are complex and usually large. Hence, there will be almost no chance to test the prototype rather than checking specific criteria on uncertain software models and limited numerical methods which are based on incomplete human knowledge to solve a real problem. Due to the presence of these uncertainties of materials property, loads on the structure during its life, structural idealization model, limitation of numerical methods, and so on, the absolute safety of a structure is impossible (Ranganathan 1999). For instance, to determine the best frame design for a building, one may be constrained by design codes as well as by the design specifications such as functional, architectural, and structural behavior requirements. Once these constraints are satisfied, the remaining problem may be to find and design that requires the least construction material.

Depending on the nature of the structure, environmental conditions, and applied actions, some types of uncertainties may become critical. The following types of uncertainties can usually be identified (Milan Holicky 2009): *natural randomness* of actions, materials properties, and geometric data; statistical uncertainties due to limited available data; uncertainties of theoretical models owing to the simplification of actual conditions; vagueness due to inaccurate definitions of performance requirements; gross errors in design, execution, and operation of the structure; and lack of knowledge of the behavior of new materials in real conditions.

Most of the uncertainties are associated with material properties, geometry, loads, and models due to a lack of knowledge and environmental factors (Matos 2007).

For the system analysis and design, the uncertainties are classified, sorted, analyzed, and used to predict system parameters and performances. The sources of uncertainties are (1) physical randomness, (2) statistical uncertainty, (3) model uncertainty, and (4) gross errors. Despite the significant success of the probabilistic methods in structural reliability assessment due to randomness, many investigators (Kai-Yuan et al. 1991a; Kai-Yuan et al. 1993; Szeliga 2004; Li et al. 2015; Tang et al. 2014; Naderpour and Alavi 2015; Fan et al. 2019) have indicated that there are other types of uncertainties, i.e., fuzziness.

2.6 Fuzziness and Fuzzy set theory

Fuzziness means an expression having an uncertain extensional denotation that has an ambiguous boundary (Zhang 1998) and also arises from inconsistency or error. Fuzziness can be applied in civil engineering disciplines with linguistic variables, words or sentences in a natural or artificial language as (Zimmermann 2011) *very cold*, *cold*, *warm*, *hot* and *very hot* for temperature; *low*, *moderate* and *high* for corrosion rate; *under-reinforced*, *balanced* and *over-reinforced* for reinforced concrete section, and *no damage*, *slight damage*, *moderate damage*, *severe damage and destructive damage* for damage assessment of earthquake effect on structures with unclear boundaries.

Fuzzy set theory is a suitable tool used for professional decision-making in structural engineering specializations such as risk assessment, reliability analysis, design optimization, and performance evaluation of the structures. These decisions are expressed in linguistic terms (e.g., "the structure is *slightly damaged*" or "the quality control is not *adequate*") with a vagueness that avoids the usual conventional set representation. For this reason, a fuzziness can be encountered to answer unclear '*how*' questions. Each form of query is intended for the reference sense of the expression and has no specific boundary in which to address the question (Zhang 1998).

The human reasoning to counteract these complexities, uncertainties, imprecision, and vagueness of data in professional judgment lead the main motivation to use fuzzy concepts. To handle this problem Zadeh (1965) introduced a fuzzy set theory, which holds a continuum of the degree of membership to model the vagueness. Many investigators proved that (Biondini et al. 2004; Fan et al. 2019; Tang et al. 2014;

Bagheri et al. 2017; Sarkar et al. 2016; Yeh and Hsu 1990) the fuzzy theory is a suitable tool and effectively well-designed for the problems in structural engineering to perform the reliability analysis and optimal design solution. Fuzzy concepts resemble human reasoning through providing a simple way to handle real problems due to their simplicity and versatility, being easy to handle problems with imprecise and incomplete data, being able to handle complexity and nonlinearity, covering a broader variety of operational conditions, and being more readily adaptable in terms of natural or linguistic words.

2.6.1 Fuzzy Sets

The fuzzy set concept was introduced by its pioneer Zadeh (1965) to represent variables with imprecise or ambiguous boundaries. Therefore, the fuzzy set theory is used to handle ambiguity, vagueness, imprecision, and insufficient level of expert knowledge on real-life phenomena as a source of uncertainty.

In ordinary set theory (Mazeika et al. 2007), the element that fulfills some defined conditions by a set is only considered as members of this set. In this case, the degree of membership is binary, i.e. either zero or one. Therefore, in ordinary set theory, there are well-defined boundaries to identify an object that belongs to a set or does not. On the contrary, the fuzzy set theory directly addresses the limitation of a crisp set by letting membership degree to which extent a variable belongs to a set. A fuzzy set is prescribed by vague or ambiguous properties; thus, its boundaries are ambiguously specified.

According to (Holicky and Schneider 2002) notion, the fuzzy set is usually represented as a set of ordered pairs of elements; each presents the element together with its membership value. A fuzzy set is represented as “A” whereas a crisp set is represented as “A”. The fuzzy sets can be represented mathematically for finite and infinite elements. The elements of the discrete fuzzy set A can be represented with its membership function as:

$$A = \left\{ \left(x, \mu_A(x) \right) \right\} \quad \text{or} \quad A = \sum_{i=1}^{n} \mu_A(x_i) / x_i \qquad (2.5a)$$

For elements of the continuous fuzzy set as:

$$A = \int \mu_A(x_i)/x_i \qquad (2.5b)$$

where, μ_A is the membership function or grade of membership of x in A that maps X to the membership space, and in the expressions, the symbol '$\sum$' or '$\int$' implies not addition or integration, respectively but union.

2.6.2 Membership function

In the fuzzy set theory, a fuzzy set A in the universe of discourse X is characterized by a characteristic function $\mu_A(x)$ (Zadeh 1965), which associates with each point in the universe of discourse X a real number in the interval [0, 1], with the value of $\mu_A(x)$ at x representing the degree of membership of x in A is called *membership function*. The universe of discourse X in concrete cases has to be chosen according to a real problem in a specific situation. The *membership function* indicates the transition of an object from not belonging to belonging is gradual, which helps us to handle impreciseness and vagueness of variables. Mathematically, the membership function can be expressed by:

$$\mu_A(x) = \begin{cases} 0 < \mu_A(x) \le 1 & \text{for } x \in A \\ 0 & \text{for x} \notin \text{A} \end{cases} \qquad (2.6)$$

in which '0' means complete exclusion from the set A, "$0 < \mu_A(x) < 1$" means partial inclusion from the set A and '1' means absolute inclusion in the set A.

Membership functions are the crucial component of fuzzy set theory, i.e., fuzziness in a fuzzy set is determined by its membership function. Accordingly, the shapes of membership functions are a useful tool for a particular problem since they affect a fuzzy inference system. It was introduced (Zadeh 1965; Zadeh 1978) and initially widely accepted as membership functions are the subjective and based context of the events, latter from measurement view, it is the connection of both subjective and objective to make a sound decision. There are numerous types of membership functions such as triangular, trapezoidal, Gaussian, singleton, bell curves, sigmoidal functions, etc. However, the only condition a membership function must satisfy is that it must vary between zero and one. To make the best choice, one needs a lot of "experience" with the given situation. Due to its simplicity, the frequently applied membership function is the triangular membership function, which is formed using straight lines.

2.6.3 Fuzzy Numbers

The fuzzy number is expressed as a fuzzy set defining a *fuzzy interval* in the real number $\Re$. Since the boundary of a *fuzzy interval* is ambiguous, the interval is also a *fuzzy set*. Generally, a *fuzzy interval* is represented by two endpoints $x_1^{-(0)}$ and $x_3^{+(0)}$, and a peak point $x_2^{(1)}$ as shown in Figure 1. *Fuzzy numbers* are a special case of fuzzy sets that have to satisfy (Lee 2004) all the conditions: convex fuzzy set; normalized fuzzy set; its membership function is piecewise continuous and it's defined in the real number. Mathematically, *fuzzy numbers are* expressed as:

$$X = \{(x, \mu_X(x)) : x \in \Re; \mu_X(x) \in [0,1]\} \qquad (2.7)$$

where X is the fuzzy number; $\mu_X(x)$ is the membership value of the element x to the fuzzy number X, and $\Re$ is the set of real numbers

The condition of normalization in the fuzzy set implies that the maximum membership value is 1.

$$\exists x \in \Re, \qquad \mu_A(x) = 1$$

The convex condition of a fuzzy set is that the line by $\alpha - cut$ is continuous and α*-cut* interval satisfies the following relation.

$$A_\alpha = \left[x_1^{(\alpha)}, x_3^{(\alpha)}\right] \qquad (2.8)$$

$$(\alpha' < \alpha) \Rightarrow \left(x_1^{(\alpha')} \le x_1^{(\alpha)}, x_3^{(\alpha')} \ge x_3^{(\alpha)}\right)$$

The convex fuzzy set condition may also be written as (Lee 2004) $(\alpha' < \alpha) \Rightarrow (A_\alpha \subset A_{\alpha'})$. If all the α*-cut* sets are convex, the fuzzy set with these α*-cut* sets is convex (see Figure 1). In other words, if a relation

$$\mu_A(x) \ge \min\left[\mu_A(x_1), \mu_A(x_2)\right] \qquad (2.9)$$

where $x = \lambda x_1 + (1-\lambda)x_2$, $x_1, x_2 \in \Re^n$, $\lambda \in (0,1)$ holds, the *fuzzy* set A is *convex*. A *fuzzy* variable X with the membership function $\mu_X(x)$ is *strongly convex* if and only if the event $\{x \mid \mu_X(x) \ge \alpha\}$ is strongly convex $\forall \alpha \in (0,1]$.

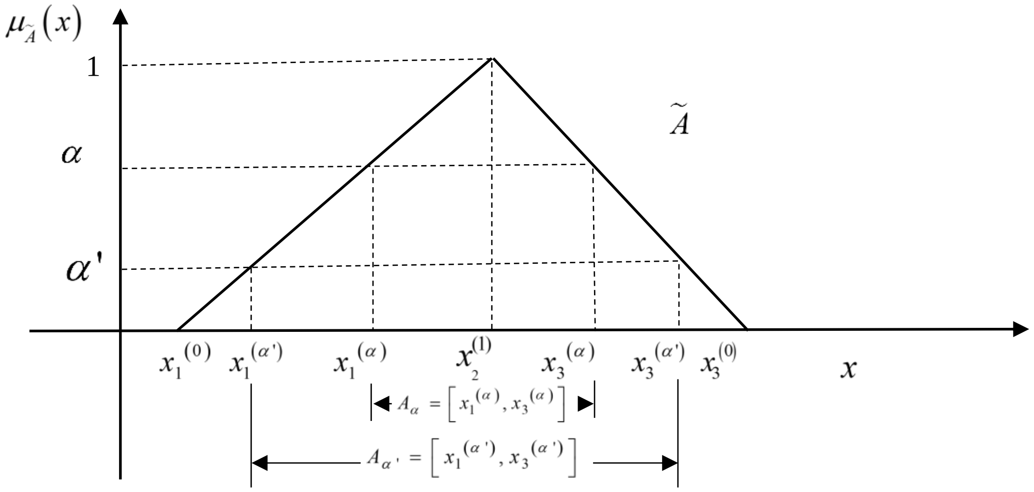


Figure 2.1: Alpha-cut fuzzy number (source: K. H. Lee 2004)

2.6.4 Standard Operation Fuzzy Set

Zadeh (1965) induced the combination of membership functions and their several properties involving fuzzy sets that are noticeable extensions of the corresponding definitions for conventional sets. The membership function is a crucial part of a fuzzy set. It is, therefore, operations with fuzzy sets are defined through their membership functions. The most widely used operations are called *standard fuzzy set operations* such as complements, intersections, and union.

Complement: The *complement* of a fuzzy set A is denoted by A' and is defined by

$$\mu_{A'}(x) = 1 - \mu_A(x), x \in X \quad (2.10)$$

Union**:** The *union* of two fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C, written as $C = A \bigcup B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}, x \in X \quad (2.11)$$

Intersection: The *intersectio*n of two fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C, written as $C = A \bigcap B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \text{Min}\{\mu_A(x), \mu_B(x)\}, x \in X \quad (2.12)$$

2.6.5 Operation of Fuzzy Interval and α-Cut Interval

The α-cut interval of fuzzy numbers is a crisp set and the operation of the fuzzy numbers can be generalized from that of a crisp interval. Suppose $A=[x_1,x_3], B=[y_1,y_3], \; \forall x_1,x_3,y_1,y_3 \in \Re$ that A and B as numbers are articulated as the fuzzy interval, and the main operations of these intervals are (Lee 2004) addition (+) and subtraction (-), in which the shape of membership function will not be changed; multiplication ($\bullet$) and division (/), in which the shape of membership function will be changed and inverse interval, $[x_1,x_3]^{-1}=[1/x_1 \wedge 1/x_3, 1/x_1 \vee 1/x_3]$ excluding the case $x_1=0$ or $x_3=0$.

The α*-cut set* is the crisp set of the elements whose degree of membership is greater than or equal to α, (Lee 2004; Zimmermann 2011) i.e., $X_\alpha=\{x,\mu_X(x)\geq\alpha : x\in\Re;\alpha\in[0,1]\}$, and $X_\alpha=\{x,\mu_X(x)>\alpha\}$ is *strong* α*-cut set* in which X_α is the crisp set at the α-level set and α is the credibility level. Note that the fuzzy operations such as addition, subtraction, multiplication, division, cross product, disjunction, conjunction, implication, relation, and composition are an extension of standard fuzzy set operations (Lee 2004).

2.7 Reliability of Structure

The structural design aims to provide the optimum level of safety and serviceability during its design life. However, design engineers try to provide the optimum design it is inevitable to avoid uncertainties of design parameters that are associated with different factors that lead to over or under-design solutions. In practice, the distribution of either aleatory or epistemic uncertainties of variables may not be appropriately represented (Balu and Rao 2012). Eventually, factitious assumptions for the distribution of the variable uncertainties will lead to inaccurate results.

Reliability is defined as the probability that a structure or system under consideration can perform the intended function under specified service conditions over a given time (Ranganathan 1999; Lemaire 2009). Conversely, the *failure probability* is the probability that a structure does not perform satisfactorily within a given time and stated conditions. Contrary to reliability, *failure possibility* is the possibility of performance less than zero under fuzzy uncertainty (Fan et al. 2019). These definitions emphasis five important elements, such as (i) because of aleatory uncertainties or epistemic uncertainties reliability is a *probability* or *possibility*, respectively; (ii) *intended function* indicates the reliable structure perform a certain function(s) satisfactorily; (iii) *time* is the design life of a structure, and (iv) *service condition* signifies the actions or stresses that may be imposed on the structure.

2.7.1 Methods of Reliability Analysis

In the real structures, both fuzzy uncertainty and random uncertainty widely exist. Thus, it is necessary to develop reliability analysis methods for guaranteeing the safe service of the designed structure under the whole life based on the existing type of uncertainty. To handle these uncertainties several investigators developed and illustrated different methods of structural reliability analysis considering probability theory (Dolinski 1982; Madsen 1985; Du and Sudjianto 2008; Barbato et al. 2008; Zhang and Du 2010). The design of the structure in the presence of the aleatory uncertainty is common practice, and probabilistic reliability analysis methods are well developed and adopted by several investigators. Lu et al. (1994) performed the reliability analysis is using first-order reliability methods (FORM) in which the formulation of the limit-state functions is consistent with the underlying design criteria. Reliability indices for various failure modes are compared and a system reliability analysis is performed to include all failure modes of the reinforced concrete beam. Val et al. (1998) elaborated several aspects of a method for reliability assessment of the RC slab bridges with corroded reinforcement using the FORM considering probabilistic models for design variables.

Abdelouafi et al. (2015) evaluated the reliability analysis of reinforced concrete structures for seismic performance-based probability theory. The study found that the reliability results are sensitive to the reliability analysis method used in that importance sampling method (ISM) is recommended over FORM because the latter overestimates the probability of failure. Pantoja et al. (2010) calculated the reliability index and probability of failure modes, considering both the safety and ductile behavior of the strut-and-tie model using Monte Carlo simulation (MCS). Mori and Ellingwood (1993) presented a method for evaluating the time-dependent reliability of a structural system subjected to stochastic loads using MCS and adaptive importance sampling (AIS) by taking account of the structural deterioration due to environmental stressors. Found that unlike systems evaluated by simple MCSs, the accuracy of the failure probability evaluated by adaptive importance sampling is relatively insensitive to the magnitude of the probability.

Stewart and Rosowsky (1998) developed a structural deterioration reliability model due to corrosion to calculate probabilities of structural failure for a typical reinforced concrete continuous slab bridge. Bhargava et al. (2011b) addressed time-dependent reliability analyses of RC beams affected by reinforcement corrosion using the MCS. In the study, initially, the predictive models are presented for the quantitative assessment of time-dependent damages in RC beams recognized as a loss of mass and cross-sectional area of reinforcing bar, loss of concrete section owing to the peeling of cover concrete, and loss of bond between corroded reinforcement and surrounding cracked concrete. The time-dependent reliability analysis was also developed (Hu and Du 2013) by a more accurate method that relaxes the assumption by using joint upcrossing rates. The method extends the existing joint upcrossing rate method to general limit-state functions with both random variables and stochastic processes employing FORM. Wang et al. (2015) presented the evaluation of the time-dependent reliability for dynamic mechanics with insufficient time-varying uncertainty information using a *non-probabilistic convex process model.*

Dealing with uncertainties in human knowledge is a difficult task within the probabilistic framework (Sexsmith 1999). The fuzzy concept has been adopted to overcome drawbacks with uncertainties in human knowledge and design parameters. The conventional reliability theory is based on the system alternates between two states, i.e., functioning or failed, and the system behavior is fully characterized in the context of probability measure assumptions. On the contrary, (Kai-Yuan et al. 1993; Kai-Yuan et al. 1991b; Kai-Yuan et al. 1991a; Kai-Yuan et al. 1991c; Kai-Yuan et al. 1991; Kai-Yuan et al. 1995) were the pioneers to introduce the possibility and fuzzy-state assumptions to replace the probability and binary-state assumptions. Based on this direction, probability and fuzzy-state assumptions reliability theory, possibility and binary-state assumptions reliability theory, and possibility and fuzzy-state assumptions reliability theory were established.

Several investigators conducted comparative studies between probability theory with possibility theory using probability theory with fuzzy set theory and found (Kai-Yuan et al. 1991a; Cremona and Gao 1997; Nikolaidis et al. 1998; Nikolaidis et al. 2003; Agarwal and Nayal 2015) that: Possibility can be less conservative than probability in risk assessment with many failure modes; In case of small sample size, possibility measure is preferable to characterize the sample particularly in place of probability measures, which measure for sample generality; In the axioms for the union of disjoint events, the probability of the union is the sum of the probabilities of these events, whereas the possibility is equal to the largest possibility; Value of each probability distribution is required to add to 1, while for possibility distributions the largest values are required to be 1; Possibility of failure is exact in the sense that no approximation is required, whereas as in probabilistic approach.

Investigators have further expanded their studies along with these directions. Utkin et al. (1995) investigated the reliability analysis of a general system. Cheng and Mon (1993) conducted a fuzzy system reliability analysis using confidence intervals. Furuta (1995) summarized fuzzy logic and its application to reliability analysis. De Cooman (1996) used the binary-state theory to model possibilistic uncertainty to conduct reliability. Cremona and Gao (1997) constructed a possibilistic alternative to the probabilistic one to provide uncertainty modeling and possibility distribution. Dodagoudar and Venkatachalam (2000) presented a possibilistic approach for the stability analysis of slopes incorporating fuzzy uncertainty. MoÈller et al. (2000) developed and formulated a general method for fuzzy structural analysis based on the fuzzy set theory in terms of $\alpha-\text{level}$ optimization with the application of a modified evolution strategy. Further described coupling between $\alpha-\text{level}$ optimization and the deterministic fundamental solution. Savoia (2002) introduced fuzzy numbers to structural reliability analysis and further extended it to stability analysis. Balu and Rao (2011) presented a practical approach based on high dimensional model representation (HDMR) for analyzing the response of structures with fuzzy parameters by integrating finite element modeling, HDMR based response surface generation, and explicit fuzzy analysis procedures. The uncertainties in the material, geometric, loading, and structural parameters represented using fuzzy concepts. Liu and Liu (2003) developed a numerical algorithm to analyze the fuzzy reliability of mechanical structures. Marano and Quaranta (2010) formulated *Cornell fuzzy reliability index* for structural reliability analysis, and proved it with illustrative examples.

Balu and Rao (2014) presented uncertainty analysis for estimating the possibility distribution of structural reliability in the presence of mixed uncertain variables i.e., random and fuzzy uncertainties. The proposed method involves high dimensional model representation for the approximation to limit state function, transformation technique to obtain the contribution of the fuzzy variables to the convolution integral, and fast Fourier transforms for solving the convolution integral. Others also investigated the possibilistic safety model of structures with fuzzy variables and also proposed a target performance-based design approach (TPBDA), which is a double-loop one, to reduce the computation cost of a triple-loop nested problem of possibilistic safety index-based design optimization (PSIBDO) by the *sequential quadratic programming (SQP)* algorithm (Tang et al. 2014; Zhangchun and Zhenzhou 2014).

The time-dependent failure possibility (TDFP) to measure the safety degree of the structure under the fuzzy uncertainty in a given time interval as, (Fan et al. 2019) the possibility of the performance less than zero under the fuzzy uncertainty in the given time interval, is proposed based on the possibility theory of the safety measure. In the study, the TDFP model has been established to measure the safety degree for the structure under the fuzzy uncertainty over a given time interval. Based on the definition investigators also established a double-loop nested optimization method (DLOM) to compute the TDFP, but the computational cost of the DLOM is too high to solve the problems of practical engineering. Therefore, the efficient single-loop optimization method (SLOM) was established based on the extreme value transformation of the time-dependent performance function to compute TDFP.

Summary

The performance of the structure depends on the strength of materials, cross-sectional dimensions, and operating conditions in its design life. Due to the presence of the uncertainties in material properties, loads on the structure during its life, structural idealization model, and limitation of numerical methods, the absolute safety of a structure is impossible.

There are many important factors, generally classified as design, construction, environment, and utilization that can significantly affect the reliability of reinforced concrete structures. Factors like design, construction, and utilization can be controlled by adopting proper design and construction techniques, and desired utilization in its design life, whereas the environmental factors are difficult to control.

The corrosion of the bar induced by carbonation is uniform and had no obvious abrupt changes in geometry along the length of the bar. By contrast, the corrosion pattern of the bar induced by chloride aggression is somewhat abrupt changes in bar geometry (localized failure) and a great variation in cross-sectional area.

However, uncertainty broadly classified as *aleatory uncertainty* and *epistemic uncertainty* that defines the method of reliability analysis, there are several sources of uncertainties. Many types of research carried out several investigations to estimate the reliability of structures using different methods like *First-Order Reliability Methods, First Order Second Moment (FOSM), Mean-value First-order Second-moment Method (MVFOSM), Second-Order Reliability Method (SORM), Advanced First Order Second Moment (AFOSM), Response Surface Method (RSM), Importance Sampling Method, Monte Carlo simulation*, etc., considering either random variables or fuzzy variables using probability theory. Probability theory is not worth to be used for a small sample size to generalize about the sample. However, the fuzzy set theory has been introduced some decades before, possibilistic theory analysis, which concerns sample particularity for small sample size, in structural engineering needs further investigations.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter mainly describes the design of the rectangular RC beam to EC2, the effects of environmental factors, computation procedure of the degradation of input variables, and consequently, the performance of the structure, and failure possibility analysis to achieve the desired objectives. It also justifies the research approach adopted over the other relevant methods.

3.2 Design Optimization of RC Beam

The design of reinforced concrete is the art of structural analysis to wisely decide the size, shape, and quantity of steel, dimension of the section, the cover thickness, the fashion how to detail the rebars to bring out a skeleton of reinforcements, and the concrete mix as well. This is typically done to ensure safety against the intended purpose and take care of the environmental effects on the concrete taking into consideration the location and mitigation measures taken to prevent the same on the structure during the design stage. The reliable design of the concrete structure is attained by considering appropriate materials properties, maximum possible actions, adequate concrete cover, appropriate methods of analysis and design, determining design life, and control of deflection and crack widths in its design life.

The design optimization was carried out by considering material properties and breadth of the beam as random variables, the total area of the reinforcement and effective depth of beam as design variables, and flexure, shear, long-term deflection, and crack were considered as design constraints based on EC2. Material properties were obtained from Eurocode, and site data has been collected from relevant sources for Addis Ababa, Ethiopia.

3.3 Time-Dependent Performance Analysis

The adequate reliability of the structure is the assured safety against the intended purpose in its design life. The reliability of the structure typically depends on the structural and mathematical models, material properties, design and construction aspects, and the structure system. In design, construction and successful operation of the structure could be exposed to different uncertainties that precisely define the efficient method of reliability analysis. The sources of uncertainties can represent insufficient knowledge of expert level on structure modeling, analysis, design, and construction; environmental factors such as creep and shrinkage that progressively reduce the bending stiffness and increase action on the structure system, and corrosion that deteriorates the material properties and steel diameter; and inappropriate utilization that leads to surpass of maximum expected load on the structure.

In this study, the three important environmental factors, i.e., creep, shrinkage, and corrosion, that degrade the performance of the concrete structures through deteriorating materials properties and the diameter of reinforcing steel, and increase action on the structure were considered. The time-variant creep coefficient and shrinkage strain model has been developed for the cross-section of the reinforced concrete beam considered in the case study and the ambient temperature and relative humidity of the considered site based on Eurocode 2 Appendix B. To account for the corrosion effect, the baseline values for parametric studies have been adopted from the relevant literature and presented in Chapter 5.

3.3.1 Identification and Estimation of Possible Failure Modes

The performance evaluation of the structure needs the estimation of the capacity (resistance) and demand (action) of the structure, in which the structure to be safe the capacity should be greater than the demand of the structure. The concrete structure resistance deteriorates and the action increases with time due to corrosion, creep, and shrinkage. Corrosion deteriorates the structure resistance through deteriorating material properties and the diameter of reinforcement steel whereas, creep and shrinkage deteriorate the structure resistance by reducing the bending stiffness and increase the action through developing additional curvature with time.

The structure system or part of the structure may fail due to the failure of one or more constraints. In general, failure occurs when the action S exceeds the resistance R for ultimate limit state constraints, and the estimated maximum value exceeds the limiting value provided in the standard code for serviceability limit state. The reinforced concrete beam, which is subjected to concentric transverse load, resist the applied load in terms of flexure and shear strength, i.e., ultimate limit state. As a result, these action effects cause deflection and crack, i.e., serviceability limit state, on the structural member in the critical sections.

As a result of the deterioration of the material properties and the variation of force with time, the performance of the structure deteriorates with time. The time-dependent performance of the structure can be obtained from Equation (3.1).

$$G_j\left(X_i,t\right)=R_j\left(X_i,t\right)-S_j\left(X_i,t\right) \tag{3.1}$$

where $G\left(X,t\right)$ is the performance of the structure with time; $R\left(X,t\right)$ is the resistance of the structure with time; $S\left(X,t\right)$ is the action on the structure with time; i represents the input variables (i.e., material properties, section dimensions, loads, etc.), and j represents constraints of the structure (i.e., flexure, shear, deflection and crack width)

3.4 Generation of Membership Functions

The membership function is an appropriate mathematical description to make a subjective evaluation of different real problems. The fuzziness of variables is characterized by membership functions, which recognize the degree of belonging of the variable to the fuzzy set of study variables. A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. There are various types of membership functions, such as triangular, trapezoidal, Gaussian, bell curves, sigmoidal functions, singleton, etc.

However, there is no specific rule and consensus to follow to generate membership, rather it depends on the expertise and the availability of data. The only condition a membership function must satisfy is that it must vary between 0 and 1. There are important steps to generate the membership function of fuzzy variables, such as (i) the identification of fuzzy variables; (ii) the determination of fuzzy interval (lower and upper limit); (iii) the selection of an appropriate type of membership function; (iv) the discretization of the degree of membership and the size of the variable to a reasonable figure; and (v) the generation of the membership function

In this study, the material properties and steel diameter that deteriorate due to corrosion, the variation of load on the structure, and consequently the deterioration of structural capacity with time are considered as fuzzy variables. To generate a fuzzy membership function, the *heuristic method*, which uses predefined shapes, is used for both input variables and output performance. Frequently used shapes of heuristic membership functions are *piecewise linear functions* and *piecewise monotonic functions*. In the case of piecewise monotonic functions, there is a smooth transition between the non-member and full-member regions (Medasani et al. 1998). The linear and piecewise linear membership functions give a reasonably smooth transition, easily handled by fuzzy operators, and easily implemented. The membership functions generated by the heuristic function have the following features (Dombi 1990): (i) all membership functions are *continuous*; (ii) all membership functions *map* an interval $\mu\left[\underline{x},\overline{x}\right]\rightarrow\left[0,1\right]$, and (iii) membership functions are either *monotonically increasing* or *monotonically decreasing* or both increasing and decreasing. Triangular and trapezoidal membership functions are generated by using this method. In this research, the triangular MF is employed because of its simplicity and frequently used in other studies.

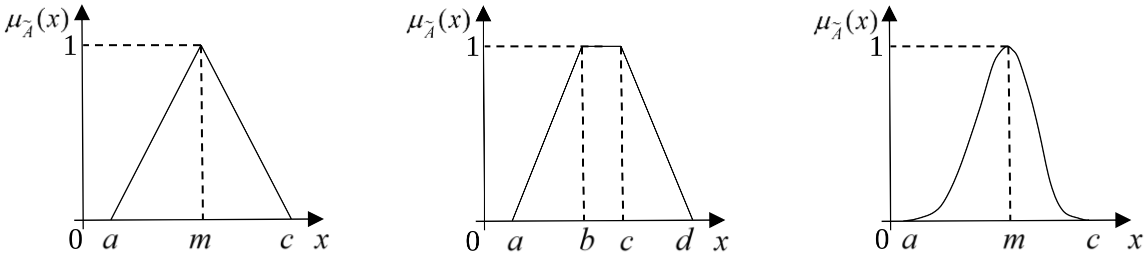
From the definition, fuzziness is the imprecision of study variables. Therefore, a small variation of the fuzzy input variables may significantly affect the output quantities. Thus, due attention is required while choosing the type of membership function. From a list of MFs, let us see the features of triangular, trapezoidal, and Gaussian membership functions. Mathematically, the functions of triangular, trapezoidal, and Gaussian membership are represented in equations (3.5a), (3.5b), and (3.5c), respectively.

$$\mu_A(x)=\begin{cases}(x-a)/(m-a) & \text{if } a\le x\le m\\(c-x)/(c-m) & \text{if } m\le x\le c\\0 & \text{Otherwise}\end{cases} \tag{3.2a}$$

$$\mu_A(x)=\begin{cases}(x-a)/(b-a) & \text{if } a< x\le b\\1 & \text{if } b< x< c\\(d-x)/(d-c) & \text{if } c\le x\le d\\0 & \text{Otherwise}\end{cases} \tag{3.2b}$$

$$\mu_A(x)=e^{\left\{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right\}} \tag{3.2c}$$

where x is the fuzzy variable within the fuzzy interval; a is the lower bound of the fuzzy variable; c is the upper bound of the fuzzy variable for triangular and Gaussian MF; d is the upper bound of the fuzzy variable for triangular and trapezoidal MF, and m is the nominal or mean value of the fuzzy variable, and σ is the standard deviation



a) Triangular MF b) Trapezoidal MF c) Gaussian MF

Figure 3.1: Features of membership functions

From the features and numerical expression of the membership function described above, we can select the appropriate one for this study. Nevertheless, for trapezoidal MF there is a defined fuzzy interval, as shown in Figure 3.1(c) the degree of membership is unity for the interval [b, c]. The input variable's sensitivity depends on the nature of the problem and error tolerance. In the case of reinforced concrete, the desired constraints such as bending moment, shear force, deflection, and crack are significantly affected by a small variation of the input variable. As expressed in Equation 3.2(c), the generation of Gaussian MF requires the mean value, standard deviation, and fuzzy interval of the study variable. If the assumed standard deviation is applied, this leads to the variables being further uncertain. To get a mean value and standard deviation, therefore, a representative sample size is necessary. Hence, it is rational for the case study to employ a triangular MF that needs a fuzzy interval and nominal value of the study variable.

3.5 Failure Possibility Analysis

Civil engineering structures are quite complex and have different size, shape, system, and performance capacity. Therefore, the performance of each for its intended purpose depends on material properties, analysis and design accuracy, section dimensions, environmental factors, construction techniques, and loading conditions. Despite using uncertain software models and limited numerical methods there is almost no chance to test the prototype of the civil engineering structure to ensure its absolute reliability.

Hence, each structure needs its version of performance and reliability analysis to ensure whether the structure is safe or in a failure state.

Based on the sample size and types of parameter uncertainty, the reliability analysis methods are classified as probability and possibility methods. The reliability analysis of the structure with random and fuzzy variables based on the probability theory is well developed. In probability theory, the fuzzy variables are converted to random variables based on their corresponding membership grade. However, it is possible to convert fuzzy variables to random variables, it is not worth using the probability theory, which requires a large sample size for sample generality. On the contrary, possibility theory is worthwhile to adopt the reliability analysis for small (even for a single sample) with fuzzy variables. Thus, the time-dependent failure possibility analysis of the reinforced concrete structure exposed to different environmental factors that deteriorate material properties, the diameter of the reinforcement bar, increase action on the structure and consequently deteriorate the performance of the structure that possesses fuzzy uncertainty of different input and output variables. This fuzziness of the section capacity leads to considering the possibility theory to accurately estimate the failure possibility of the structure.

In this study, the failure possibility, complementary to the reliability, analysis is employed to estimate the reliability of concrete structure because a single structure member is considered and fuzziness of input and output variables. To evaluate the time-dependent failure possibility analysis, the time-dependent performance and membership functions of the input variables and defined constraints were determined for the specified time interval $t \in [t_s, t_e]$. In a specified time interval, the minimum performance of the structure governs the reliability of the structure. Thus, using the *extreme value transformation* method, $\underset{t\in[t_s,t_e]}{Min}\, G(X,t), t \in [t_s, t_e]$ and $\mu_{G_{\min}(Y,t)}$ used to estimate the time-dependent failure possibility of the concrete structure from the following expression (Fan et al. 2019).

$$\pi_{f_j t\in[t_s,t_e]} = Poss\left\{ \underset{t\in(t_s,t_e)}{G_{\min}}\left(X,t\right) \le 0, \exists t \in \left[t_s, t_e\right] \right\} \tag{3.3}$$

The time-dependent failure possibility of the reinforced concrete structure is estimated using a numerical algorithm, presented in Chapter 6. The generalized methodology of this study is shown in Figure 3.2.

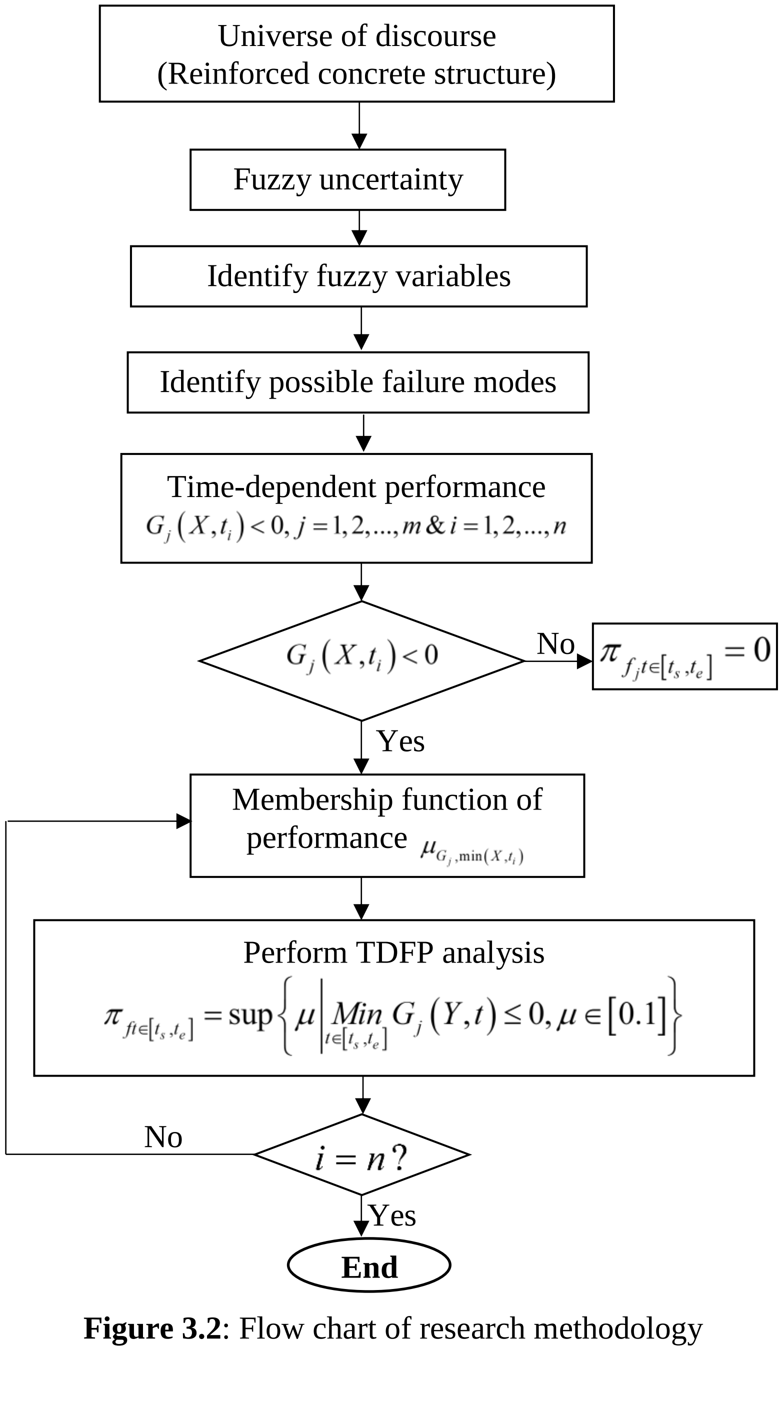


Figure 3.2: Flow chart of research methodology

CHAPTER 4

TIME-DEPENDENT PERFORMANCE OF CONCRETE STRUCTURES

4.1 Introduction

The performance of a structure can be assessed for its safety, serviceability, and economy criteria in design life to know whether the system is reliable. Despite the expectation of long service of civil engineering structures, the resistance deteriorates with time due to loads exceeding the expected design load and the various environmental factors that will occur within the design life.

This chapter presents the conventional design optimization of the RC beam to Eurocode, the general performance of the structure for various resistance and actions. It predominantly portrays time-variant materials properties, the time-dependent performance of concrete structures of both safety and serviceability criteria.

4.2 General Performance of Structures

The structure performance can be evaluated based on its capacity-demand ratio. Let us take R to be the resistance (capacity) (e.g., flexure, shear, axial, torsion, or combination of two or more) of the structure and S be the demand (action) (e.g., dead loads, imposed loads, environmental loads, or combination of two or more) of the structure. In a real problem, both resistance and action may be a combination of random and fuzzy uncertain variables. Based on appropriate consideration of the uncertainty of variables and design methods to achieve reliable (i.e., capacity greater than demand) structure. The structure may fail by implementing poor design methods via insufficient knowledge, deterioration of constituent material strength along with design life. Suppose the space D in a two-dimensional plane is a plot of capacity and demand. In reliability analysis, the space D of the capacity-demand curve may divide into the failure and safe region shown in Figure 4.1. The mathematical expression of the performance function is the difference between the capacity $R(X)$ and demand $S(X)$ of the structure.

$$G(X) = R(X) - S(X) \tag{4.1}$$

Three critical safety states from Equation (4.1) of the performance of the structure are: when the performance $G(X,t) \leq 0$ falls in the *failure state*; the performance $G(X,t) > 0$ falls in the *safe state,* and the performance $G(X,t) = 0$ is the *limit state*. The degree of curve in Figure 4.1 depends on the type of loading, type of constraints (e.g., flexure, shear, axial load, deflection or crack), the shape of the structure, structural system (i.e., support condition), and so on.

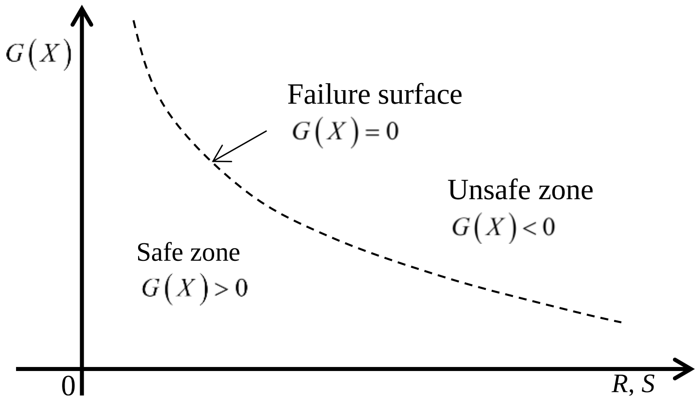


Figure 4.1: Shows limit state, safe and unsafe regions

4.3 Time-Dependent Performance of RC Structure

The design of a structure is to satisfy the intended purpose in its design life. However, the performance of the structure deteriorates with time due to operating conditions and environmental factors that deteriorate material properties and induce additional load with time. Properties of the material deteriorate over time shown in Table 4.1 results degradation in the performance of the structure. In general, the time-dependent performance of the structure obtained from the expression:

$$G(X,t) = R(X,t) - S(X,t) \tag{4.2}$$

where, $G(X,t)$ is the performance of the structure with time; $R(X,t)$ is the resistance of the structure with time, and $S(X,t)$ is the action on the structure with time. The representation Equation (4.2) applies to all desired ultimate and serviceability limit state constraints. Thus, the failure of the structure is progressive and time-dependent. To explore the time-dependent failure possibility of the concrete structure the following case study has been employed through the procedures shown in Figure 3.2.

A simply supported 6 m span reinforced concrete beam provided in the salt storage building in Addis Ababa, Ethiopia, is subjected to a permanent load of 12 kN/m and an imposed load of 18 kN/m is considered. Addis Ababa, the capital of Ethiopia, lies at an elevation of 2355 m above mean sea level and is located at $9^{0}01^{'}48^{''}$N $38^{0}42^{'}24^{''}$E, where the average annual temperature and relative humidity of $15.9\ ^{o}C$ and $60.7\ \%$, respectively. The average minimum temperature of consecutive three months (i.e., November, December, and January) is $7\ ^{o}C$, which indicates below $7\ ^{o}C$ also exists that can meltdown the salt deposit.

The core value of loading $w = 1.35g_k + 1.5q_k$ is equal to 43.20 kN/m at the time $t = 0$, whose design bending moment and shear force are $M_{Ed} = 194.40$ kNm and $V_{Ed} = 129.60$ kN, respectively. The design of flexural reinforcement is carried out based on EC2 shown in Figure 4.2 using materials of concrete grade C25/30, steel grade 460 MPa for longitudinal reinforcement, cement type is class N and maximum size of aggregate, $d_a = 25$ mm. The design optimization of flexural reinforcement and the effective depth of the beam section has been maintained by Equation (4.3) constraint expressions.

Objective: Mimimize $A_{st} = M_{Ed}/zf_{yd}$ and d

Constraints:

$$0.87A_{st} f_{yd}\left(d - 0.652\frac{A_{st} f_y}{bf_{ck}}\right) - M_{Ed} \le 0 \qquad (4.3a)$$

$$\text{Basic } \frac{L}{d}.\frac{310}{\sigma_s} - \frac{L}{d} \le 0 \qquad (4.3b)$$

$$0.3 - w_{k,\max} \le 0 \qquad (4.3c)$$

$$A_{st,\min} \le A_{st} \le 0.04A_c;\ 300 \le d \le 500 \qquad (4.3d)$$

where: M_{Ed} is the design bending moment; b, d are breadth (300 mm is considered) and effective depth of the beam, respectively; L is the effective span length of the beam, i.e., 6000 mm; the limiting deflection is span/250, i.e., $\delta_{\lim} = 6000/250 = 24$ mm in EC2 (Clause 7.4.1); basic span/depth ratio and σ_s are provided in EC2 (clause 7.4.2

); and $A_{st,\min}$ is the minimum area of steel required in the section, i.e., $0.26 f_{ctm} bd / f_{yk}$.

Similarly, the design optimization of shear reinforcement of the beam section for a steel grade of links 250 MPa is maintained by satisfying Equation (4.4).

Objective: Mimimize A_{sw} and s

Constraints:

$$s - \frac{0.9 A_{sw} d f_{ywd}}{V_{Rd,s}} \leq 0 \qquad (4.4a)$$

$$s_{\min} \leq s \leq s_{\max} \qquad (4.4b)$$

where: $V_{Rd,s}$ is the design shear resisted by stirrups i.e., $V_{Rd,s} = V_{Ed} - V_{Rd,c}$ in which V_{Ed} is the design shear force due to applied load at d distance from the face of support; $V_{Rd,c}$ is the shear resistance of the concrete section; A_{sw} is the area of two-legged shear reinforcement; f_{ywd} is the design yield stress of the shear reinforcement; $s_{\min}$ is the minimum spacing shear reinforcement, i.e., $A_{sw} f_{yk} / (0.08 b \sqrt{f_{ck}})$; $s_{\max}$ is the maximum spacing of shear reinforcement which is equal to $0.75d$.

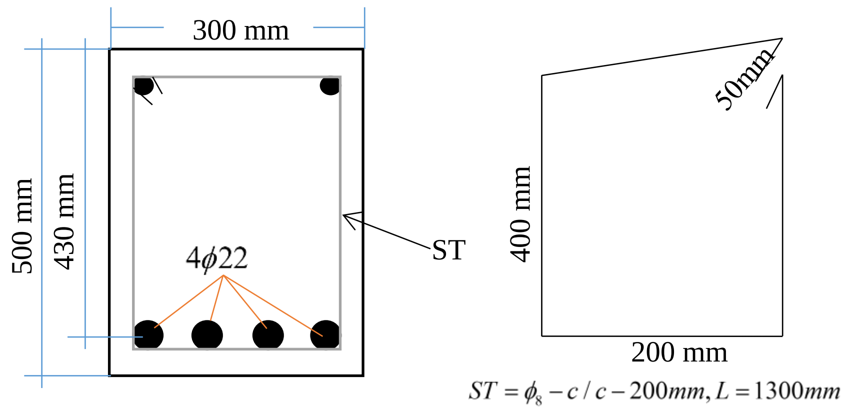


Figure 4.2: Detail of reinforced concrete beam section

4.3.1 Evaluation of Time-Variant Performance of Input Variables

Concrete, reinforcing steel, and loads are key input variables in reinforced concrete structures. The material properties deteriorate over time, even under normal environmental conditions, due to creep, shrinkage, and cyclic loading.

4.3.1.1 *Effects of Creep and Shrinkage*

The time-variant creep coefficient and shrinkage strain model have been developed for the concrete beam using empirical expressions provided in Eurocode 2 Annex B (ES EN 1992-1-1 2004). Input parameters considered for time-variant creep coefficient and shrinkage strain model were the cross-sectional area of concrete $A_c = 0.15\ \text{m}^2$, three parts of cross-section perimeter exposed to drying $u = 1.3\ \text{m}$, age of concrete loading $t_o = 7\ \text{days}$, the ambient temperature and relative humidity of Addis Ababa are $15.9\ ^oC$ and $RH = 60.7\ \%$, respectively, the type of cement considered is $Class\ N$, and the age of the concrete at the end of curing $t_s = 7\ \text{days}$. The creep coefficient and shrinkage strain increase rapidly at an early stage and attain their maximum intensity slowly as shown in Figure 4.3. The final creep coefficient and shrinkage strain at the infinite time are $\varphi(\infty, t_0) = 3.159$ and $\varepsilon_{cs}(\infty, t_s) = 40.97 \times 10^{-5}$, respectively. The detail of the model is provided in Appendix A.

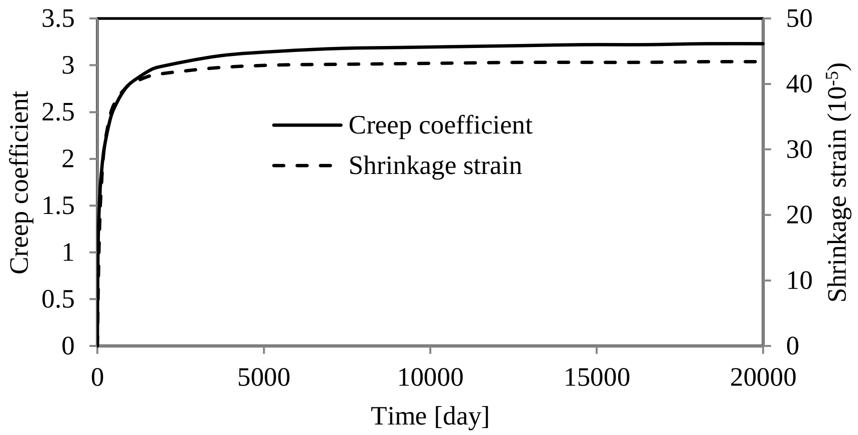


Figure 4.3: Variation of creep coefficient and shrinkage strain with time

4.3.1.2 *Deterioration of Input Variables due to Corrosion*

Corrosion is a significant factor in a hostile environment, which adversely affects the material properties, diameter of steel reinforcement, and changes the color of the concrete. The corrosion initiation time is an important parameter to consider its effect on the structure's design life. To estimate the corrosion initiation time, parameters such as the chloride diffusion coefficient, chloride concentrations, and the concrete cover of the structure member are required. The baseline values of the corrosion parameters used for parametric studies proposed by Enright and Frangopol (1998) were: chloride diffusion coefficient $D_c = 1.29\ \text{cm}^2/\text{year}$; surface chloride concentration $C_o = 0.10$ (percentage to the weight of concrete), and critical chloride concentration $C_{cr} = 0.04$ (percentage to the weight of concrete) and these parameters later adopted by different researchers (François et al. 2013; Kliukas et al. 2015; Chehade et al. 2018). The rate of corrosion is also an important parameter required to estimate the corrosion initiation time. The corrosion rate r_{corr} is obtained from the corrosion current density, i_{corr} which is given by $\mu\text{A/cm}^2$ (Enright and Frangopol 1998; Val et al. 1998) in which $1\mu\text{A/cm}^2$ is equal to $11.6\mu\text{m/year}$. In this research, a moderate corrosion rate is considered as the current density of $i_{corr} = 0.75\ \mu\text{A/cm}^2$ that is $i_{corr} = 8.7\ \mu\text{m/year}$.

$$r_{corr} = 0.0232 i_{corr} \tag{4.5}$$

In addition to corrosion parameters, concrete cover, i.e., $5.8\ \text{cm}$ and $5\ \text{cm}$ for flexural and shear reinforcement, respectively, provided in Figure 4.2 is required to determine the corrosion initiation time. Thus, the corrosion initiation time from Equation (2.1) is 18.410th year and 13.368th year for flexural and shear reinforcement, respectively, using the MATLAB program. Thus, the time-variant diameter of the flexural reinforcement is determined using Equation (2.2) for various corrosion rates,

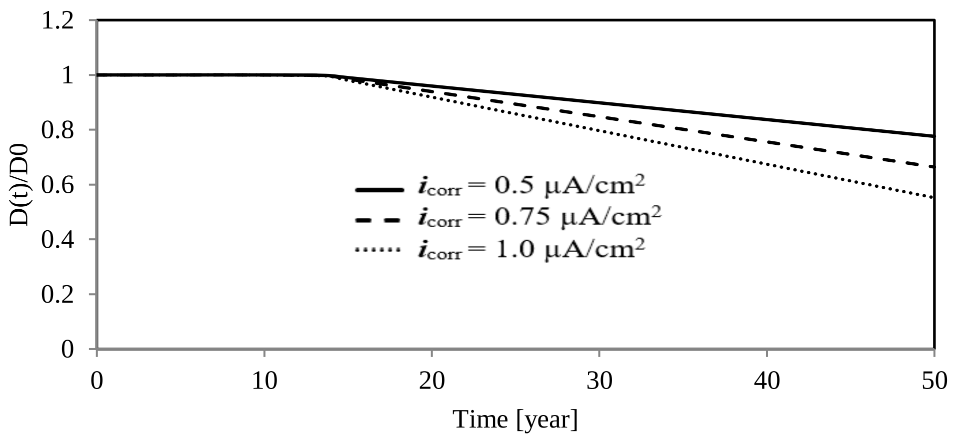


Figure 4.4: Reduction of reinforcement bar diameter vs time for various corrosion rates

and the reduction of steel diameter increases with increased corrosion rate as shown in Figure 4.4.

A moderate corrosion current density of $0.75\mu A/cm^2$ which is $8.7\mu\text{m}/\text{year}$ considered in this research. The deterioration of steel diameter, concrete strength, and yield stress of steel due to corrosion are obtained by using Equation (2.2), (2.3), and (2.4), respectively, and results for the corrosion current density $0.75\mu A / cm^2$ shown in Table 4.1.

Table 4.1: Time-variant c/s area and yield stress of steel, strength and modulus of elasticity of concrete due to corrosion

Time (year)	$D_i(t)$ (mm)	$A_{sl}(t)$ (mm^2)	$f_{ck}(t)$ (MPa)	$E_{cm}(t)$ (GPa)	$f_{yl}(t)$ (MPa)	$A_{sw}(t)$ (mm^2)	$f_{yw}(t)$ (MPa)
0	22	1520.531	25	31	460	100.531	250
5	22	1520.531	25	31	460	100.531	250
10	22	1520.531	25	31	460	100.531	250
15	22	1520.531	25	31	460	93.946	249.989
20	21.679	1476.493	20.307	29.530	459.989	71.027	249.746
25	20.670	1342.226	20.288	29.518	459.815	51.307	249.184
30	19.661	1214.358	20.267	29.506	459.428	34.787	248.305
35	18.651	1092.889	20.245	29.492	458.829	21.467	247.107
40	17.642	977.820	20.222	29.479	458.015	11.346	245.590
45	16.633	869.150	20.198	29.464	456.989	4.425	243.756
50	15.624	766.880	20.172	29.449	455.750	0.703	241.603

4.3.2 Safety Performance Criteria

The ultimate limit state (ULS) deals with the strength and stability of the structure under the maximum overload it is expected to carry. That implies that the system should not fail under any combination of anticipated maximum stress. The safety performance evaluation is an examination of the carrying capacity and the internal design force of the structure. A reinforced concrete beam may be subjected to bending, shear, torsional stress, or their combination in a lifetime. Bending stress is the critical stress in the design of a reinforced concrete member (Koteš et al. 2016) to decide the size of the section, area, and configuration of the steel reinforcement. Then, the member is designed for shear to ensure the safety against the shear stress (Koteš et al. 2015; ES EN 1992-1-1 2004; Mosley et al. 2012) in a whole lifetime because a shear failure is mostly sudden and brittle.

A concrete structure performs its intended purpose satisfactorily over its design life against all anticipated stresses to achieve safety performance. The two essential safety criteria, such as flexure and shear, are explored by the time-varying parameters in this study.

4.3.2.1 Flexural Performance

Adequate design loads, suitable material properties, corrosion, creep coefficient, and shrinkage strain are critical factors that need consideration for assessing the time-dependent flexural resistance of the section. The flexural capacity of concrete structures degrades over time because of increased creep and shrinkage and degradation of the steel reinforcement area by corrosion shown in Table 4.1. The time-dependent flexural performance of the beam section estimated from the general expression:

$$G(X_i(t)) = M_R\left(A_s, f_y, f_{ck}, d, b, ...\right)(t) - M_a\left(w_{DL}, w_{LL}, sh, ...\right)(t) \qquad (4.6)$$

where, $M_R(t)$ is the ultimate bending moment capacity of the beam, and $M_a(t)$ is the actual bending moment load at mid-span

The resistance and action in the beam section vary with time shown in Equation (4.6). The time-variant flexural resistance of the RC beam section that considers the effect of corrosion in terms of material properties and area of steel reinforcement is from Equation (4.7).

$$M_R(t) = 0.87 A_s(t) f_y(t)\left[d - 0.652\frac{A_s(t) f_y(t)}{b f_{ck}(t)}\right] \qquad (4.7)$$

Creep affects the concrete property by reducing the modulus of elasticity of the concrete and consequently, the flexural rigidity of the member reduces. The effective time-variant modulus of the elasticity of the concrete due to corrosion and creep is obtained from:

$$E_{c,ef}(t) = E_{cm}(t) / \left(1 + \varphi(t, t_0)\right) \qquad (4.8)$$

$$E_{cm}(t) = 22\left[\frac{f_{cm},(t)}{10}\right]^{0.3} \qquad (4.9)$$

where $E_{cm}(t)$ is the time-dependent elastic modulus of concrete in [GPa], and $\varphi(t, t_0)$ is the time-variant creep coefficient

In combination, creep and shrinkage induce additional curvature on the member, which also induces additional bending moment and shear force on the member (Mosley et al. 2012).

$$r_{t,sh} = \frac{E_{c,ef}(t) I_c(t)}{M_{sh}(t)} \qquad (4.10)$$

Hence, the flexural force induced by creep and shrinkage is time-dependent and can be obtained from the moment-curvature relationship as expressed in Equation (4.11).

$$M_{sh}(t) = \frac{E_{c,ef}(t) I_c(t)}{r_{t,sh}} \tag{4.11}$$

in which $1/r_{t,cs}$ is the curvature from shrinkage and expressed as:

$$\frac{1}{r_{t,sh}} = \xi\left(\frac{1}{r}(t)\right)_{sc} + (1-\xi)\left(\frac{1}{r}(t)\right)_{su} \tag{4.12}$$

where ξ is the coefficient given by $1-\beta(\sigma_{sr}/\sigma_s)$ allowing for tension stiffening; $(1/r(t))_{su}$ and $(1/r(t))_{sc}$ are the time-variant curvature of the uncracked and cracked section, respectively; $\sigma_{sr}/\sigma_s = M_{cr}/M_{Qp}$; σ_{sr} is the steel stress at first cracking; σ_s is the steel stress of quasi-permanent service load; β is the coefficient takes into account duration of loading and which is 0.5 for sustained loads; M_{cr} is the crack moment of the concrete section, $M_{cr} = f_{ctm}.I_u/(h-x_u)$; M_{Qp} is the moment of the quasi-permanent moment at the critical section

Therefore, the time-variant bending moment (or action) of a simply supported reinforced concrete beam subject to uniformly distributed load and creep and shrinkage effect can be determined from:

$$M_a(t) = \frac{(w_{d,DL} + w_{d,LL}(t))L^2}{8} + \frac{E_{c,ef}(t) I_c(t)}{r_{t,sh}} \tag{4.13}$$

The curvature of both uncracked and cracked conditions of creep and shrinkage are estimated from the Equations (4.14) and (4.15), respectively (ES EN 1992-1-1 2004; Mosley et al. 2012).

$$\left(\frac{1}{r}(t)\right)_{su} = \frac{\varepsilon_{cs}(t).\alpha_e(t).S_u}{I_u} \tag{4.14}$$

$$\left(\frac{1}{r}(t)\right)_{sc} = \frac{\varepsilon_{cs}(t).\alpha_e(t).S_c(t)}{I_c} \tag{4.15}$$

where $\alpha_e(t)$ is the effective time-variant modular ratio given by $\alpha_e(t) = E_s/E_{c,eff}(\mathrm{t})$; E_s is the elastic modulus of steel reinforcement (200GPa); $\varepsilon_{cs}(t)$ is the time-variant shrinkage strain; I_u and I_c are the second moment of area for an uncracked and cracked condition, respectively; x_u and x_c are the neutral axis for an uncracked and cracked condition, respectively; Su and $Sc(t)$ are the first moments of area of the reinforcement about the centroid of the uncracked and fully cracked section, respectively

A sufficiently accurate calculation is performed for neutral axis depth, the first and second moment of area of the section (Mosley et al. 2012; MacGregor et al. 1997), by considering the transformed section as shown in Figure 4.5. The neutral axis depth, the first and second moment of area of the transformed section determined from Equations (4.16) - (4.21).

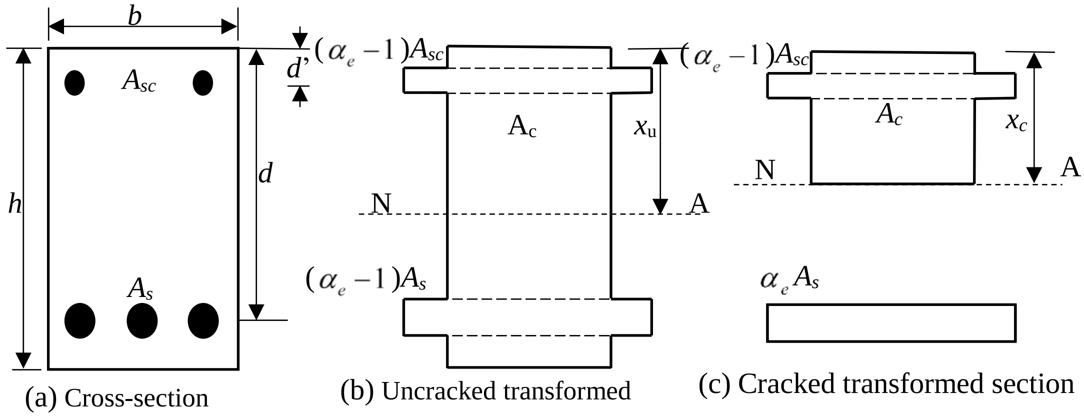


Figure 4.5: Shows the transformed uncracked and cracked sections

Uncracked section properties

Depth of the neutral axis, x_u :

$$x_u = \text{Area Moment / Area} \tag{4.16a}$$

where:

$$\text{Area Moment} = (\alpha_e - 1) A_{sc} d' + (\alpha_e - 1) A_s d + bh^2 / 2 \tag{4.16b}$$

$$\text{Area} = (\alpha_e - 1) A_{sc} + (\alpha_e - 1) A_s + bh \tag{4.16c}$$

The first-moment area S_u of the reinforcement about the centroid of the section:

$$S_u = A_s.(d - x_u) \tag{4.17}$$

The second-moment area, I_u :

$$I_u = I_{u,conc} + I_{u,A_{sc}} + I_{u,A_s} \tag{4.18a}$$

Where:

$$I_{u,conc} = bh^3/12 + bh\left(x_u - h/2\right)^2 \tag{4.18b}$$

$$I_{u,A_{sc}} = \left(\alpha_e - 1\right) A_{sc}\left(x_u - d'\right)^2 \tag{4.18c}$$

$$I_{u,A_s} = \left(\alpha_e - 1\right) A_s\left(d - x_u\right)^2 \tag{4.14d}$$

Cracked section properties

Depth to the neutral axis, x_c:

$$x_c = \left[-b_{eq} + \left(b_{eq} - 4ac\right)^{0.5}\right]\Big/2a \tag{4.19a}$$

Where:

$$a = b/2 \tag{4.19b}$$

$$b_{eq}\left(t\right) = \left(\alpha_e\left(t\right) - 1\right) A_{sc} + \alpha_e\left(t\right) A_s \tag{4.19c}$$

$$c = -\left[\left(\alpha_e\left(t\right) - 1\right) A_{sc} d' + \alpha_e\left(t\right) A_s d\right] \tag{4.19d}$$

The first-moment area $S_c\left(t\right)$ of the reinforcement about the centroid of the section:

$$S_c\left(t\right) = A_s\left(t\right).\left(d - x_c\left(t\right)\right) \tag{4.20}$$

The second moment of area, I_c:

$$I_c\left(t\right) = I_{c,conc}\left(t\right) + I_{c,A_{sc}}\left(t\right) + I_{c,A_s}\left(t\right) \tag{4.21a}$$

Where:

$$I_{c,conc}\left(t\right) = bx_c^{\,3}\left(t\right)/12 + bx_c^{\,3}\left(t\right)/4 = bx_c^{\,3}\left(t\right)\Big/3 \tag{4.21b}$$

$$I_{c,A_{sc}}\left(t\right) = \left(\alpha_e\left(t\right) - 1\right) A_{sc}\left(t\right)\left(x_c\left(t\right) - d'\right)^2 \tag{4.21c}$$

$$I_{c,A_s}\left(t\right) = \alpha_e\left(t\right) A_s\left(t\right)\left(d - x_c\left(t\right)\right)^2 \tag{4.21d}$$

As expressed in Equation (4.7), the flexure resistance of reinforced concrete structure depends on material properties, cross-sectional dimensions, area of steel reinforcement, structural system, loading condition, creep, and shrinkage. The increment of applied bending moment from creep and shrinkage grow rapidly and reach the highest intensity at an early age (see Figure 4.6), due to the maturity of concrete and the effect of creep, which grows rapidly in early age shown in Figure 4.3 that reduces the elastic modulus of concrete. Besides, the performance of reinforced concrete beam significantly decreases with time due to corrosion of reinforcement steel that deteriorates the effective diameter of reinforcement steel and concrete strength and reduces the elastic modulus of concrete with creep increases.

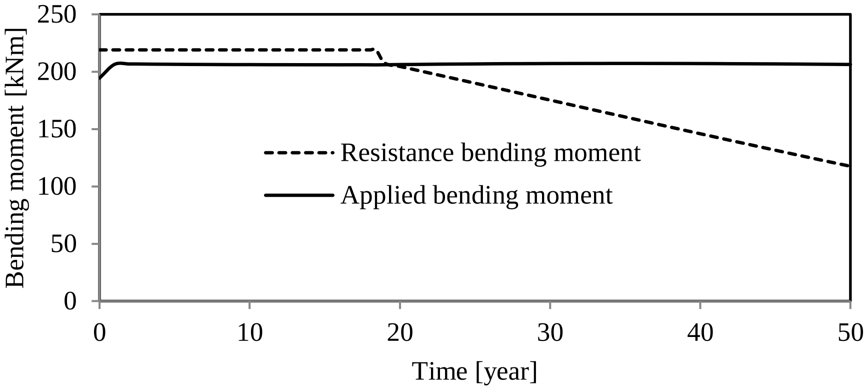


Figure 4.6: Time-dependent flexural performance of the RC beam

The flexural capacity and demand of the section coincide, i.e., the capacity to demand ratio is equal to *unity*, at a time of 19.5th year shown in Figure 7. In other words, the point at which demand and capacity coincide that point is said to be a limit state, i.e., $G_M(X,t) = M_R\left(X,t\right) - M_a(X,t) = 0$. Therefore, at the time after the 19.5th year, the structure becomes unsafe due to its limited flexural capacity, i.e., demand exceeds the capacity of the section.

4.3.2.2 Shear Performance

The transfer of shear in reinforced concrete members occurs by a combination of the: shear resistance of the *uncracked* concrete in compression, *aggregate interlock* force that can be developed tangentially along with the expected crack propagation, and similar to a frictional force due to the irregular interlocking of aggregates along the rough concrete surface on each side of the crack, *dowel action* of the longitudinal reinforcement is the resistance of the longitudinal bars to transverse force, and *shear reinforcement* resistance from stirrups. From these, the shear resisted by concrete is the sum of shear in the uncracked compression zone, aggregate interlock force, and dowel action of the longitudinal reinforcement taken as $V_{Rd,c}$ (ES EN 1992-1-1 2004; Mosley et al. 2012), and the remaining shear force $V_{Rd,s}$ resisted by stirrups.

The performance of the reinforced concrete structure for shear estimated from the expression:

$$G\left(X_i\left(t\right)\right)=V_R\left(V_{Rd,c}\left(t\right),V_{Rd,s}\left(t\right)\right)-V_a\left(V_{DL},V_{LL},...,V_{sh}\left(t\right)\right) \tag{4.22}$$

where $V_{Rd,c}\left(t\right)$ is the design shear resistance of the member without shear reinforcement; $V_{Rd,s}\left(t\right)$ is the shear resistance of the member by stirrups; V_{DL},V_{LL} is the applied shear from the live load and dead load, respectively; and $V_{sh}\left(t\right)$ is the additional load-induced from creep and shrinkage

The time-variant shear resistance and the applied shear force of the reinforced concrete section obtained from the following equations.

$$V_{Rd,c}\left(t\right)=\left[C_{Rd,c}k\left(t\right)\left(100\rho_l\left(t\right)f_{ck}\left(t\right)\right)^{1/3}\right]b_w d \geq v_{\min}bd \tag{4.23}$$

$$V_{Rd,s}\left(t\right)=\frac{A_{sw}\left(t\right)}{s}zf_{ywd} \tag{4.24}$$

$$V_{DL},V_{LL}=\frac{w_{DL}L}{2}+\frac{w_{LL}L}{2} \tag{4.25}$$

$$V_{sh}\left(t\right)=\frac{4E_{c,ef}\left(t\right)I_c\left(t\right)1/r_{t,sh}}{L} \tag{2.26}$$

Corrosion affects the shear resistance of the reinforced concrete beam mainly through the loss of effective area of reinforcement and degrading the strength of the concrete as provided in Equations (4.23) - (4.25). Thus, the shear resistance of the section decreases aggressively. Besides, the concrete cover of the stirrups is lower than that of flexural reinforcement, which means that the beam is more susceptible to shear failure due to corrosion than to moment failure in time shown in Figure 4.6 and Figure 4.7.

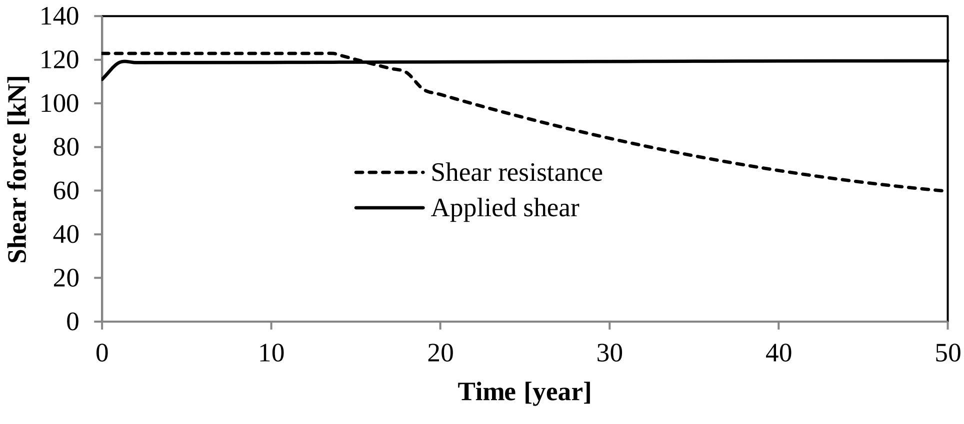


Figure 4.7: Time-dependent shear performance of reinforced beam concrete

time 15.8th year, the shear capacity and demand of the section coincide, so the limit state of the section takes place (see Figure 4.7). Therefore, at the time after the 15.8th year, the structure becomes unsafe due to its limited shear capacity.

4.3.3 Serviceability Performance Criteria

Serviceability limit state (SLS) deals with conditions such as deflection, crack of a structure under service loads, excessive vibration, fatigue, etc. The SLS performance measurement is an evaluation of the maximum effects of action (e.g., deflection, crack width, etc.) and their limit values in standard codes. The serviceability of the structural system ensured by a part/member or whole structural system that has been serviceable against deflection, crack, and vibration, i.e., the action effect should be less than or equal to the limiting values of each constraint provided in design codes. A serviceable structural system has to be well designed, constructed, and operated for the intended purpose. In addition to adopting the poor design, construction, and utility the environmental factors such as corrosion, creep, and shrinkage also significantly affect the serviceability of the structure.

Creep and shrinkage occur simultaneously and jointly influence the behavior of reinforced concrete members. Creep and shrinkage increase rapidly and reach the highest intensity at an early age shown in Figure 3 and then continue to increase gradually approach their limiting value, .i.e., time-dependent process. In the limit state method, it is necessary to assess deformations due to creep and shrinkage from serviceability considerations. In addition to that, in an aggressive environment, concrete structures face reinforcement corrosion, which is a significant factor impacting the durability of reinforced concrete through severe cracking of the concrete, and severe structural damage (Malhas and Mahamid 2016). In this study, two critical serviceability criteria, deflection, and crack were investigated by considering time-variant parameters.

The expressions provided in EC2 (ES EN 1992-1-1 2004) were modified to estimate the time-dependent performance of both deflection and crack, to consider the environmental factors. These expressions account for the effect of creep and shrinkage that reduces the effective modulus of elasticity of the concrete induces additional curvature that increases the long-term deflection of the concrete structure.

4.3.3.1 Deflection Performance

Important factors that need consideration in the computation of time-dependent deflection are criteria defining the limiting deflection, appropriate design loads, material properties, creep, and shrinkage. In concrete structures, deflections increase with time due to the reduction of reinforcing steel area through corrosion as presented in Table 4.1, creep under sustained load reduces the modulus of elasticity with time, and shrinkage induces additional curvature of section.

The design load to estimate long-term deflections is the quasi-permanent load, G_k and $\psi_2 Q_k$ in which ψ_2 is equal to 0.8 for storage purposes. The total time-dependent deflection of the section is a combined effect of flexure and shrinkage and which can be estimated from the following expressions:

$$\delta(t) = kL^2 \left(1/r_{t,Qp} + 1/r_{t,cs}\right) \tag{4.27}$$

where $1/r_{t,Qp}$ is the flexural curvature due to quasi-permanent load and estimated from Equation (4.28) and $1/r_{t,cs}$ is the curvature from shrinkage and estimated from Equation (4.12)

$$\frac{1}{r_{t,Qp}} = \zeta\left(\frac{1}{r}(t)\right)_c + (1-\zeta)\left(\frac{1}{r}(t)\right)_u \tag{2.28}$$

The curvature of both uncracked and cracked condition for the quasi-permanent load estimated from the Eqs. (2.29) and (2.30), respectively.

$$\left(\frac{1}{r}(t)\right)_u = \frac{M_{Qp}}{E_{c,eff}(t).I_u} \tag{2.29}$$

$$\left(\frac{1}{r}(t)\right)_c = \frac{M_{Qp}}{E_{c,eff}(t).I_c(t)} \tag{2.30}$$

where M_{Qp} is the moment of the quasi-permanent moment at critical section; k is the factor taken in to account the distribution of bending moment and is equal 0.104 for a simply supported beam subjected to *uniformly distributed load,* and L is the beam span

As expressed in Equations, the deflection of reinforced concrete structures depends on material properties, cross-sectional dimensions, area of reinforcing steel, structural system (i.e., supporting system), loading condition, creep, and shrinkage.

From the detailed computation, the majority of deflection induced due to sustained loads. The effect of creep rapidly increased at an early shown in Figure 4.3, due to the immaturity of the concrete, which decreases the concrete elastic module and subsequently increases the deflection rapidly to the highest intensity early shown in Figure 4.8.

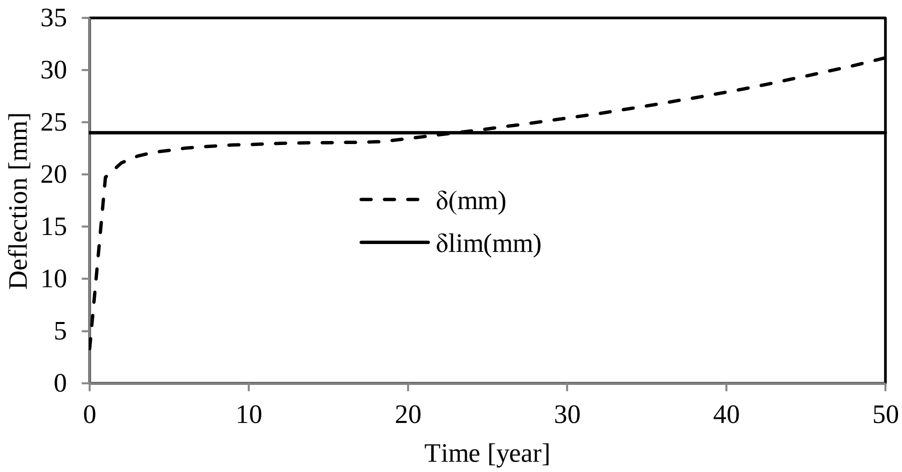


Figure 4.8: Time-dependent and limit deflection of the RC beam

However, the deflection occurs in rapid intensity early, slowly increases in normal conditions but, the intensity of deflection rapidly changes due to corrosion of steel reinforcement shown in Figure 4.8 (i.e., after 18.410th year). Therefore, environmental factors significantly affect the built structures through deteriorating material strength. The expressions provided in design codes estimate the deflection of the structural member with no variation with overtime, whereas, in a real problem, the deflection of the structure is progressive and increases with time.

The limiting value of deflection $\delta_{\lim}$ of a beam subjected to quasi-permanent load is span/250, i.e., $l/250 = 24$ mm. As shown in Figure 4.8, at the time of the 23rd year, the limiting deflection and estimated deflection coincide, i.e., the limiting value and the demand of the section are equal at the time shown in Figure 4.8. Therefore, at the time after 23-year, the structure becomes unserviceable due to excessive deflection that denies the comfort of the occupants.

4.3.3.2 Crack Performance

The performance of the reinforced concrete structure for crack width estimated from the expression:

$$G\left(X_i(t)\right) = w_{k,\,\mathrm{lim}} - w_{k,ap}\left(b, d, f_c\left(t\right), f_y\left(t\right), E_c\left(t\right), A_s(t), \ldots\right) \qquad (4.31)$$

Crack width due to applied load before corrosion

The crack of the concrete section begins early due to service load because of the limited tensile strength of the concrete. In a simply supported beam subjected to uniformly distributed load, the maximum bending stress $\sigma = M_{\max} y / I$ occurs at mid-span that is much greater than the tensile strength f_{ct} of the concrete. Therefore, the concrete section cracks before corrosion take place. The maximum crack width of the section before corrosion takes place is estimated from the expression provided in EC2 (ES EN 1992-1-1 2004) as:

$$w_{k,ap} = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})(t) \qquad (4.32)$$

$$(\varepsilon_{sm} - \varepsilon_{cm})(t) = \frac{\sigma_s\left(t\right) - k_t \dfrac{f_{ct,eff}\left(t\right)}{\rho_{p,eff}\left(t\right)}\left(1 + n\rho_{p,eff}\left(t\right)\right)}{E_s\left(t\right)} \geq 0.6\frac{\sigma_s\left(t\right)}{E_s\left(t\right)} \qquad (4.33)$$

$$s_{r,max} = 3.4c + 0.425k_1k_2\phi / \rho_{p,eff} \qquad (4.34)$$

where $w_{k,ap}$ is the maximum crack width from service load; $s_{r,max}$ is the maximum crack spacing; $(\varepsilon_{sm} - \varepsilon_{cm})(t)$ is the difference mean strain in the reinforcement under relevant load combination and the mean strain in the concrete between cracks, and all other variables and relevant constants in the expressions were obtained from EC2

As expressed in Equations (2.32)–(4.34), crack width performance depends on materials properties, cross-sectional dimensions, applied load, and environmental factors. Besides these factors, the way the concrete structures are designed, detailed and constructed also affects crack performance. However, once the maximum crack is formed due to loading, the stress in the section increases, which can not change the crack spacing; however, the crack width increases because of variations in other parameters.

Crack width after corrosion of embedded reinforcement takes place

The thick wall cylinder with embedded reinforcement, subject to internal corrosion pressure from the corrosion product, between the concrete-bar-interface, was modeled to investigate the combined effect of applied load and corrosion (Bazant 1979) as shown in Figure 4.9.

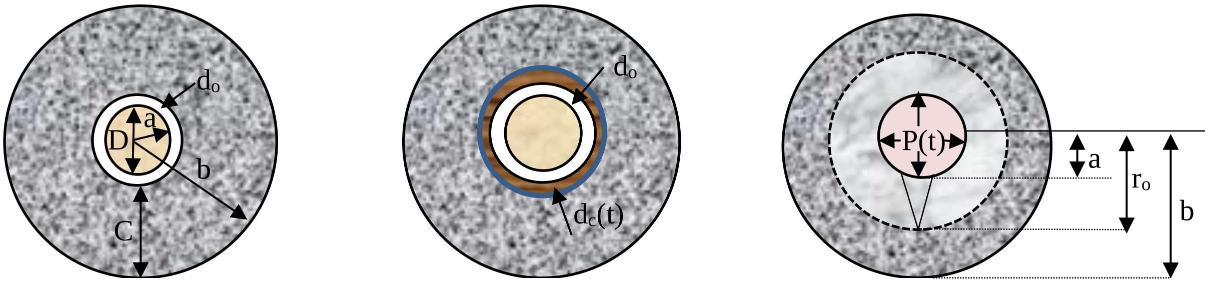


Figure 4.9: Illustration of the internal pressure-induced concrete cracking process due to corrosion

where D is the diameter of the embedded bar; d_0 is the thickness of the annular layer of concrete pores at the concrete-bar interface; C is the thickness of the concrete cover, and $a = D/2 + d_0$ and $b = C + D/2 + d_0$ are the inner and outer radii of the thick-wall cylinder, respectively

In the thick-wall concrete cylinder, the internal stress $f(t)$ is induced from (1) the corrosion products of the embedded reinforcement, which increases along time and denoted as $f_1(t)$, and (2) the radial component of the stress from applied load between the reinforcing bar and the concrete and denoted as f_2 (Yang 2010). Therefore, the total spread-out stress $f(t)$ is given by:

$$f(t) = f_1(t) + f_2 \qquad (4.35)$$

The corrosion-induced expansion along time due to the internal pressure induced from corrosion products along time which needs to generate the radial pressure (Liu and Weyers 1998) determined as:

$$d_c(\mathrm{t}) = \frac{W_{rust}(\mathrm{t})}{\pi(D + 2d_o)}\left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}}\right) \qquad (4.36)$$

where α_{rust} is the coefficient of corrosion product; ρ_{rust} is the density of corrosion products; ρ_{st} is the density of the steel, and $W_{rust}(\mathrm{t})$ is the mass of corrosion products which is time-dependent and obtained from the expression provided in (Liu and Weyers 1998)

After corrosion products fill the annular pores it induces stress and partially cracks the thick-wall concrete cylinder along a concrete cylinder radius r_0, which varies between the radii a and b (Yang 2010) shown in Figure 4.9(c). The radial displacement ***u(r)*** and the stress components developed from the theory of elasticity.

$$u(r) = c_1(r_o)r + \frac{c_2(r_o)}{r} \tag{4.37}$$

$$\varepsilon_r(r) = \frac{du(r)}{dr} \text{ and } \varepsilon_\theta(r) = \frac{u(r)}{r} \tag{4.38}$$

The stress components from the radial equilibrium are obtained (Sadd 2009) from Figure 4.11 in the finite element form of the thick-wall concrete cylinder:

$$\sigma_\theta(r) - \sigma_r(r) - \frac{rd\sigma_r(r)}{dr} = 0 \tag{4.39}$$

$$\sigma_\theta(r) = \frac{E_{ef}}{1-v_c^2}\left(c_1(r_o)(1+v_c) + c_2(r_o)\frac{1-v_c}{r^2}\right) \tag{4.40}$$

$$\sigma_r(r) = \frac{E_{ef}}{1-v_c^2}\left(c_1(r_o)(1+v_c) - c_2(r_o)\frac{1-v_c}{r^2}\right) \tag{4.41}$$

The crack can propagate to the area of the concrete cylinder to completely crack the section as corrosion products increase over time. he section's radial displacement $u(r)$ is expressed as follows (Yang 2010):

$$u(r) = c_3 r^{\sqrt{\alpha}} + c_4 r^{-\sqrt{\alpha}} \tag{4.42}$$

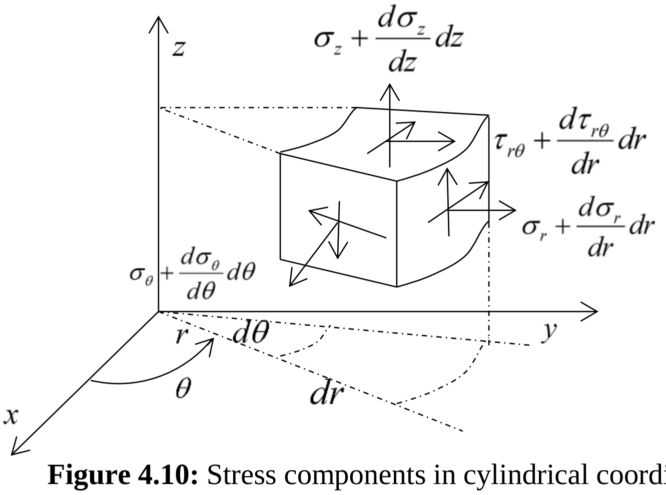


Figure 4.10: Stress components in cylindrical coordinates

where ε_r and ε_θ are the radial and tangential strain, respectively; E_{ef} is the effective modulus of elasticity taking into account the time-variant creep effect; v_c is the Poisson's ratio of the concrete; and $c_1(r_o)$, $c_2(r_o)$ are the coefficients, which are a function of r_0, which varies between a and b as shown in Figure 4.10

The constants $c_1(r_o)$ and $c_2(r_o)$ be obtained by considering boundary conditions for the concrete cylinder, i.e., $\sigma_r(a) = f_t$ and $\sigma_r(b) = 0$ at the stage of crack initiates. The constants c_3 and c_4 are coefficients to be determined by setting the boundary conditions for the cracked section with fully corroded reinforcement in the beam section as $u(a) = d_c(t) + d_p$ and $\sigma_r(b) = 0$. *in which*: d_p is the displacement caused by f_2 at a point $r = a$ in the uncracked concrete cylinder that satisfy the boundary conditions, i.e., $\sigma_r(a) = -f_2$ and $\sigma_r(b) = 0$, had been estimated from the expression provided (Liu and Weyers 1998):

The stiffness reduction factor α is to account for the residual stiffness and residual stress, which varies with time (not with radius) is proposed in (Yang 2010). Having the coefficients obtained by setting desired boundary conditions, the stiffness reduction factor expressed as:

$$\alpha = \frac{f_t \exp\left\{-\frac{\pi f_t(a+b)}{G_f}\left[\frac{\left(b^{\sqrt{\alpha}} - a^{\sqrt{\alpha}}\right)\left[c_3 + c_4 \big/ (ab)^{\sqrt{\alpha}}\right]}{\sqrt{\alpha}(b-a)} - \frac{1}{b-a}\int_a^b \left(c_3(\xi) + \frac{c_4(\xi)}{\xi^2}\right) d\xi\right]\right\}}{\frac{E_{ef}\left(b^{\sqrt{\alpha}} - a^{\sqrt{\alpha}}\right)\left[c_3 + c_4 \big/ (ab)^{\sqrt{\alpha}}\right]}{\sqrt{\alpha}(b-a)}}$$

(4.43)

G_f - is the fracture energy, which is determined from a base value of G_{fo} and the mean compressive strength of concrete f_{cm} *according to* (Pantazopoulou and Papoulia 2001)

$$G_f = G_{fo}\left(f_{cm} \big/ f_{cm0} \right)^{0.7} \qquad (N/mm) \qquad (4.44)$$

in which $f_{cm0} = 10\ \text{MPa}$; $f_{cm} = 33\ \text{MPa}$ for C25/30 concrete grade from EC2 (Table 3.1), and G_{fo} is obtained by Lagrange interpolation based on the maximum aggregate size, d_a between the values of 0.025N/mm, 0.030N/mm, and 0.038N/mm corresponding to d_a = 8mm, 16mm and 32mm, respectively.

In this research, the maximum aggregate considered size is d_a = 25mm provided in the case study, and the corresponding value of fracture energy G_f has been estimated and provided in Table 4.2.

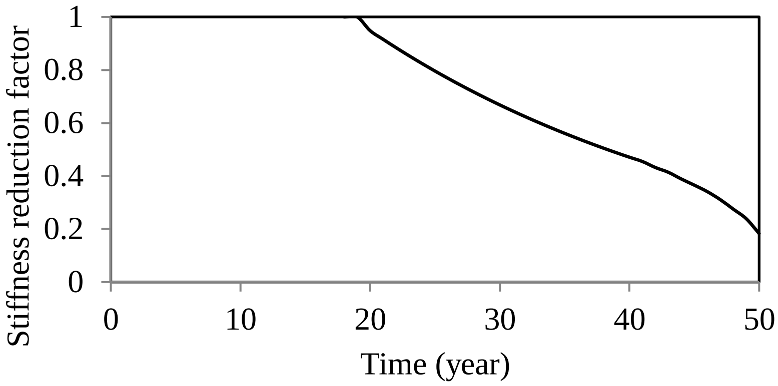


Figure 4.11: Variation of stiffness reduction factor with time

By solving coefficients of boundary conditions, fracture energy, and stiffness reduction factor in Equation (4.43) simultaneously α has been determined using the MATLAB. The reduction of the stiffness reduction factor for specified material properties and solving boundary conditions shown in Figure 4.11.

By applying the boundary conditions of stress, $\sigma_\theta(r,t) = f_t$ and the surface crack width of the thick-wall concrete cylinder estimated from the expression (Yang 2010):

$$w_c = 2\pi\left[\varepsilon_\theta(b) - \varepsilon_\theta^{e,m}(b)\right] \tag{4.45}$$

in which $\varepsilon_\theta^{e,m}(b)$ is the maximum elastic strain of concrete at the radius of the thick-wall concrete cylinder, $r = b$ as (Timoshenko and Goodier 1970) becomes $\varepsilon_\theta^{e,m}(b) = \dfrac{f_t - v_c\sigma_r(b)}{E_{ef}}$; and $\sigma_r(b) = 0$, the crack width w_c of the concrete section expressed as (Yang 2010):

$$w_c = 2\pi b\left[c_3 b^{(\sqrt{\alpha}-1)} + c_4 b^{(-\sqrt{\alpha}-1)} - \frac{f_t}{E_{ef}}\right]$$

$$= \frac{4\pi d_c(t)}{(1-v_c)(ab)^{\sqrt{\alpha}} + (1+v_c)(ab)^{\sqrt{\alpha}}} + \frac{4\pi a\left[(1+v_c)b^2 + (1-v_c)a^2\right]f_2}{(b^2-a^2)\left[(1-v_c)(a/b)^{\sqrt{\alpha}} + (1+v_c)(b/a)^{\sqrt{\alpha}}\right]E_f} - \frac{2\pi b f_t}{E_{ef}}$$

(4.46)

Table 4.2: Values of basic variables used in crack width computation

Parameter	Symbol	Unit	Values	Sources
Diameter of rebar	D	mm	24	Case study (Figure 4.2)
Concrete cover	C	mm	58	Case study (Figure 4.2)
Compressive strength of concrete	f_{ck}	MPa	25	ES EN 1992-1-1(2004)
Mean compressive strength at 28 days, i.e., $f_{cm} = f_{ck} + 8$ MPa	f_{cm}	MPa	33	ES EN 1992-1-1(2004)
Modulus of elasticity of concrete	E_{cm}	GPa	31	ES EN 1992-1-1(2004)
Yield stress of rebar	f_y	MPa	460	ES EN 1992-1-1(2004)
Yield stress of stirrups	f_{yw}	MPa	250	ES EN 1992-1-1(2004)
Modulus of elasticity of steel	E_s	GPa	200	ES EN 1992-1-1(2004)
Fracture energy	G_f	N/mm	0.0803	Pantazopoulou (2001)
The current density of corrosion	i_{corr}	$\mu A/cm^2$	0.75	Enright and Frangopol (1998)
Poisson's ratio of concrete	v_c	-	0.18	Liu and Weyers (1998)
Poisson's ratio of steel	v_s	-	0.3	ES EN 1992-1-1(2004)
The thickness of the annular layer at the concrete-bar interface	d_o	μm	12.5	Liu and Weyers (1998)
Coefficient of corrosion product	α_{rust}	-	0.57	Liu and Weyers (1998)
The density of corrosion products	ρ_{rust}	kg/m^3	3600	Liu and Weyers (1998)
The density of the steel	ρ_{st}	kg/m^3	7850	ES EN 1992-1-1(2004)

In normal conditions, the crack width is estimated empirically due to quasi-permanent load, which is time-invariant before corrosion initiates as shown in Equations (4.32)-(4.34) and Figure 4.13. Hence, corrosion reduces the cross-sectional area of reinforcing steel, and the strength of concrete, the term $(\varepsilon_{sm} - \varepsilon_{cm})(t)$ expressed in Equation 4.33 increases with time due to corrosion, eventually, the crack width increases with time shown in Figure 4.12.

The total crack width of the section under the combined effect of embedded reinforcement corrosion and applied load was estimated using Equation (4.46) by considering a thick-walled concrete cylinder. Therefore, we can conclude that the crack width of the reinforced concrete beam subjected to chloride diffusion is related to corrosion products, the applied load, the concrete properties, and geometry of the section as indicated in Equation (4.46), in which the crack width increases with time as shown in Figure 4.12.

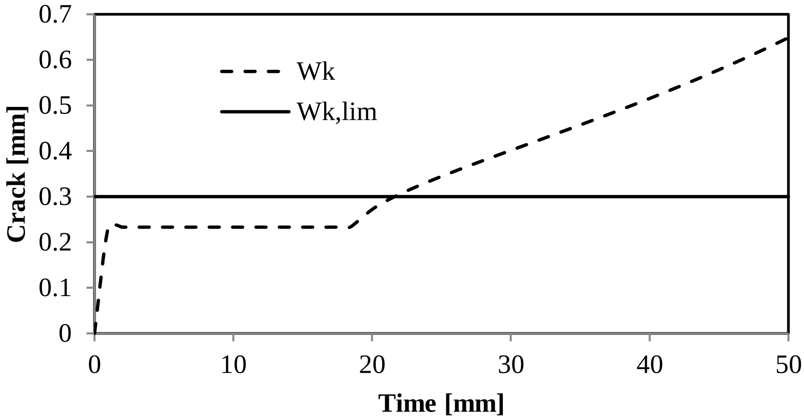


Figure 4.12: Time-dependent and limit crack width of the RC beam

The corrosion crack commences at the time of 0.654th year after the initiation of corrosion, derived from the expression in (Liu 1996) and Table 4.2 data. The limiting value of crack width $w_{k,\,\mathrm{lim}}$ is the limit value and depends on the exposure condition of the structural member exposed to chloride in moderate humidity (wet, rarely dry, XD2) is **0.3**mm (ES EN 1992-1-1 2004). At the time of 21.5th year, the limiting crack width, 0.3 mm, and estimated crack width coincide as shown in Figure 4.12. Therefore, the limit state of crack width occurs at the time of the 21.5th year.

Note that the limit state of the considered reinforced concrete beam against shear, flexure, crack, and deflection occurs at 15.8, 19.5, 21.5, and 23rd year, respectively. Therefore, among the ultimate limit state, the reinforced concrete beam section is more susceptible to the shear failure, because the concrete cover of the links is smaller than that of longitudinal reinforcement in which corrosion of links begins before the longitudinal reinforcement. Moreover, the effective area as well as strength of links and flexural reinforcement, and concrete strength reduce due to corrosion, eventually; the shear capacity of the section significantly decreases with time. From the time-dependent performance evaluation, considering creep and shrinkage for specified ambient temperature and relative humidity, and moderate corrosion rate of the current density of 0.75 μA/cm^2, the service life of the reinforced concrete beam is reduced to less than 50% of its design life.

CHAPTER 5

UNCERTAINTIES OF PARAMETERS

5.1 Introduction

The structural engineering activities are broadly categorized as (Savoia 2012) (a) to *predict the behavior* of the structure under fully and partially known conditions and (b) to design a structural system that satisfies the desired criteria for the intended purpose in its lifetime. While designing and predicting the performance of the structure, there will be unavoidable uncertainties. However, avoiding uncertainty can be minimized by developing adequate knowledge to establish parameters, boundary conditions, and constructing accurate mathematical and physical models is a difficult task. Most of the uncertainties are associated with material properties, geometry, loads, and models due to a lack of knowledge and environmental factors (Matos 2007).

This chapter presents uncertainty in parameters, fuzzy concepts and set theory, generation of membership functions, fuzzy operators, uncertainty propagation, and numerical examples for relevant sections.

5.2 Uncertainty in Parameters

The structural reliability analysis involves the rational treatment of parameter uncertainty in structural analysis and the design of sound decisions. Basic parameters (except for physical properties and mathematical constants, e.g., the density of materials, modulus elasticity of steel, partial safety factors, etc.) that involve in structural analysis and design associated with some degree of different uncertainties (Thoft-Christensen and Baker 1982). Uncertainties in reinforced concrete occur due to the possibility of deviation of materials strength, deviation of magnitude and distribution of loads, deviation of the sectional dimensions, impacts of environmental factors, improper position of reinforcement, inaccuracy of calculation procedures, and discontinuity of adjacent strips (or bond problem).

In general, the source of uncertainties may be (i) inherent randomness, i.e., *physical uncertainty* from material properties, and geometric data, (ii) limited information, i.e., *statistical uncertainty* due to limited available data and economic constraint to test more specimens, (iii) imperfect knowledge, i.e., *model uncertainty* from limited numerical models, vagueness due to inaccurate definitions of performance requirements, and inappropriate structural modeling, and (iv) *gross errors* in design, execution, and operation of the structure; and lack of knowledge of the behavior of new materials in real conditions.

5.2.1 Uncertainty in Materials

The physical and mechanical properties of materials must be defined to undertake a structural design and performance evaluation. The uncertainty of the mechanical properties of concrete mainly depends on properties of ingredients, concrete composition, testing procedure, the variation of concrete being in the structure than in control specimens, material degradation, execution, etc. Therefore, variability in the mechanical properties of concrete is considerably higher.

On the other hand, the variability of the mechanical properties of steel is usually negligible in production due to higher industrialization and manufacturing quality control. The desired properties of steel are preserved by proper handling, transport, and protection from aggressive chemicals to satisfy the anticipated requirements of the structure. Steel variations exist because of the cross-section and yield stress deterioration resulting from corrosion, load experience, etc.

5.2.2 Uncertainty in Geometry

The variability in the geometry of concrete elements is due to the deviation of the cross-sectional dimension and shape specified in the design document due to the misalignment of member(s), the inappropriate position of reinforcement, etc.

5.2.3 Uncertainty in Loads

A designed structure should withstand the maximum expected load in its lifetime. In the design of a reliable structural system, it is necessary to consider all possible actions that can apply to the structural system in a specified location. Loads that act on the system broadly categorized based on magnitude and location with respect to time as dead loads, live loads, and environmental loads.

Dead loads are loads that do not change its magnitude and location with time. Dead load is generated from the self-weight of the structure itself and fixed equipment (if applicable). It is possible to determine dead load with better accuracy from the dimension of the structural members and the specific weight of materials. Therefore, its variability with time is negligible, and the uncertainty in magnitude is insignificant compared with other kinds of loads.

Live loads consist of occupancy loads (i.e., human occupation, movable equipment, and materials) in building and traffic loads on bridges. They may be either fully or partially preset or not at all and may also change in location. Their magnitude and distribution are uncertain with respect to time. Their maximum intensities throughout the life of the structure is not known with precision, but occurs only on special occasions during a short or even moderate period and significantly considered.

Environmental *loads* consist of earthquake loads, wind loads, snow loads, soil pressure, loads caused by temperature differentials (e.g., contraction and relaxation, creep and shrinkage), etc. Environmental loads at any given time are uncertain in intensity, distribution, and location.

5.3 Types of Uncertainties

Based on the sources, uncertainties broadly classified as aleatory uncertainties and epistemic uncertainties. Aleatory (or random) uncertainty is associated with the inherent randomness in the physical properties and the system environment, whereas epistemic (or fuzzy) uncertainty arises from insufficient knowledge and imprecision of information about a problem that is going to be studied (Kai-Yuan et al. 1991a; Kai-Yuan et al. 1993; Szeliga 2004; Li et al. 2015; Tang et al. 2014; Naderpour and Alavi 2015; Fan et al. 2019). In general, aleatory uncertainty is data-based, whereas epistemic uncertainty is knowledge-based. However, both types of uncertainties may be combined and analyzed as total uncertainty or treated separately depending on the chosen analysis method.

In a stochastic analysis, variables that involve aleatory uncertainty represented by a probability distribution, which provides information on their mean values, standard deviations, and correlation with other variables. However, observation of specific variable quantities will not always provide sufficient data to allow for an interpretation of the distribution type of uncertain quantities. The results of the reliability analysis are sensitive to the probability distribution. Therefore, the selection of an appropriate distribution type is a crucial tool. When there is no detailed information about variables, normal and lognormal distributions are used. In structural engineering, mechanical properties of materials and load intensity are non-negative, whereas performance variables may be negative values. Therefore, the lognormal distribution is used to describe load and material variables, whereas the normal distribution is used to describe resistance (performance) variables. Furthermore, the reliability analysis of a structure that involves aleatory uncertainty of variables carried out by *probability theory*.

Epistemic uncertainty of variables is represented by possibility distribution using the membership function. The choice of membership function depends on available data and experience of the expert knowledge level. In a possibility analysis, the reliability analysis of a structure that involves the epistemic uncertainty of variables evaluated by using possibility theory. In this study, epistemic uncertainty is explored well and applied to achieve the desired objectives.

5.3.1 Fuzzy Variables

Suppose Y be a linguistic variable with the label "reinforcement" with the universe of discourse, $U = \left[A_{s,\min}, A_{s,\max} \right]$. For the linguistic variable reinforcement, the terms that are fuzzy sets referred to as *under-reinforcement*, *balanced reinforcement*, and *over-reinforcement* may be considered. Here, the base variable x is the area of reinforcement in mm^2 or cm^2. The expression $M\left(X\right)$ is the mathematical rule that assigns a fuzzy set to the terms x as expressed in Equation (5.1).

$$M\left(\text{under reinforcemet}\right) = \left\{ \left(x, \mu_u\left(x\right) \right) \mid x \in U \right\} \tag{5.1}$$

where

$$\mu_u\left(x\right) = \begin{cases} \left(x - A_{s,\min}\right) / \left(A_{s,cal} - A_{s,\min}\right) & \text{if } A_{s,\min} \le x \le A_{s,cal} \\ \left(A_{s,bal} - x\right) / \left(A_{s,bal} - A_{s,cal}\right) & \text{if } A_{s,cal} \le x \le A_{s,bal} \\ 0 & \text{Otherwise} \end{cases} \tag{5.2}$$

Thus, reinforcement is a fuzzy variable with the universe of discourse and expressed in linguistic terms based on the amount of the steel reinforcement, i.e., satisfy interval (lower and upper bound) provided in design codes, provided in the designed section.

5.3.2 Relationship Between Fuzzy Number and a Real Number

The α is the threshold for two adjacent data to be considered belonging to the same class. Note that the small α will have a large fuzzy interval, and the large α will have a smaller fuzzy interval. Let us consider the fuzzy set of concrete strength:

$$F_{ck} = \{0/20 + 0.2/21 + 0.4/22 + 0.6/23 + 0.8/24 + 1/25 + 0.8/26 + 0.6/27 + 0.4/28 + 0.2/29 + 0/30\}$$

This fuzzy set of concrete strength can be set into several α*-cut sets* and *strong* α*-cut sets*, all of which are crisp for the arbitrary values of α = 0, 0.2, 0.4, 0.6, 0.8, and 1.

The α***-cut sets are:*** $F_{ck0.2} = \{21\ 22\ 23\ 24\ 25\ 26\ 27\ 28\ 29\}$; $F_{ck0.4} = \{22\ 23\ 24\ 25\ 26\ 27\ 28\}$; $F_{ck0.6} = \{23\ 24\ 25\ 26\ 27\}$; $F_{ck0.8} = \{24\ 25\ 26\}$, and $F_{ck1} = \{25\}$

Strong α***-cut sets are****:* $F_{ck0.2} = \{22\ 23\ 24\ 25\ 26\ 27\ 28\}$; $F_{ck0.4} = \{23\ 24\ 25\ 26\ 27\}$; $F_{ck0.6} = \{24\ 25\ 26\}$, and $F_{ck0.8} = \{25\}$

For a fuzzy *number* denoted by X and a real number denoted by r_0, Figure 5.1 (*a* and *b*) show their possible order relations using α*-cut*. The mathematical notations, $\mu_X(r)$ represents the membership function of X; m is the mean of X with $\mu_X(m)$ =1; the vertical axis denotes the degree of membership; α_0, α_1 and α_2 are three degrees of membership with the condition $\alpha_1 < \alpha_0 < \alpha_2$; α_0 is the membership degree of a real number r_0, i.e., $\alpha_0 = \mu_X(r_0)$, X_α represents the α-cut of X satisfying the condition $X_\alpha = \{r \mid \mu_X(r)\ \geq \alpha\}$ (Tang et al. 2014). For each $\alpha_i \in (0,1]$, $X_{\alpha_i} = \{r \mid \mu_X(r)\ \geq \alpha i\}$ is an interval defined as $\left[x^{-\alpha_i}, x^{+\alpha_i}\right]$, as shown in Figure 5.1 (*a* and *b*). We can obtain three important points from Figure 2 (*a*): the equality $x^{-\alpha_0} = r_0$ supports; $x^{-\alpha}$ is larger than r_0 when $\alpha > \alpha_0$, such as α_2 where $\alpha_2 > \alpha_0$ and $x^{-\alpha_2} > r_0$ hold; and $x^{-\alpha}$ is less than r_0 when $\alpha < \alpha_0$, such as α_1 where $\alpha_1 < \alpha_0$ and $x^{-\alpha_1} < r_0$ hold. Thus, the critical degree of membership α_0 can be used to define the possibility that (Tang et al. 2014) X is less than r_0, i.e., $Poss\{X \leq r_0\} = \mu_0$.

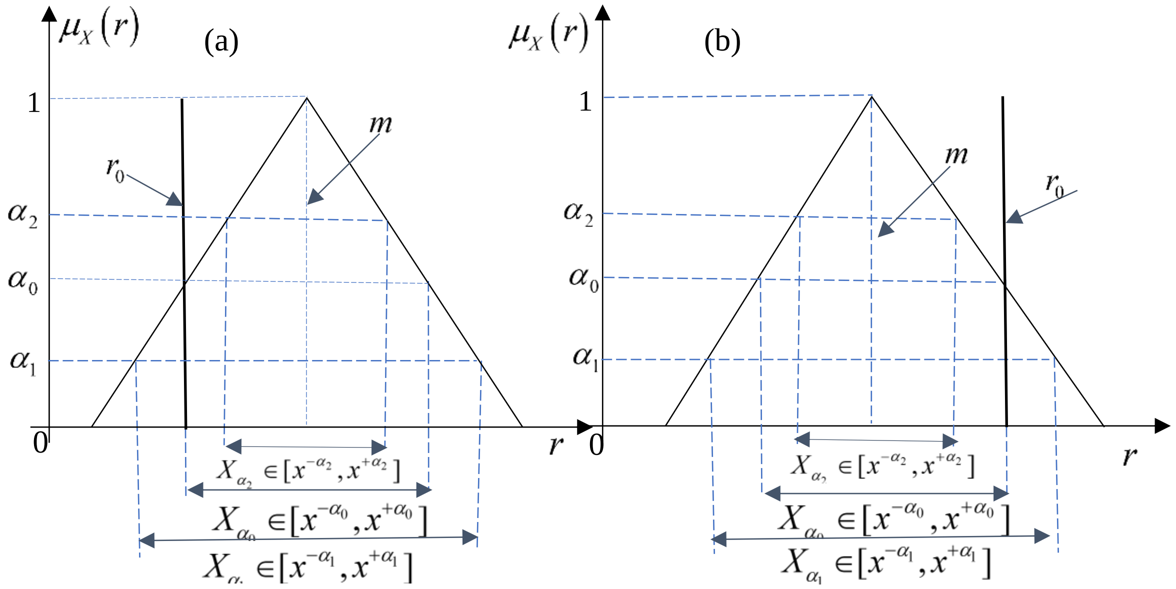


Figure 5.1: The order relation between fuzzy and real numbers

5.3.3 Generation of Membership Function of Fuzzy Variables

The fuzziness of the variable is expressed by its degree of membership. The generation of membership functions for imprecise data is a basic stage in applications of fuzzy concepts (Medasani et al. 1998). To generate the membership function of the variable, its ranges i.e., the lower and upper limit should be identified. Besides, the range of variables is an appropriate limit to handle the variation of parameters easily and precisely. To do so, the value obtained through the deterministic design procedure considered as the core value, whose degree of membership is *unity*, of the study parameter (Lu et al. 1994). And the upper and lower bound of the variables obtained from code provisions and the evaluation of the existing situation.

5.3.3.1 Membership Function of Corrosion Rate

Suppose that I_{corr} is the *universe of discourse* of the corrosion density. Let us classify the level of corrosion current density as low, moderate, and high in the linguistic variable. From statistical investigations, these *low*, *moderate*, and *high* rates of corrosion current density are a subset of I_{corr} with the interval of i_{corr} = 0.1– 0.5 μA/cm^2 for *low* corrosion rate; $i_{corr} = 0.5 - 1$ μA/cm^2 for *moderate* corrosion rate; and $i_{corr} > 1$ μA/cm^2 for *high corrosion* rate as shown in Table 2.1.

The classical approach, the probability, one way to describe the classical set is a *low* rate. Suppose a low rate, corrosion current density membership to a low rate set that belongs to the universal set I_{corr}, such that the corrosion current density is between 0.1 μA/cm^2 and 0.5μA/cm^2. Similarly, the member of corrosion current density belongs to *moderate* rate, if it is between 0.5μA/cm^2 and 1μA/cm^2. Besides, when the density of the corrosion current is greater than 1 μA/m^2, the member's corrosion current density is at a *high rate*. In the classical approach, it is obvious that 0.49 μA/m^2 is a low rate whereas 0.51 μA/m^2 is a moderate rate which means the classic sets are rigidly bounded, making it very difficult to express the imprecise data. However, representing them in a fuzzy set based on their respective degrees membership is very simple.

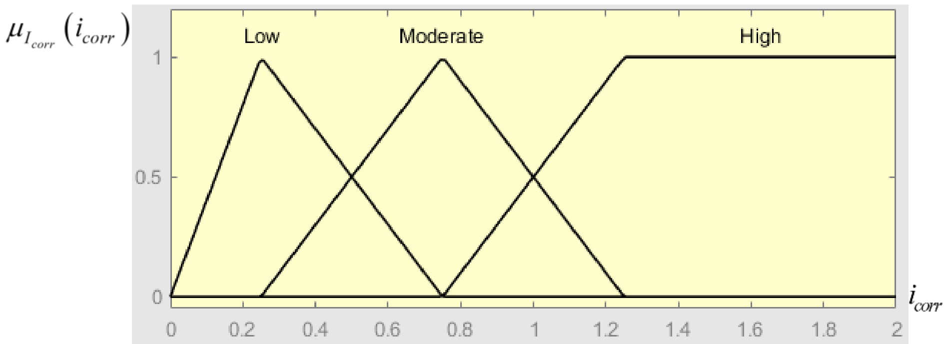


Figure 5.2: Fuzzy membership function of corrosion current density

If the corrosion current density is around 0.25 μA/cm^2, it is a low rate; corrosion current density is around 0.75 μA/cm^2, it is a moderate rate, and when the corrosion current density is around 1.25 μA/cm^2, it is a high rate as shown in Figure 5.2. In this sense, the fuzzy sets have no rigid boundary. Let us consider a corrosion current density of 0.5μA/cm^2 that can simultaneously belong to a *low* rate as well as a *moderate* rate, with a fuzzy membership grade of 0.5. When 0.625μA/cm^2 is considered, it is likely in the category of *moderate* rate with a membership degree of 0.75, whereas the 0.35μA/cm^2 is with a membership degree of 0.8 at a *low* rate and 0.2 in *moderate* rate. In this study, the upper bound of corrosion current density is taken as 2μA/cm^2 for illustration but its value may be more or less than 2μA/cm^2 in a real problem. This illustrates how imprecise data can be categorized in a clear way using fuzzy sets.

5.3.3.2 Membership Function of Input and Output Variables of RC Structure

The corrosion parameters, cross-section dimensions are considered as the input variables, whereas the deterioration of material properties as output variables as shown in Table 5.1 from the detailed calculation. Furthermore, the material properties considered as input variables, and the safety and serviceability constraints are the output variables. The time-variant concrete strength and effective steel reinforcement area due to the moderate corrosion rate are determined, and the result is shown in table 5.1 during design life. The corrosion rate, a fuzzy variable, is considered as an input variable that deteriorates the strength of concrete, the diameter of steel reinforcement, and the yield strength of reinforcement steel. The fuzziness of the corrosion rate propagates to the output variables along with the design life of the concrete structure.

Table 5.1. Intervals of input and output variables

Variables	Parameter	Unit	Abbreviation	Database range	
				Minimum	Maximum
Input	Corrosion current density	μA/cm^2	r_{corr}	0.75	
	Chloride diffusion coefficient	cm^2/year	D_c	1.29	
	Surface chloride concentration (%)	-	C_0	0.10	
	Critical chloride concentration (%)	-	C_{cr}	0.04	
	Concrete cover	cm	C	5.8	
	Time	year	t	0	50
	Design load	kN/m	w	16	46.04
Output	Compressive strength of concrete	MPa	$f_{ck}(t)$	20.167	25
	Modulus of elasticity of concrete	GPa	$E_{cm}(t)$	29.501	31
	Yield stress of steel (flexural)	MPa	$f_y(t)$	455.374	460
	Yield stress of steel (stirrups)	MPa	$f_{yw}(t)$	233.206	250
	The effective area of rebar	mm^2	$A_{st}(t)$	766.880	1520.531
	The effective area of a stirrup	mm^2	$A_{sw}(t)$	0.704	100.531

Note that the database with single values is the constant values of design parameters and baseline values of corrosion parameters.

To generate the membership function of the fuzzy input variables, their interval determined as presented in Table 5.1. As presented in Section 4.3.1.2, reasoning and citations about corrosion parameters, and from Figure 4.2, the concrete cover do not have minimum and maximum ranges. The initially provided material properties of the concrete beam are $f_{ck} = 25$ MPa, $f_y = 460$ MPa and the area of steel is $1520.53\ \text{mm}^2$. At the end of the design life, the material properties deteriorated to concrete strength of

$20.167\ \text{MPa}$, the yield stress of steel is $455.75\ \text{MPa}$ and the area of steel reinforcement is $766.88\ \text{mm}^2$ due to corrosion.

In design codes of concrete structure, the concrete grades such as C20/25, C25/30, C30/37, etc., in which the intermediate grades are not recognized. However, the desired grade of concrete is achieved during construction due to the aggressive environment the strength of concrete may decrease along with the design life of the structure and these intermediate strengths can be recognized by fuzzy set theory with their respective membership degrees. Therefore, the fuzzy interval of the concrete strength is limited to the lower and upper bound lies in between $20\ \text{MPa}$ and $30\ \text{MPa}$ from EC2 (Table 3.1).

The area of steel reinforcement in a singly reinforced concrete beam section is limited to $A_{st,\min} \le A_{st,prov} \le 0.04A_c$ as provided in EC2, in which $A_{st,\min}$ is $189.57\ \text{mm}^2$; $A_{st,prov}$ is $1520.53\ \text{mm}^2$, and $0.04A_c$ is $6000\ \text{mm}^2$. However, the maximum limit of the steel area in a singly reinforced concrete beam is $0.04A_c$ it should not exceed $A_{st,bal} = 1935.04\ \text{mm}^2$ in the under-reinforced section. Therefore, the fuzzy interval of the steel area is limited to the lower and upper bound lies in between $189.57\ \text{mm}^2$ and $1935.04\ \text{mm}^2$ with the core value of $1378.56\ \text{Nmm}^2$.

On the contrary, the design load varies from a dead load of $16\ \text{kN/m}$, dead load and live load $43.2\ \text{kN/m}$ to $46.04\ \text{kN/m}$ as Eurocode. This variation of input variables possesses fuzziness. Therefore, for the fuzzy input variables $X = \left\{ f_{ck0}, E_{cm0}, f_{yo}, A_{sto}, F \right\}$ provided in Table 5.1, the triangular membership functions are in Equations (5.3) – (5.7), respectively.

$$\mu_{\tilde{F}_{ck}}(f_{ck}(t)) = \begin{cases} \dfrac{f_{ck}(t) - 20}{25 - 20} & \text{if } 20 \le f_{ck}(t) \le 25 \\ \dfrac{30 - f_{ck}(t)}{30 - 25} & \text{if } 25 \le f_{ck}(t) \le 30 \\ 0 & \text{Otherwise} \end{cases} \tag{5.3}$$

$$\mu_{\tilde{F}_y}(f_y(t)) = \begin{cases} \dfrac{f_y(t) - 420}{460 - 420} & \text{if } 420 \le f_y(t) \le 460 \\ \dfrac{500 - f_y(t)}{500 - 460} & \text{if } 460 \le f_y(t) \le 500 \\ 0 & \text{Otherwise} \end{cases} \tag{5.4}$$

$$\mu_{\tilde{A}_{st}}(A_{st}(t)) = \begin{cases} \dfrac{A_{st}(t) - 189.57}{1378.56 - 189.57} & \text{if } 189.57 \le A_{st}(t) \le 1378.56 \\ \dfrac{1935.04 - A_{st}(t)}{1935.04 - 1378.56} & \text{if } 1378.56 \le A_{st}(t) \le 1935.04 \\ 0 & \text{Otherwise} \end{cases} \tag{5.5}$$

$$\mu_{\tilde{W}}(w) = \begin{cases} \dfrac{w(t) - 16}{43.2 - 16} & \text{if } 16 \le w(t) \le 43.2 \\ \dfrac{46.04 - w(t)}{46.04 - 43.2} & \text{if } 43.2 \le w(t) \le 46.04 \\ 0 & \text{Otherwise} \end{cases} \tag{5.6}$$

$$\mu_{E_{cm}}(E_{cm}) = \begin{cases} \dfrac{E_{cm}(t) - 29}{31 - 29} & \text{if } 29 \le E_{cm}(t) \le 31 \\ \dfrac{31 - E_{cm}(t)}{33 - 31} & \text{if } 31 \le E_{cm}(t) \le 33 \\ 0 & \text{Otherwise} \end{cases} \tag{5.7}$$

Note: Variables with a constant value (for example, density and modulus of elasticity of steel) are treated as a fuzzy variable with the membership function of *singleton*, whose degree of membership is *unity*.

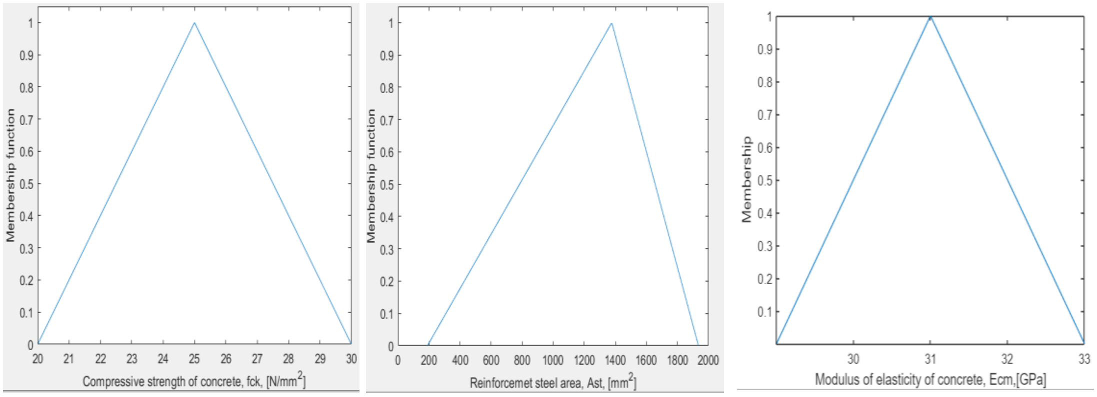


Figure 5.3: Membership function of time-variant input variables

A triangular membership function of the flexural capacity developed for the lower bound is $31.89\,\text{kNm}$ from the concrete strength of $20\,\text{MPa}$ and steel area $189.57\ \text{mm}^2$. The core value is $206.82\ \text{kNm}$ which is a deterministic design moment from applied and long-term effect loading, and the upper bound (or limiting bending moment) is $273.01\,\text{kNm}$, which is from concrete strength of $25\,\text{MPa}$ and the balanced section steel area of 1935.04 mm^2 of the same cross-sectional dimension shown in Figure 3.1. Similarly, the membership functions of other constraints were developed in Chapter 6 to perform an analysis of the possibility of failure.

5.4 Uncertainty Propagation

Structural engineers have developed analytical models to represent systems by incorporating physical laws, empirical tools, and experimental results based on observing the system output variables. These models intend to relate input-output variables (i.e., the relationship between independent and dependent variables). For instance, the deflection of the girder is an output of the function of the input variables, such as loads, material properties, span length, and support conditions. Similarly, other relevant output variables (i.e., ultimate and serviceability limit state criteria) depend on the controllable and uncontrollable input variables. In the modeling, the uncertainty of the physical laws, material mechanics, sustained and environmental loads, and boundary conditions lead to the uncertainty of the output variables. This functional relationship problem is known as the *uncertainty propagation* of the input to output variables (Ayyub and Klir 2006; Li 2013; Caoa et al. 2019). The complexity of *uncertainty propagation* increases by considering nonlinearity in behavior, bifurcation, instability, logic rules, and across-discipline interactions. In general, uncertainty propagation can be expressed by using the function of *random variables* and *fuzzy variables* in the input-out relationship.

Several methods used to solve the propagation of the aleatory input-output uncertain variables provided in (Ayyub and Klir 2006), and in general expressed by:

$$f = g(x) \tag{5.8}$$

where x is the input variable, f is the output variable, and g is the function that relates input-output variables

Based on Zadeh's extension principle, the propagation of an m - dimensional vector of fuzzy input variables $\left(X\right)$ expressed by (Adhikari and Khodaparast 2014):

$$f = g\left(X\right) \tag{5.9}$$

where f is the n - dimensional vector of fuzzy output variables that can be obtained from appropriate numerical optimization $g(X): R^m \rightarrow R^n$ applied to m fuzzy input variables X

The function $g(\bullet)$ depends on the material properties, types, and behavior of structure or system, constraints, etc., of a real problem under consideration. This study mainly focused on epistemic uncertainty; hence, the propagation of the epistemic uncertainty has been discussed herein wide.

5.4.1 A Numerical Approach for Fuzzy Uncertainty Propagation

The numerical strategy for propagating uncertainty, characterized as fuzzy sets, is presented in a three-step procedure by using the *extension principle* (Chen et al. 2015; Jakeman et al. 2010). These three-step procedures used to solve the propagation of epistemic uncertain variables involve: i) identifying the ranges of the uncertain inputs, ii) generating an accurate numerical approximation of the solution within the estimated ranges, and iii) post-processing the results.

Extension principle: Let $A_1, A_2, ..., A_m$ be fuzzy sets with the corresponding membership functions $\mu_{A_1}, \mu_{A_2}, ..., \mu_{A_m}$ defined on their corresponding universe of discourse $X_1, X_2, ..., X_m$, respectively, and let X be the *Cartesian product* $X = X_1 \text{ x } X_2 \text{ x }, ..., \text{x } X_m$. Let f be a mapping from X to a set Y, i.e., $y = f\left(x_1, x_2, ..., x_m\right)$, i.e., $x \in A, y \in B, y = f\ \ x$ or $x = f^{-1}\ \ y$. Then, the extension principle allows us to define a fuzzy set B in Y by (Zimmermann 2011; Lee 2004):

$$B = \left\{\left(y, \mu_B\left(y\right)\right) | \ y = f\left(x_1, x_2, ..., x_m\right), \left(x_1, x_2, ..., x_m\right) \in X\right\} \tag{5.10}$$

where

$$\mu_B(y)=\begin{cases}\sup_{(x_1,x_2,\ldots,x_m)\in f^{-1}(y)}\min\left\{\mu_{A_1}(x_1),\mu_{A_2}(x_2),\ldots,\mu_{A_m}(x_m)\right\} & \text{if } f^{-1}(y)\neq\varnothing\\ 0 & \text{if } f^{-1}(y)=\varnothing\end{cases}$$

(5.11)

where

f^{-1} is the inverse of f

For $m=1$, the extension principle reduces to

$$B=\left\{\left(y,\mu_B(y)\right)\mid y=f(x),x\in X\right\} \quad (5.12)$$

where

$$\mu_B(y)=\begin{cases}\sup_{x\in f^{-1}(y)}\mu_A(x) & \text{if } f^{-1}(y)\neq\varnothing\\ 0 & \text{if } f^{-1}(y)=\varnothing\end{cases} \quad (5.13)$$

Example 5.1: Let $A=\{(-1,0.3),(0,0.7),(1,1),(2,0.5)\}$ be a fuzzy set and $f(x)=x^2$, find $B=f(A)$ by applying the extension principle. Since the problem belongs to $m=1$, then B can be obtained by using Equations (5.12) and (5.13).

$B=\{(0,\mu_B(0)),(1,\mu_B(1)),(4,\mu_B(4))\}$

$\mu_B(0)=\sup_{0\in f^{-1}(0)}(0.7)=0.7$; $\mu_B(1)=\sup_{-1,1\in f^{-1}(1)}(1)=1$; $\mu_B(4)=\sup_{2\in f^{-1}(4)}(0.5)=0.5$

Therefore, $B=\{(0,0.7),(1,1),(4,.5)\}$

Example 5.2: For $m=2$

$A_1=\{(-1,1),(0,0.4),(1,0.2),(2,0.5)\}$ and $A_2=\{(-1,0.5),(0,0.08),(1,1),(2,0.4)\}$, if $f(x_1,x_2)=x_1^2+x_2^2$. Find $B=f(A_1,A_2)$. By applying the extension principle, B obtained by using Equations (5.10) and (5.11).

$f(0,0)=0^2+0^2=0$, $\mu_B(0)=\sup_{(0,0)\in f^{-1}(0)}\min(0.4,0.08)=0.08$

$$\mu_B(1)=\sup_{\substack{(-1,0)\in f^{-1}(1)\\(1,0)\\(0,-1)\\(0,1)}}\left(\min(1,0.08),\min(0.2,0.08),\min(0.4,0.5),\min(0.4,1)\right)$$

$=\sup(0.08,0.08,0.4,0.4)=0.4$

$$\mu_B(2) = \sup_{\substack{(-1,-1)\in f^{-1}(2)\\(-1,1)\\(1,-1)\\(1,1)}} \{\min(1,0.5), \min(1,1), \min(0.2,0.5), \min(0.2,1)\}$$

$$= \sup\{0.5, 1, 0.2, 0.2\} = 1$$

$$\mu_B(4) = \sup_{\substack{(0,2)\in f^{-1}(4)\\(2,0)}} \{\min(0.4,0.4), \min(0.4,0.4)\}$$

$$= \sup\{0.4, 0.4\} = 0.4$$

$$\mu_B(8) = \sup_{(2,2)\in f^{-1}(8)} \{\min(0.5,0.4)\} = 0.4$$

$$\mu_B(5) = \sup_{\substack{(-1,2)\in f^{-1}(5)\\(1,2)\\(2,-1)\\(2,1)}} \{\min(1,0.4), \min(0.2,0.4), \min(0.5,0.5), \min(0.5,1)\}$$

$$= \sup\{0.4, 0.2, 0.5, 0.5\} = 0.5$$

Therefore, $B = \{(0, 0.08), (1, 0.4), (4, 0.4), (5, 0.5), (8, 0.4)\}$

The commonly used numerical approach to solve uncertainty propagation is a *global optimization approach* (GOA). In GOA, each $\alpha-\text{cut}$ used to solve both minimum and maximum values of the output quantities. A $\alpha-\text{cut}$ value of 0 is used to obtain the interval (or support), beyond which the output variable is not the issue, of the fuzzy output variables. The nominal value of output quantities can be obtained by setting the value of $\alpha=1$. Thus, by combining all possible results for all $\alpha-\text{cuts}$, one can obtain the fuzzy description of the output quantities. Although several tools are available for numerical optimization, its computational cost is high to solve two optimization problems for each $\alpha-\text{cut}$. Therefore, adopting an efficient numerical method is necessary to employ the approach.

5.5 Application of Fuzzy Concepts in Reinforced Concrete Structures

5.5.1 Design of RC Structure Using Fuzzy Set Theory

In structural engineering, the design parameters are not certain due to different factors. Some uncertainties can be estimated by preparing sufficient sample data, but human knowledge is limited to define and handle all uncertainties. The design optimization of the reinforced concrete structure possesses various constraints and bound of design variables (objectives). The lower and upper bound of the design variable implies the fuzzy interval of the variables. Within the fuzzy interval, the specific value of the variable has its corresponding membership function.

In reinforced concrete structure, the presence of fuzzy uncertainty of input variables such as material properties, cross-sectional dimensions, load uncertainties, and model uncertainties (Fan et al. 2019) also propagates to the output performance of the structure by the function relationship between the input variables and output performance. In this research, the optimum design of reinforced concrete structure based on the membership degree of input variables (concrete strength and reinforcement steel area) and the flexural performance of the structure as an output variable is carried out using fuzzy relation and fuzzy composition.

To illustrate the problem, let us consider the same problem used in Chapter 4 and taking some elements of concrete strength and reinforcement steel with their respective degree of membership from Figure 5.3, and the flexural performance of the RC beam section. Let's take fuzzy sets of steel area:

$$A_{st} = \sum \mu_{A_{st}} \left(A_{st,i} \right) / A_{st,i} \tag{5.14}$$

The compressive strength of concrete

$$F_{ck} = \sum \mu_{F_{ck}} \left(f_{ck,j} \right) / f_{ck,j} \tag{5.15}$$

The flexural resistance of the beam section

$$M_R = \sum \mu_{M_R} \left(M_{R,k} \right) / M_{R,k} \tag{5.16}$$

The fuzzy relation of steel area and the flexural resistance of the section is given by:

$$R = A_{st} \mathrm{x} M_R = \sum_i^m \sum_k^p \mu_R \left(A_{st,i}, M_{R,k} \right) \Big/ \left(A_{st,i}, M_{R,k} \right) \tag{5.17}$$

in which $\mu_R \left(A_{st,i}, M_{R,k} \right) = \mu_{A_{st}} \left(A_{st,i} \right) \wedge \mu_{M_R} \left(M_{R,k} \right) = \min \left[\mu_{A_{st}} \left(A_{st,i} \right), \mu_{M_R} \left(M_{R,k} \right) \right]$.

Similarly, the fuzzy relation of concrete strength and the flexural resistance of the section is:

$$S = F_{ck} \mathrm{x} M_R = \sum_i^m \sum_k^p \mu_R \left(f_{ck,j}, M_{R,k} \right) \Big/ \left(f_{ck,j}, M_{R,k} \right) = \min \left[\mu_{F_{ck}} \left(f_{ck,j} \right), \mu_{M_R} \left(M_{R,k} \right) \right]$$

(5.18)

As if R and S are two fuzzy relations the fuzzy composition, which is used to obtain optimum design solution and to evaluate the performance of the structure (Brown et al. 1983), is given by:

$$R \circ S = \left\{ \max - \min \left[\mu_R \left(A_{st,i}, M_{R,k} \right), \mu_S \left(f_{ck,j}, M_{R,k} \right) \right] \right\} \tag{5.19}$$

Let us consider the input variables of concrete strength and area of reinforcement with $\alpha - cut$ of 0.6 in which the combination of the lower and upper bound of variables to be considered. From the triangular membership function of concrete strength and area of reinforcement from Figure 5.3, let us consider the α -cut of 0.6 for these input variables. The fuzzy sets of concrete strength, area of reinforcement, and the flexural capacity of the section which is computed from the section for corresponding strength of concrete and area of steel, respectively, as follows:

$F_{ck} = \{0.6/23 \quad 0.8/24 \quad 1/25 \quad 0.8/26 \quad 0.6/27\}$

$A_{st} = \{0.6/902.96 \quad 0.934/1300 \quad 1/1378.56 \quad 0.934/1415.29 \quad 0.6/1601.15\}$

$M_R = \{0.62/141.13 \quad 0.94/195.43 \quad 1/206.71 \quad 0.94/210.03 \quad 0.52/237.42\}$

The fuzzy relation of the area of reinforcement and flexural capacity of the section is obtained by the cross product of the column vector of the area of reinforcement and row vector flexural capacity using Equation (5.17). Similarly, the fuzzy relation of concrete strength and flexural capacity of the section is obtained by the cross product of a column vector of concrete strength and row vector flexural capacity using Equation (5.18). Then finally, from Equation (5.19) the design optimization can be decided from the fuzzy composition of two relations. Using a similar procedure of fuzzy concepts, the level of time-dependent performance of the structure can also be evaluated.

$$R = A_{st} \text{x} M_R = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.934 & 0.934 & 0.934 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.934 & 0.934 & 0.934 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix};$$

$$S = F_{ck} \text{x} M_R = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix}$$

$$D = R \circ S = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix} \rightarrow \text{ is fuzzy composition}$$

The membership function of input variables and the fuzzy composition matrix are closely related. When the degree of membership approaches *unity* the design is good and the degree of membership approaches zero (0) design becomes poor. For a single α-cut there is one lower and one upper bound of both input variables and output variables except for $\alpha = 1$, which has a single value. If $\alpha - \text{cut}$ approaches zero (0), the lower bound of the input variable is very small and the performance of the structure becomes unsafe. For example, for the input variables, a concrete strength of 0.6/23, and the steel area of 0.6/902.96 the capacity of the section is $141.13\,\text{kNm}$, which is very small compared with the demand of the section. The upper bound of the section's performance is larger than the core value leads to a safe section but it leads to over-strength deign and the economy can be under question. For example, the 0.6/27 concrete strength and 0.6/1601.15 steel area are the input variables, and the capacity of the section becomes $237.42\,\text{kNm}$, whose membership is 0.52, which is significantly larger than the demand of the section; consequently, the section becomes uneconomical. Except for the core value of the structural performance, the other membership degrees have both lower and upper bounds of the section capacity.

However, the degree of membership of the lower bound approaches to unity, the capacity is less than the demand of the section but, the structure will not collapse because the reinforced concrete structure fails after attaining its possible plastic mechanisms. In optimum design, the combination of concrete strength with membership degree 0.6 and steel area with a membership degree of *unity* both the lower and upper bound of flexural capacity are close to the demand of the section. This implies and strengthens the principle that the capacity of a singly reinforced section governed by steel and wise consideration of fuzzy interval for a reasonable solution.

5.5.2 Performance Evaluation of RC Structure Using Fuzzy Set Theory

All the possible combination of input variables (concrete strength in N/mm^2 and area of reinforcement in mm^2) with their respective membership function and the flexural capacity in kNm of the reinforced concrete beam section is represented in Table 5.2. The flexural capacity of the section is obtained by using Equation (4.5).

Table 5.2: Membership function, the fuzzy interval of input variables, and the section capacity

Material	Membership function	Interval	Flexural capacity Interval of section	Membership of flexural capacity
Concrete	0.6	[23, 27]	[141.13, 230.81]	[0.62, 0.62]
Steel	0.6	[902.96, 1601.15]	[143.23, 237.42]	[0.64, 0.52]
Concrete	0.6	[23, 27]	[194.21, 208.58]	[0.93, 0.96]
Steel	0.934	[1300,1415.15]	[198.56, 213.74]	[0.95, 0.89]
Concrete	0.6	[23, 27]	[204.06, 208.96]	[0.98, 0.95]
Steel	1	[1378.56, 1378.56]		
Concrete	0.8	[24, 26]	[141.72, 232.67]	[0.63, 0.60]
Steel	0.6	[902.96, 1601.15]	[142.77, 235.97]	[0.63, 0.54]
Concrete	0.8	[24, 26]	[195.43, 210.03]	[0.94, 0.94]
Steel	0.934	[1300,1415.15]	[197.60, 212.60]	[0.95, 0.90]
Concrete	0.8	[24, 27]	[205.44, 208.96]	[0.99, 0.97]
Steel	1	[1378.56, 1378.56]		
Concrete	1	[25, 25]	[206.71, 206.71]	[1, 1]
Steel	1	[1378.56, 1378.56]		

Note that, in concrete steel combination, the interval of section capacity and membership function the first row and the second row of each combination rather than a membership grade is unity was determined by considering the lower bound and the upper bound of concrete strength, respectively.

CHAPTER 6

TIME-DEPENDENT FAILURE POSSIBILITY ANALYSIS

6.1 Introduction

This chapter mainly presents the basic concepts of time-dependent failure possibility, estimation of possibilistic safety index, and finally estimation of time-dependent failure possibility.

6.2 Basic Concepts of Time-Dependent Failure Possibility

The time-dependent failure possibility is the possibility of performance less than zero under fuzzy uncertainty in a specified time interval. The failure possibility is defined as the maximum membership function of the performance of all possible constraints at which $f_G^{-\alpha} = 0$ (Tang et al. 2014; Zhangchun and Zhenzhou 2014). The method of failure possibility evaluation depends on the type of MF selected for structure performance. As membership functions are complex, the computational cost of the failure possibility analysis increases and leads to inaccurate results. For instance, pseudo exponential, and general bell membership functions lead to high computational cost and inaccurate results, whereas the triangular membership functions lead to simple failure possibility analysis such as simple interpolation or bisection method that has a low computational cost and gives accurate results.

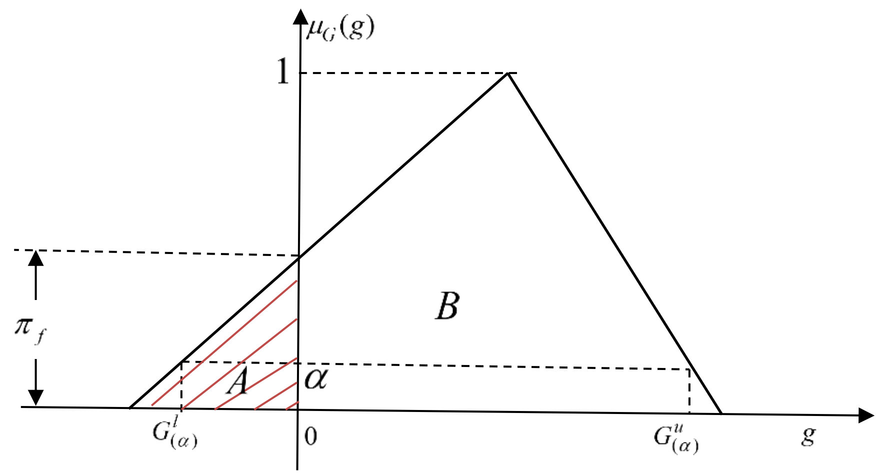


Figure 6.1: The relationship between fuzzy intervals of performance, membership function of performance and the failure possibility index

where: $G^{l}_{(\alpha)}$ and $G^{u}_{(\alpha)}$ are the lower and upper bounds of $G_{(\alpha)}$, which is $\alpha-\text{cut}$ of $G(X,t)$; $G_j(X,t)$ is the performance of a specified constraint; $f_G^{-\alpha}$ is the lower bound of F_{G_α} which is $\alpha-\text{cut}$ of F_G and π_f is the failure possibility of the structure

As shown in Figure 6.1 the performance of structure less than zero, which is indicated in part A (hatched), and greater than zero is indicated in part B. Any combination of input variables, output responses, and performance of different constraints in part A leads to failure of the structure whereas, in part B, these combinations give reliable (safe) structure and similarly, the interface that separate two parts A and B the combination of parameters yield $P=0$, at which the safety of the structure ceases or attains limit state.

Suppose that $X=\{X_1,X_2,X_3,...,X_m\}$ is the fuzzy input variables with the MF of $\mu_{X_i}(x)\ (i=1,2,...,\text{m})$. Assume that the time-dependant performance (TDP) of the j^{th} constraint of the structure is given by $G_j(X,t)$, which is a function of the fuzzy input variables Y and the time t. In the structure system, the presence of fuzzy uncertainty of input variables (Fan et al. 2019) also propagates to the output responses and performance of the structure by the function relationship between the input variables, output responses, and performance by *extension principle*. The MF $\mu_{G_j}(g)(j=1,2,...,n)$ of the structure performance can be evaluated by using the optimization algorithm.

In the study, the time-dependent performance of different constraints, i.e., flexure, shear, deflection, and crack of the beam subjected to sustained load, corrosion, creep and shrinkage have been performed analytically to check whether the beam is safe using Equation (3.4). Since deterioration of material properties and variation of action lead to the imprecision of data that possesses fuzziness. The fuzzy uncertainty of input variables propagates to the output response; eventually, the output responses and performances are fuzzy variables.

Table 6.1: Time-dependent resistance, action, limit values and performance of the RC beam

Time (year)	Resistance		Action		Action effects		Limiting values		Performance			
	Flexure (kNm)	Shear (kN)	Flexure (kNm)	Shear (kN)	Deflection (mm)	Crack (mm)	Deflection (mm)	Crack (mm)	Flexure (kNm)	Shear (kN)	Deflection (mm)	Crack (mm)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(2)-(4)	(3)-(5)	(8)-(5)	(9)-(7)
0	218.99	122.71	72.00	48.00	3.32	0.00	24.00	0.30	146.99	74.71	20.68	3.00
5	218.99	122.71	206.02	118.77	22.32	0.23	24.00	0.30	12.97	3.94	1.68	0.07
10	218.99	122.71	206.08	118.81	22.87	0.23	24.00	0.30	12.91	3.90	1.13	0.07
15	218.99	122.03	206.27	118.94	23.05	0.23	24.00	0.30	12.72	3.09	0.95	0.07
20	203.77	106.03	206.42	119.04	23.43	0.27	24.00	0.30	-1.86	-13.00	0.57	0.03
25	189.42	95.18	206.58	119.14	24.37	0.34	24.00	0.30	-16.63	-23.97	-0.37	-0.04
30	174.89	85.63	206.74	119.25	25.41	0.40	24.00	0.30	-31.52	-33.62	-1.41	-0.10
35	160.31	77.41	206.93	119.38	26.58	0.45	24.00	0.30	-46.44	-41.97	-2.58	-0.15
40	145.81	70.50	207.06	119.46	27.88	0.51	24.00	0.30	-61.18	-48.96	-3.88	-0.21
45	131.52	64.92	207.13	119.51	29.43	0.57	24.00	0.30	-75.60	-54.59	-5.43	-0.27
50	117.54	60.63	207.18	119.44	31.18	0.64	24.00	0.30	-89.64	-58.92	-7.18	-0.34

The TDFP analysis can be performed by fuzzy operations and numerical algorithm based on possibility theory as follows:

$$\begin{aligned}\pi_{f_j t\in[t_s,t_e]} &= Poss\left\{G_j\left(t\right)\leq 0\right\} \\ &= \sup\left\{\mu \middle| G_j\left(t\right)\leq 0, t\in\left[t_s,t_e\right]\right\}\end{aligned} \tag{6.1}$$

where $\pi_{f_j t\in[t_s,t_e]}$ is the TDFP at the time instant $t\in\left[t_s,t_e\right]$; $Poss\{\bullet\}$ is the possibility of the event; $G(t)$ is the TDP of the j^{th} constraint; $\sup\{\bullet\}$ represents the supremum of the set and μ is the membership degree of the performance at the time instant $t\in[t_s,t_e]$

The constraints of the concrete structures depend on material properties that deteriorate with time and the structural system (e.g., member arrangement and support condition, span length of the structural members, and types of load). Due to environmental factors such as corrosion, creep and shrinkage, and residual stress from improper utilization, the performance of the structure deteriorates with time. These reduced performances of the structure with time $G(X,t)$ increase the failure possibility with time. Figure 6.2 shows the increment of TDFP of the structure. This shows, the service life of a structure increases the deterioration of material properties, and creep and shrinkage increases, eventually the resistance of the structure decreases. Consequently, the performance of the structure decreases with a time that increases failure possibility. To show the increment of the TDFP, the distribution of the MF $\mu_{G_j}(g)$ of the structure performance is arranged in descending order of time because the performance reduction increase with time (i.e., negative part), and failure possibility increases with time.

Similar to the MF of the fuzzy variables, the value of the TDFP of the structure should satisfy the inequality $0 \le \pi_{f_j t \in [t_s, t_e]} \le 1$. For boundaries of the time interval, i.e., $[t_s, t_{ei}]$ and $[t_s, t_{ej}]$ satisfy the inequality $t_{ei} < t_{ej}$, the time-dependent failure possibility $\pi_{ft \in [t_s, t_{ei}]}$ and $\pi_{ft \in [t_s, t_{ej}]}$ should satisfy the following inequality (Fan et al. 2019).

$$\pi_{ft \in [t_s, t_{ei}]} \le \pi_{ft \in [t_s, t_{ej}]} \quad \text{for} \quad t_{ei} < t_{ej} \tag{6.2}$$

Therefore, for every instant of time $t_i \in [t_1, t_2, t_{3,} ..., t_n]$, the performance of the structure $G(t_i)(i = 1, 2, ..., n)$ satisfies the inequality $G(t_1) < G(t_2) <, ..., < G(t_n)$; consequently, the TDFP has to satisfy the following inequality.

$$\pi_{ft_{(1)}} > \pi_{ft_{(2)}} > \pi_{ft_{(3)}} >, ..., > \pi_{ft_{(n)}} \tag{6.3}$$

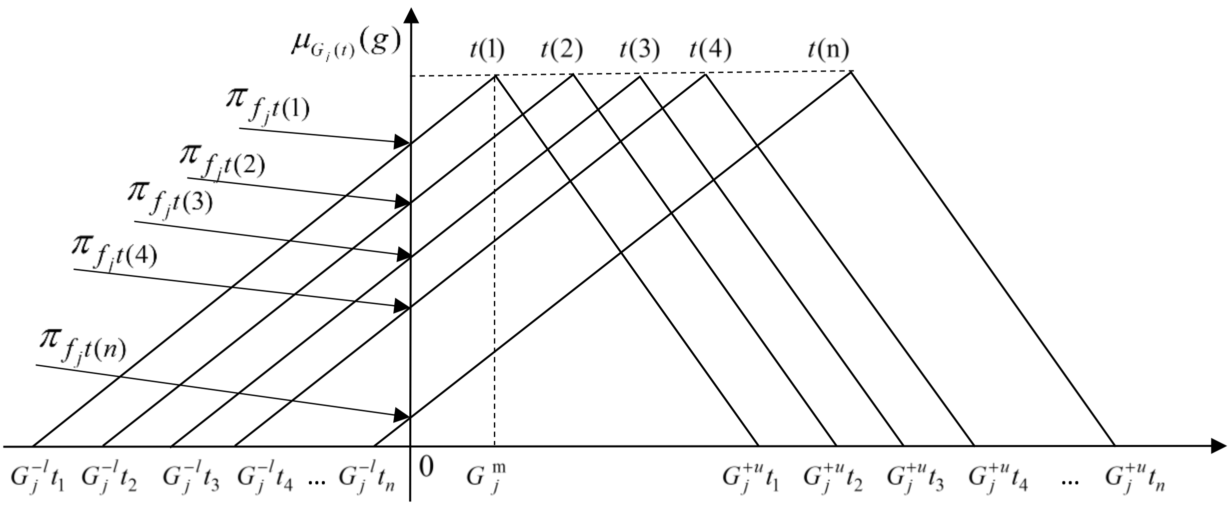


Figure 6.2: The relationship between TDFP and MF of the performance in the time interval

where, $G_j^{-l} t_i$ and $G_j^{+u} t_i$ are the lower and upper bounds of $G_j(t)$ the interval, respectively; $\mu_{G_j(t)}(g)$ is the MF of the j^{th} constraint performance at the time t; $\pi_{f_j t(\mathrm{i})}$ is the failure possibility of the j^{th} constraint at the time instant $t(i)$; G_j^{m} is the nominal value of the j^{th} constraint, and $[t_1, t_2, t_{3,} ..., t_n]$ represents the descending order of the time set.

6.3 Estimation of Possibilistic Safety Index

Form the definition of failure possibility π_f expressed mathematically in Equation (6.1): if the specific constraint's performance with time satisfies the inequality $0 \geq G^{\mathrm{m}}_{j(t)}$ then $\pi_{f(t)} = 1$, which implies complete failure (unreliable); if $0 \leq G^l_j\left(t \in [t_s, \mathrm{t}_e]\right)$ then $\pi_{f_j t \in [t_s, t_e]} = 0$ implies no failure (reliable), and if $G^l_j\left(t \in [t_s, \mathrm{t}_e]\right) \leq G_j\left(t \in [t_s, \mathrm{t}_e]\right) \leq G^{\mathrm{m}}_j\left(t \in [t_s, \mathrm{t}_e]\right)$, the time-dependent failure possibility $\pi_{f_j t \in (t_s, t_e)}$ can be obtained from Equation (6.4) (Tang et al. 2014).

$$\pi_f = \begin{cases} 0 & \text{if } f^{-\alpha}_{G(t)} \geq 0 \\ Poss\left\{\alpha \,\middle|\, f^{-\alpha}_{G(t)} \leq 0, \mu \in [0,1]\right\} & \text{if } 0 \leq f^{-\alpha}_{G(t)} \leq G^m_j \\ 1 & \text{if } 0 \geq G^{\mathrm{m}}_j \end{cases} \tag{6.4}$$

To estimate the failure possibility of the structure, the performance function should satisfy the inequality $0 \leq f^{-\alpha}_{G(t)} \leq G^m$ and its solution obtained from an expression:

$$f^{-\alpha}_{G_j t \in [t_s, \mathrm{t}_e]} = 0 \tag{6.5}$$

To obtain TDFP $\pi_{f_j t \in [t_s, t_e]}$ from Equation (6.5), one should first determine the lower bound $f^{-\alpha}_{G_j t \in [t_s, \mathrm{t}_e]}$ of $F_{G_\mu(t)}$ which is $\alpha-$cut of $F_{G_j(t)}$. The performance function may or may not be linear, depending on input variables and type of constraints. For a reinforced concrete structure involving fuzzy input variables, the output variables (i.e., responses) obtained from Equation (6.5) are nonlinear. For instance, flexural, shear, deflection, and crack performances of the reinforced concrete structure possess nonlinearity depending on material properties, loading type, and structure system. The nonlinearity of performance complicates the solution methods, consequently, increases computational cost.

In general, the time-dependent performance, i.e., $G_j(t) = G(X, t)$ is a function of the fuzzy input variables X and the time t. Hence, $G_j(t)$ is also a fuzzy variable because of fuzzy uncertainty propagation with time through the *extension principle*. Eventually, the MF of the performance $G_j(t)$ is also time-dependent. Similar to time-independent failure possibility (TIFP), the TDFP is the maximum failure possibility of the structure over a specified time interval $t \in [t_s, t_e]$. If the membership degree of TDFP is greater than $\pi_{f_j t\in[t_s,\mathrm{t}_e]}$, then $G_j(t)$ always could be nonnegative over the time interval $t \in [t_s, t_e]$, which guarantees the reliability of the structure. For a specific time instant t , we could have the time-invariant material properties, sectional dimensions, and loading. Therefore, the performance $G_j(t)$ of the structure system becomes time-invariant. Thus, based on the *extreme value transformation* method, the minimum performance $G_{\min}(X)$ is considered to evaluate the time-dependent failure possibility π_f at a specific time interval $t \in [t_s, t_e]$. Therefore, the TDFP is equivalent to the TIFP of the minimum performance function $\underset{t\in[t_s,t_e]}{G_{\min}}(Y)$ of $P_j(t) = P(Y,t)$ estimated from the numerical algorithm (Fan et al. 2019):

$$\begin{aligned} \pi_{f_j t\in[t_s,\mathrm{t}_e]} &= Poss\left\{ \underset{t\in(t_s,t_e)}{G_{\min}}(X,t) \le 0, \exists t \in [t_s, t_e] \right\} \\ &= \sup\left\{ \mu \,\middle|\, \underset{t\in[t_s,t_e]}{Min} G(\mathrm{Y},t) \le 0, \mu \in [0.1] \right\} \end{aligned} \tag{6.6}$$

From the time-dependent performance analysis, we can have the lower bound $f^{-\alpha}_{G_j t\in[t_s,\mathrm{t}_e]}$, the core value $G^{\mathrm{m}}_{jt\in[t_s,\mathrm{t}_e]}$, and the upper bound $f^{+\alpha}_{G_j t\in[t_s,\mathrm{t}_e]}$ of the structural performance. To generate a triangular MF of performance obtained from Equation (6.5), the lower bound and nominal value of a defined constraint is sufficient. After obtaining the lower bound, whose value is less than zero at a specified time instant, and the nominal value of performance function, the failure possibility $\pi_{f_j t\in[t_s,\mathrm{t}_e]}$ is computed by using the optimization algorithm by setting $f^{-\alpha}_{G_j t\in[t_s,\mathrm{t}_e]} = f^{-\pi_f}_{G_j t\in[t_s,\mathrm{t}_e]} = 0$. Because a membership function of performance for a specific constraint is continuous from lower bound to a core value, consequently f is a continuous function defined on the membership interval $[a,b]$, with $f(a)$ and $f(b)$ of opposite sign. The intermediate value theorem denotes that a number c exists in (a,b) with $f(c) = 0$. In this procedure, there is only one possible root in the interval (a,b) because there is only one point at which the MF crosses the ordinate axis.

From the intermediate value theorem (Burden and Faires 2011); (Hoffman 2001), the numerical analysis is employed to obtain the solution for Equation (6.4). The procedure is summarized as follows:

i. *Initialization*: set $\alpha_1^0 = 0$ and $\alpha_2^0 = 1$, and terminating condition as $\varepsilon = 1\text{x}10^{-8}$

ii. *Iteration 1*: compute $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-\alpha_1^0}$ and $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-\alpha_2^0}$, and if $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-\alpha_2^0} = f_{G_j t\in[t_s,\mathrm{t}_e]}^{-1} = m_{G_j} \le 0$ holds, the structure is considered as unreliable (complete failure), no iteration is required and the failure possibility of the structure can be taken as $\pi_{f_j t\in[t_s,\mathrm{t}_e]} = 1$, else, evaluate $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-\left(\alpha_1^0+\alpha_2^0\right)/2}$, and go to the next step.

iii. *Iteration* k $\left(k \ge 1\right)$: If $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-\alpha_1^{k-1}}$ and $f_{G_j t\in[t_s,\mathrm{t}_e]}^{-(\alpha_1^{k-1}+\alpha_2^{k-1})/2}$ have the same sign, set $\alpha_1^k = \left(\alpha_1^{k-1} + \alpha_2^{k-1}\right)/2$, and $\alpha_2^k = \alpha_2^{k-1}$. Otherwise, update the values as $\alpha_1^k = \alpha_1^{k-1}$ and $\alpha_2^k = \left(\alpha_1^{k-1} + \alpha_2^{k-1}\right)/2$. Go to the next step.

iv. *Terminating condition:* calculate $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right|$, and if the absolute value is less than or equal to the stopping condition that set in the initialization step holds, stop the iteration, and the time-dependent failure possibility $\pi_{f_j t\in[t_s,\mathrm{t}_e]}$ can be estimated as $\pi_{f_j t\in[t_s,\mathrm{t}_e]} = \left(\alpha_1^{k-1} + \alpha_2^{k-1}\right)/2;$ else, go to step 3 and carry out the iterative procedure until the terminating condition satisfied.

6.4 Estimation of Time-Dependent Failure Possibility

To perform the TDFP analysis, the estimation of the nominal value and the lower bound of performance $G_j^{-l} t_i$, which is less than or equal to zero, of the corresponding constraint at a specified time interval $t \in [t_s, t_e]$ carried out. From the analysis, the nominal value of flexure is $206.82\ \text{kNm}$, the shear force is $111\ \text{kN}$, as per EC2 the nominal value of deflection is a limit deflection of the horizontal element, $L/250 = 24\ \text{mm}$, and the limit crack width is 0.3 mm. The result of TDFP of each constraint at the specified time interval $t \in [t_s, t_e]$ is shown in Table 6.2 and Figure 6.3 using the triangular MF, which is simple to apply.

The TDFP of the concrete structure increases with time due to deterioration of material properties and loss of steel cross-section due to corrosion, reduction of bending stiffness, and additional action from creep and shrinkage that reduces the performance of the structure. As shown in Figure 6.2, for each MF, a point at which it crosses the ordinate axis is taken as a failure possibility π_f for each time instant $t \in [t_s, t_e]$. For different constraints of the structure, the membership function of performance could be different. Therefore, each constraint has different membership functions and time-dependent failure possibilities shown in Figures 6.2 and 6.3, respectively.

The failure possibility of the structure is the maximum failure of all possible constraints in its design life as expressed in Equation (6.7) and its result is represented in the last column of Table 6.2

$$\pi_{ft\in[t_s,t_e]} = Max\left\{\mu_{\substack{G_{\min}\\ t\in(t_s,t_e)}}(flexure) \le 0, \mu_{\substack{G_{\min}\\ t\in(t_s,t_e)}}(shear) \le 0, \mu_{\substack{G_{\min}\\ t\in(t_s,t_e)}}(deflection) \le 0, \mu_{\substack{G_{\min}\\ t\in(t_s,t_e)}}(crack) \le 0\right\} \quad (6.7)$$

Table 6.2: TDFP for different constraints of the RC beam

Time instant (yr.) $t \in [t_s, t_e]$	FP of constraints $(\pi_{f_j\, t\in[t_s,t_e]})$				FP of structure $\pi_{f\, t\in[t_s,t_e]}$
	Flexure	shear	Deflection	Crack	Max$(\pi_{f_j\, t\in[t_s,t_e]})$
[0, 0]	0	0	0	0	0
[0, 5]	0	0	0	0	0
[0, 10]	0	0	0	0	0
[0, 15]	0	0	0	0	0
[0, 16]	0	0.00946	0	0	0.00946
[0, 20]	0.01209	0.11718	0	0	0.11718
[0, 22]	0.03742	0.15807	0	0.02000	0.15807
[0, 24]	0.06895	0.19726	0.00833	0.10000	0.19726
[0, 25]	0.08297	0.21581	0.01542	0.13333	0.21581
[0, 30]	0.15400	0.30282	0.05875	0.33333	0.33333
[0, 35]	0.22454	0.37803	0.10750	0.50000	0.50000
[0, 40]	0.29615	0.44099	0.16167	0.70000	0.70000
[0, 45]	0.36529	0.49170	0.22625	0.90000	0.90000
[0, 47]	0.39295	0.50872	0.254167	0.99333	0.99333
[0, 48]	0.40654	0.51656	0.26875	1.00000	1.00000
[0, 50]	0.43342	0.53062	0.29917	1.00000	1.00000

For the specified site condition and corrosion current density $0.75\ \mu A/cm^2$, the reinforced concrete beam is unsafe from the 16th year in its lifetime due to the performance of the beam against shear criterion, the other criteria follow, as shown in

Table 6.2. The susceptibility of the RC beam section to the shear failure over the flexural criteria is due to the smaller concrete cover of the links that initiate the corrosion of links before longitudinal reinforcement (ES EN 1992-1-1 2004; IS 456 2000), and shear performance highly depends on the area and yield strength of links, the strength of concrete, and the dowel action of the longitudinal reinforcements.

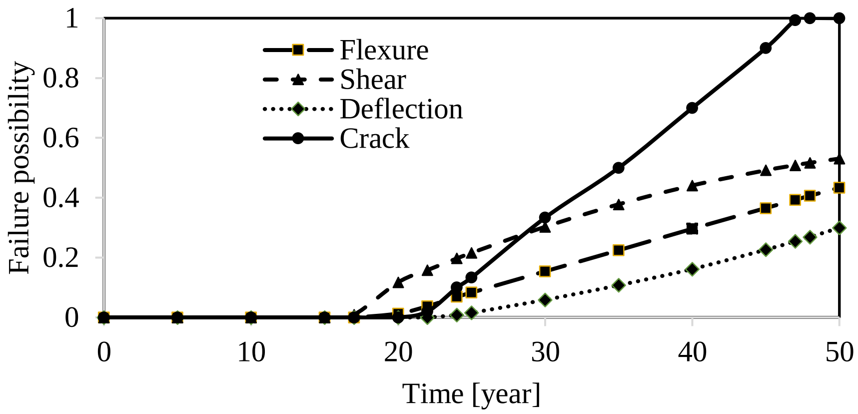


Figure 6.3: TDFP for different constraints of the reinforced concrete beam

The TDFP as shown in Figure 6.3 and Table 6.2, the failure initiation time and the degree of failure of each constraint were different due to the functional relationship between input variables and the performance of the structure. However, each constraint owns its MF of the performance, only the worst-case governs the failure possibility of the structure. For example, the structure is safe against flexure, deflection, and crack before the 19.5th year of its design life whereas, the structure failure against shear initiates in the 15.8th year of its design life. The reinforced concrete beam, which is considered in this study, failure is governed by shear stress up to the 29th year, and then by crack failure until the design life. For a specific time instant $t \in [t_s, t_e]$ for instance [0, 25], the failure possibility π_f of the RC beam against shear is 0.21581, flexure is 0.08297, deflection is 0.01542, and crack is 0.13333 for the specified environmental factors. Therefore, in the 25th year, the possibility of failure is 0.21581, which is governed by shear stress. Similarly, for a specific time instant [0, 40], the failure possibility π_f of the RC beam against shear is 0.44099, flexure is 0.29615, deflection is 0.16167, and crack is 0.70000 for the specified environmental factors. Therefore, in the 40th year of the structure, the possibility of failure is 0.70000, governed by crack, i.e., due to the combined effect of applied load and the corrosion products. The complete failure of the RC beam, considered in this study, occurs in the 48th year.

6.5 Validation of the Study

The time-invariant performance analysis methods expressed in standard codes and other literature do not consider the deterioration of material properties and load with respect to time that contrasts the real problem. In this study, the expressions with time-variant have been developed to account for the deterioration of the input variables and variation load over time and compared with the relevant literature. To perform the time-variant performance of the structure, site data, material properties, and sectional dimension of the RC beam using the EC2 Appendix B, a time-variant creep coefficient and shrinkage strain model has been established. The corrosion data has been. The baseline values of corrosion parameters for parametric studies proposed by (Enright and Frangopol 1998) and adopted by (Kliukas et al. 2015; Chehade et al. 2018) has been applied in this study.

Possibility analysis applies to the system problem processes epistemic uncertainty. Zadeh (1978); Kai-Yuan et al. (1991, 1993 & 1995) introduced the possibility theory and fuzzy-state assumptions to replace conventional probability theory and binary-state assumptions to handle imprecise information and small sample size of the real problem. The failure possibility analysis was introduced by (Zhangchun and Zhenzhou 2014; Tang et al. 2014) to perform design optimization of mechanical structure based on the possibility safety index method. Besides, the time-dependent failure possibility analysis introduced by (Fan et al. 2019) and several numerical examples and steel beam subjected to time-variant load and corrosion were used to verify the rationality of the established method. This study is carried out based on theories and examples articulated in literature and the results obtained were crosschecked using Excel Sheet and Matlab.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

The study was carried out by reviewing literature in the area of structural reliability to identify the causes and types of uncertainties. Further, the literature conducted on factors accelerating failure possibility, and reliability analysis methods. After identification of time-variant uncertainty of input variables and its propagation to the output performance, the time-dependent failure possibility analysis of the reinforced concrete beam was performed. To achieve this, four specific objectives were set as presented in Section 1.3 and performed through the methodology provided in Chapter 3.

Uncertainties specifically emerging in environmental dynamics are difficult to identify, predict, evaluate, and mitigate. The effects of such uncertainties need to be examined for decision-making so as to minimize the negative impacts. However, different types of uncertainties complicate the assessment of safety levels based on predictions, and in civil engineering structures, it is addressed by means of safety factors that are necessarily derived based on precise information or data. Nevertheless, all information may not be precise or complete, hence probability theory may not be handy to treat such uncertainties. This paper considers fuzzy uncertainties characterized by membership functions to handle incomplete or imprecise data, which leads to the assessment of safety in terms of failure possibility.

To carry out failure possibility analysis, firstly, the possible failure modes of the structure such as flexure, shear, deflection, and crack were identified and their time-dependent performance is estimated with an appropriate numerical model that varies with time. If the nominal value of collapse criteria and the limit value of the serviceability criteria are below applied stress and the stress effects, the failure possibility of the structure is zero. Therefore, the failure possibility evaluation is performed when the performance function is less than or equal to zero.

7.2 Conclusions

The results indicate that the performance of the reinforced concrete beam significantly decreases with the increase of corrosion rate, creep, and shrinkage in its design life, whereas the action on the structure increases due to creep and shrinkage. As the corrosion rate increases the diameter loss of reinforcement and strength loss of concrete increases, consequently the flexural and shear resistance decrease, whereas the deflection and crack increase. The empirical expressions provided in design codes estimate both deflection and the crack width constant, but in a real problem, both constraints of the reinforced concrete structure are progressive and significantly increase with time due to environmental factors. For the considered beam, the long-term deflection is 21.99 mm, the crack width is 0.233 mm, and the design load is 43.2 kN/m under normal conditions, while the deflection, crack, and the design load increased to 31.18 mm, 0.637 mm, and 46.04 kN/m (increased by 6.57 % of the factored load), respectively at the end of design life due to corrosion, creep and shrinkage.

The fuzzy uncertainty treats the partial belonging of the parameter to handle imprecise information. For example, the concrete strength in its design life has declined from 25 N/mm^2 to 20.167 N/mm^2, which lost its degree of membership to about zero (0), but this zero degree of membership does not mean that the concrete has no strength; rather its strength has fallen fully to the next lower grade given in EC2. Likewise, the steel area deteriorates from 1520.31 mm^2 to 766.88 mm^2, whose degree membership at the end of its design life becomes 0.48555. This degradation of material properties and variation load with time propagates to the performance of the structure.

The failure possibility overshoots due to deterioration of material properties and cross-section caused by environmental factors such as corrosion, creep and shrinkage, inappropriate consideration of mathematical and structural models. Analogous to the series circuit system, the failure of a reinforced concrete structure occurs even due to the failure against a single constraint in its design life. Among the constraints, the performance of structure against shear force, whose failure possibility initiates early and has high intensity, governs the service life whereas the deflection contributes the least for specified environmental factors. The partial failure of the concrete beam is governed by shear stress till the 25th year, then governed by crack and the complete failure occurs in the 48th year. The majority of failure intensity is caused by corrosion that deteriorates both concrete and steel grade and diameter of reinforcing bars whereas, creep and shrinkage that reduce bending stiffness and induce extra deflection. Hence, imprecise uncertainties originating from the environmental factors that cause corrosion need to be appropriately characterized for uncertainty quantification.

7.3 Practical Implications

The performance of structure degrades with time due to corrosion from aggressive chemical attack, creep, shrinkage, and inappropriate utility. Therefore, the time-dependent performance evaluation of the structure gives important inputs to know the level of damage and decide the maintenance strategy for preventing the premature failure of structures due to environmental effects.

From the extension principle approach, a straightforward approach, the fuzzy set theory is applicable in design optimization or performance level evaluation of reinforced concrete structure in which both system safety and economic requirements can be fulfilled when the degree membership of input variables approaches *unity*. However, the computational cost is high the inverse reliability analysis is used for design optimization based on the possibility safety index.

7.4 Recommendations

The concrete structure is safe against collapse state; excessive crack width imparts the serviceability of the structure in terms of the loss of water tightness, decay of aesthetic value, and damage of protection against corrosion of reinforcement. Therefore, during the construction of concrete structures to be built in the area of aggressive chemicals (e.g., the underground water tank and offshore structures), the treatment of steel, improving the quality of concrete, and providing adequate concrete cover is required to control corrosion. Similarly, excessive deflection affects the serviceability in terms of inducing extra moment and crack, deny occupants comfort, deny operation of mechanical equipment (if any), and may stagnate water in a low-lying area. Thus, appropriate design and detailing is required to improve the serviceability.

In this study, the RC beam with the flexure, shear, deflection, and crack constraints with material and load uncertainty due to corrosion, creep, and shrinkage considered. Hence, uncertainty from other environmental loads (e.g., earthquake and wind), soil-structure interaction, 3D structures, i.e., structure with multiple constraints need further investigation. Because, there is no specific rule to generate membership functions that leads membership function itself to fuzzy, thus, fuzzy variables with an *n-type* fuzzy need to be investigated for accurate results.

The spalling o concrete cover leading to the reduction in cross-sectional area of member, but no investigation was done before to quantify the spalling size of concrete. Therefore, in further studies it is recommended to consider the spalling of concrete cover to obtain more accurate results. In addition, the phenomena of creep and shrinkage leads to nano/micro-cracks formation in RC and thus could accelerate the corrosion of the steel in concrete. In this study the variables involved in the above-mentioned two phenomena are considered as statistically independent due to lack of related work (either analytical or experimental), therefore, this area to be addressed in further studies.

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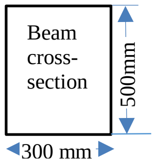
Appendix-A: Time-variant creep and shrinkage model



Cement type: Class N Age of concrete at end of curing: $t_s = 7$ days.

$T = 15.9\ °C$ Age of concrete at loading: $t_0 = 7$ days,

$RH = 60.7\ \%$ Concrete characteristic strength: $f_{ck} = 25$ MPa

Three parts of cross-section perimeter exposed to drying.

The coefficient k_h depending on the notional size h_0 calculated as a function of h_0 in accordance with EN1992-1-1 Table 3.3. With linear interpolation, the following value obtained k_h = 0.82. The adjusted value of concrete age at loading t0 by taking into account both contributions of ambient temperature and cement type is t_0 = 5.756 days.

Description	Formula	Estimated value
Coefficients and that consider the influence of concrete strength:	$\alpha_1 = (35\ MPa / f_{cm})^{0.7}$, $\alpha_2 = (35\ MPa / f_{cm})^{0.2}$ $\alpha_3 = (35 MPa / f_{cm})^{0.5}$	$\alpha_1 = 1.042$ $\alpha_2 = 1.012$ $\alpha_3 = 1.030$
Coefficient α depends on the cement type	-	$\alpha = 0$
Factor accounts for the effect of relative humidity	$\varphi_{RH} = 1 + (1 - RH / 100) / 0.1.\sqrt[3]{h_0}$	$\varphi_{RH} = 1.641$
Coefficient allows for the effect of concrete strength	$\beta(f_{cm}) = 16.8 / f_{cm}^{0.5}$	$\beta(f_{cm}) = 2.925$
Coefficient allows for the effect of concrete age at loading.	$\beta(t_0) = 1 / (0.1 + t_0^{0.2})$	$\beta(t_0) = 0.658$
Coefficient depends on the RH and h_0	$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500$	$\beta_H = 597.307$
Notional creep coefficient	$\varphi_0 = \varphi_{RH}.\beta(f_{cm}).\beta(t_0)$	$\varphi_0 = 3.259$
	-	$\beta(\infty, t_0) = 1$
Final creep coefficient at infinite time	$\varphi(\infty, t_0) = \varphi_0.\beta(\infty, t_0)$	$\varphi(\infty, t_0) = 3.159$
Coefficients depend on the cement type	-	$\alpha_{ds1} = 4$ $\alpha_{ds2} = 0.12$
Factor accounts for the effect of relative	$\beta_{RH} = 1.55[1 - (RH / RH_0)^3]$	$\beta_{RH} = 1.203$
Basic drying shrinkage	$\varepsilon_{cd,0} = 0.85[(220 + 110.\alpha_{ds1}).\exp(-\alpha_{ds2}.f_{cm} / f_{cm0})].10^{-6}.\beta_{RH}$	$\varepsilon_{cd,0} = 45.43x10^{-5}$
Coefficient to describe the development of creep with time after loading	$\beta(t, t_0) = [(t - t_0) / (\beta H + t - t_0)]^{0.3}$	$\beta(\infty, t_0) = 1$
Autogenous shrinkage strain at infinite time	$\varepsilon_{ca}(\infty) = 2.5 \cdot (fck - 10MPa) \cdot 10^{-6}$	$\varepsilon_{ca}(\infty) = 3.75 \times 10^{-5}$
Total shrinkage strain	$\varepsilon_{cs} = k_h.\varepsilon_{cd,0} + \varepsilon_{ca}$	$\varepsilon_{cs}(\infty, t_s) = 40.97 \times 10^{-5}$

Time, t (day)	$\beta_c(t, t_0)$	$\beta_{ds}(t, t_s)$	$\beta_{as}(t, t_s)$	$\varphi(t, t_0)$	$\varepsilon_{cs}(t, t_s)$
0	0.00000000	0	0.0000000	0.000000000	0.000000000
7	0.10598482	1.004628	0.4108947	0.334491743	0.000384684
200	0.672481995	0.998405	0.9408943	2.122376341	0.000402241
400	0.773581515	0.999334	0.9816844	2.441449907	0.000404116
600	0.825239784	0.999579	0.9925458	2.604485183	0.000404615
800	0.857271907	0.999692	0.9965065	2.705579667	0.000404806
1000	0.879228131	0.999757	0.9982082	2.774874268	0.000404894
1200	0.895264539	0.999800	0.9990202	2.825485723	0.000404940
1400	0.907509567	0.999830	0.9994376	2.864131453	0.000404967
1600	0.917173988	0.999852	0.9996645	2.894632699	0.000404984
1800	0.924999948	0.999869	0.9997935	2.919331697	0.000404995
2000	0.931468682	0.999882	0.9998695	2.939747245	0.000405003
2200	0.936906336	0.999893	0.9999157	2.956908668	0.000405008
2400	0.941541982	0.999902	0.9999444	2.971538927	0.000405013
2600	0.945541332	0.999910	0.9999628	2.984161014	0.000405016

2800	0.949027281	0.999917	0.9999747	2.995162789	0.000405019
3000	0.952092924	0.999922	0.9999825	3.004838063	0.000405022
3200	0.954810090	0.999927	0.9999878	3.013413532	0.000405024
3400	0.957235096	0.999932	0.9999914	3.021066935	0.000405026
3600	0.959412728	0.999935	0.9999939	3.027939615	0.000405027
3800	0.961379048	0.999939	0.9999956	3.034145392	0.000405028
4000	0.963163429	0.999942	0.9999968	3.039776958	0.000405030
4200	0.964790032	0.999945	0.9999977	3.044910573	0.000405031
4400	0.966278918	0.999947	0.9999983	3.049609549	0.000405032
4600	0.967646883	0.999950	0.9999987	3.053926892	0.000405033
4800	0.968908094	0.999952	0.9999990	3.057907320	0.000405033
5000	0.970074590	0.999954	0.9999993	3.061588820	0.000405034
5200	0.971156660	0.999955	0.9999995	3.065003871	0.000405035
5400	0.972163157	0.999957	0.9999996	3.068180409	0.000405035
5600	0.973101735	0.999959	0.9999997	3.071142593	0.000405036
5800	0.973979049	0.999960	0.9999998	3.073911426	0.000405037
6000	0.974800910	0.999961	0.9999998	3.076505250	0.000405037
6200	0.975572419	0.999963	0.9999999	3.078940157	0.000405037
6400	0.976298066	0.999964	0.9999999	3.081230326	0.000405038
6600	0.976981826	0.999965	0.9999999	3.083388295	0.000405038
6800	0.977627225	0.999966	0.9999999	3.085425195	0.000405039
7000	0.978237404	0.999967	0.9999999	3.087350941	0.000405039
7200	0.978815170	0.999968	1.0000000	3.089174393	0.000405039
7400	0.979363042	0.999969	1.0000000	3.090903493	0.000405040
7600	0.979883280	0.999970	1.0000000	3.092545383	0.000405040
7800	0.980377925	0.999970	1.0000000	3.094106501	0.000405040
8000	0.980848821	0.999971	1.0000000	3.095592666	0.000405041
8200	0.981297639	0.999972	1.0000000	3.097009150	0.000405041
8400	0.981725896	0.999973	1.0000000	3.098360743	0.000405041
8600	0.982134974	0.999973	1.0000000	3.099651807	0.000405041
8800	0.982526133	0.999974	1.0000000	3.100886318	0.000405042
9000	0.982900526	0.999974	1.0000000	3.102067915	0.000405042
9200	0.983259208	0.999975	1.0000000	3.103199928	0.000405042
9400	0.983603148	0.999975	1.0000000	3.104285416	0.000405042
9600	0.983933237	0.999976	1.0000000	3.105327189	0.000405042
9800	0.984250296	0.999977	1.0000000	3.106327836	0.000405043
10000	0.984555080	0.999977	1.0000000	3.107289746	0.000405043
10200	0.984848290	0.999977	1.0000000	3.108215127	0.000405043
10400	0.985130573	0.999978	1.0000000	3.109106021	0.000405043
10600	0.985402528	0.999978	1.0000000	3.109964320	0.000405043
10800	0.985664712	0.999979	1.0000000	3.110791782	0.000405043
11000	0.985917642	0.999979	1.0000000	3.111590039	0.000405044
11200	0.986161801	0.999979	1.0000000	3.112360611	0.000405044
11400	0.986397636	0.999980	1.0000000	3.113104915	0.000405044
11600	0.986625566	0.999980	1.0000000	3.113824270	0.000405044
11800	0.986845982	0.999981	1.0000000	3.114519911	0.000405044
12000	0.987059250	0.999981	1.0000000	3.115192992	0.000405044
12200	0.987265712	0.999981	1.0000000	3.115844593	0.000405044
12400	0.987465688	0.999981	1.0000000	3.116475725	0.000405044
12600	0.987659480	0.999982	1.0000000	3.117087339	0.000405045
12800	0.987847370	0.999982	1.0000000	3.117680327	0.000405045
13000	0.988029624	0.999982	1.0000000	3.118255526	0.000405045
13200	0.988206491	0.999983	1.0000000	3.118813725	0.000405045
13400	0.988378207	0.999983	1.0000000	3.119355667	0.000405045
13600	0.988544994	0.999983	1.0000000	3.119882053	0.000405045
13800	0.988707061	0.999983	1.0000000	3.120393542	0.000405045
14000	0.988864606	0.999984	1.0000000	3.120890758	0.000405045

14200	0.989017815	0.999984	1.0000000	3.121374291	0.000405045
14400	0.989166865	0.999984	1.0000000	3.121844697	0.000405045
14600	0.989311923	0.999984	1.0000000	3.122302504	0.000405046
14800	0.989453147	0.999984	1.0000000	3.122748212	0.000405046
15000	0.989590687	0.999985	1.0000000	3.123182294	0.000405046
15200	0.989724686	0.999985	1.0000000	3.123605199	0.000405046
15400	0.989855278	0.999985	1.0000000	3.124017354	0.000405046
15600	0.989982593	0.999985	1.0000000	3.124419162	0.000405046
15800	0.990106751	0.999985	1.0000000	3.124811010	0.000405046
16000	0.990227869	0.999986	1.0000000	3.125193262	0.000405046
16200	0.990346057	0.999986	1.0000000	3.125566268	0.000405046
16400	0.990461420	0.999986	1.0000000	3.125930358	0.000405046
16600	0.990574058	0.999986	1.0000000	3.126285848	0.000405046
16800	0.990684067	0.999986	1.0000000	3.126633040	0.000405046
17000	0.990791538	0.999986	1.0000000	3.126972221	0.000405046
17200	0.990896557	0.999987	1.0000000	3.127303665	0.000405046
17400	0.990999207	0.999987	1.0000000	3.127627633	0.000405046
17600	0.991099569	0.999987	1.0000000	3.127944377	0.000405047
17800	0.991197716	0.999987	1.0000000	3.128254134	0.000405047
18000	0.991293723	0.999987	1.0000000	3.128557134	0.000405047
18200	0.991387657	0.999987	1.0000000	3.128853595	0.000405047
18400	0.991479587	0.999988	1.0000000	3.129143727	0.000405047
18600	0.991569574	0.999988	1.0000000	3.129427730	0.000405047
18800	0.991657680	0.999988	1.0000000	3.129705796	0.000405047
19000	0.991743964	0.999988	1.0000000	3.129978110	0.000405047
19200	0.991828481	0.999988	1.0000000	3.130244848	0.000405047
19400	0.991911285	0.999988	1.0000000	3.130506181	0.000405047
19600	0.991992427	0.999988	1.0000000	3.130762269	0.000405047
19800	0.992071958	0.999988	1.0000000	3.131013271	0.000405047
20000	0.992149924	0.999989	1.0000000	3.131259335	0.000405047

Appendix-B: Time-variant material properties

Time (year)	$D(t)$ (mm)	$A(t)$ (mm^2)	$f_y(t)$ (N/mm^2)	$D_{ws}(t)$ (N/mm^2)	$A_{ws}(t)$ (mm^2)	$f_{yw}(t)$ (N/mm^2)	$f_{ck}(t)$ (N/mm2)	$E_{cm}(t)$ (GPa)
0	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
1	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
2	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
3	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
4	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
5	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
6	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
7	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
8	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
9	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
10	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
11	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
12	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
13	22.000	1520.531	460.000	8.000	100.531	250.000	25.000	31.000
14	22.000	1520.531	460.000	7.935	98.914	249.999	25.000	31.000
15	22.000	1520.531	460.000	7.734	93.946	249.978	25.000	31.000
16	22.000	1520.531	460.000	7.532	89.107	249.931	25.000	31.000
17	22.000	1520.531	460.000	7.330	84.395	249.860	25.000	31.000
18	22.000	1520.531	460.000	7.128	79.811	249.762	25.000	31.000
19	21.881	1504.114	459.998	6.926	75.355	249.64	20.307	29.587
20	21.679	1476.493	459.989	6.724	71.027	249.491	20.303	29.584
21	21.477	1449.127	459.971	6.523	66.827	249.318	20.300	29.584
22	21.275	1422.018	459.945	6.321	62.755	249.119	20.296	29.582
23	21.074	1395.165	459.910	6.119	58.811	248.894	20.292	29.582
24	20.872	1368.567	459.867	5.9179	54.995	248.644	20.288	29.578
25	20.670	1342.226	459.815	5.715	51.307	248.369	20.284	29.578
26	20.468	1316.140	459.755	5.513	47.747	248.068	20.28	29.576
27	20.266	1290.311	459.686	5.311	44.315	247.741	20.275	29.573
28	20.064	1264.737	459.608	5.110	41.011	247.389	20.271	29.571
29	19.863	1239.419	459.522	4.908	37.835	247.012	20.267	29.572
30	19.661	1214.358	459.428	4.706	34.787	246.609	20.263	29.572
31	19.459	1189.552	459.325	4.504	31.867	246.181	20.258	29.590
32	19.257	1165.003	459.213	4.302	29.075	245.727	20.254	29.588
33	19.055	1140.709	459.093	4.100	26.411	245.248	20.25	29.585
34	18.853	1116.671	458.965	3.899	23.875	244.743	20.245	29.583
35	18.651	1092.889	458.828	3.697	21.467	244.213	20.241	29.565
36	18.450	1069.364	458.682	3.495	19.187	243.658	20.236	29.564
37	18.248	1046.094	458.528	3.293	17.034	243.077	20.231	29.561
38	18.046	1023.080	458.365	3.091	15.010	242.470	20.227	29.559
39	17.844	1000.322	458.194	2.889	13.114	241.838	20.222	29.558
40	17.642	977.820	458.015	2.688	11.346	241.181	20.217	29.557
41	17.440	955.574	457.827	2.486	9.706	240.498	20.212	29.556
42	17.239	933.584	457.630	2.284	8.194	239.789	20.208	29.555

43	17.037	911.850	457.425	2.082	6.809	239.056	20.203	29.551
44	16.835	890.372	457.211	1.880	5.553	238.296	20.198	29.550
45	16.633	869.150	456.989	1.678	4.425	237.512	20.193	29.549
46	16.431	848.184	456.758	1.477	3.425	236.701	20.188	29.549
47	16.229	827.474	456.519	1.275	2.552	235.866	20.183	29.543
48	16.028	807.020	456.271	1.073	1.808	235.005	20.177	29.544
49	15.826	786.822	456.015	0.871	1.192	234.118	20.174	29.542
50	15.624	766.880	455.750	0.669	0.703	233.206	20.167	29.541

Appendix-C: Time-variant flexural and shear capacity of the RC beam

Time (year)	x (mm)	a (mm)	M_R (kNm)	$V_{Rd,s}$ (kN)	$V_{Rd,c}$ (kN)	V_{Rd} (kN)
0	174.861	139.889	218.990	42.289	80.422	122.711
1	174.861	139.889	218.990	42.289	80.422	122.711
2	174.861	139.889	218.990	42.289	80.422	122.711
3	174.861	139.889	218.990	42.289	80.422	122.711
4	174.861	139.889	218.990	42.289	80.422	122.711
5	174.861	139.889	218.990	42.289	80.422	122.711
6	174.861	139.889	218.990	42.289	80.422	122.711
7	174.861	139.889	218.990	42.289	80.422	122.711
8	174.861	139.889	218.990	42.289	80.422	122.711
9	174.861	139.889	218.990	42.289	80.422	122.711
10	174.861	139.889	218.990	42.289	80.422	122.711
11	174.861	139.889	218.990	42.289	80.422	122.711
12	174.861	139.889	218.990	42.289	80.422	122.711
13	174.861	139.889	218.990	42.289	80.422	122.711
14	174.861	139.889	218.990	41.608	80.422	122.0308
15	174.861	139.889	218.990	39.517	80.422	119.9395
16	174.861	139.889	218.990	37.478	80.422	117.9002
17	174.861	139.889	218.990	35.491	80.422	115.9133
18	174.861	139.889	218.990	33.557	80.422	113.9791
19	212.947	170.358	207.460	31.675	74.767	106.442
20	209.073	167.259	204.561	29.847	74.301	104.1485
21	205.221	164.177	201.655	28.073	73.836	101.9083
22	201.410	161.128	198.738	26.352	73.368	99.71911
23	197.63	158.104	195.814	24.684	72.898	97.58233
24	193.883	155.106	192.883	23.071	72.427	95.49814
25	190.167	152.134	189.947	21.512	71.955	93.46668
26	186.483	149.187	187.006	20.007	71.481	91.48807
27	182.841	146.273	184.060	18.557	71.004	89.56126
28	179.223	143.378	181.113	17.161	70.527	87.68869
29	175.637	140.509	178.165	15.820	70.049	85.86926
30	172.084	137.667	175.216	14.534	69.569	84.10303
31	168.573	134.858	172.266	13.303	69.086	82.38891
32	165.086	132.069	169.318	12.126	68.603	80.72921
33	161.633	129.307	166.373	11.004	68.119	79.12279
34	158.222	126.578	163.430	9.9374	67.631	77.56852
35	154.837	123.869	160.492	8.9255	67.143	76.06864
36	151.493	121.194	157.558	7.9685	66.652	74.62086
37	148.183	118.547	154.630	7.0663	66.160	73.22626
38	144.901	115.920	151.710	6.219	65.667	71.88583
39	141.659	113.328	148.796	5.4264	65.171	70.5973
40	138.453	110.762	145.892	4.6885	64.673	69.36167
41	135.281	108.225	142.997	4.0051	64.174	68.17881
42	132.137	105.710	140.113	3.3762	63.673	67.04962
43	129.035	103.228	137.240	2.8017	63.170	65.97182

44	125.968	100.774	134.379	2.2812	62.665	64.94631
45	122.936	98.3489	131.531	1.8148	62.158	63.97289
46	119.940	95.9518	128.697	1.4022	61.649	63.05134
47	116.979	93.5831	125.878	1.0433	61.138	62.18143
48	114.059	91.2474	123.074	0.7377	60.624	61.36192
49	111.159	88.9269	120.288	0.4854	60.111	60.59653
50	108.316	86.6528	117.517	0.2859	59.592	59.87801

Appendix-D: Additional flexure and shear on RC beam due to creep and shrinkage

Time (year)	$E_{c,ef}$ (GPa)	*m*	*Sc (mm3)*	*Ic (10^9mm)*	*(1/r)sc (/mm)*	*(1/r)su (/mm)*	(1/r)sn (/mm)	M_{shc} (kNm)	V_{shc} (kN)
0	31.5	6.354	400777	1.1E+9	0.00000	0.00000	0.00000	0.00	0.00
1	9.66	20.71	196933	3.2E+9	3.83E-7	3.39E-7	3.79E-7	11.52	7.68
2	8.84	22.62	176373	3.6E+9	3.91E-7	4.25E-7	3.94E-7	11.57	7.72
3	8.48	23.57	166418	3.7E+9	3.87E-7	4.66E-7	3.94E-7	11.59	7.73
4	8.28	24.15	160541	3.8E+9	3.82E-7	4.90E-7	3.92E-7	11.61	7.74
5	8.18	24.46	157306	3.9E+9	3.79E-7	5.03E-7	3.90E-7	11.62	7.74
6	8.07	24.78	154093	3.9E+9	3.75E-7	5.17E-7	3.88E-7	11.62	7.74
7	8.01	24.97	152174	4.0E+9	3.73E-7	5.25E-7	3.86E-7	11.64	7.76
8	7.95	25.16	150263	4.0E+9	3.70E-7	5.32E-7	3.84E-7	11.65	7.76
9	7.91	25.29	148993	4.0E+9	3.68E-7	5.37E-7	3.83E-7	11.66	7.77
10	7.89	25.35	148359	4.0E+9	3.69E-7	5.41E-7	3.84E-7	11.68	7.79
11	7.85	25.48	147094	4.1E+9	3.66E-7	5.45E-7	3.82E-7	11.74	7.83
12	7.83	25.54	146463	4.1E+9	3.65E-7	5.48E-7	3.81E-7	11.75	7.83
13	7.81	25.61	145832	4.1E+9	3.65E-7	5.50E-7	3.81E-7	11.80	7.86
14	7.79	25.67	145202	4.1E+9	3.64E-7	5.53E-7	3.80E-7	11.80	7.87
15	7.79	25.67	145202	4.1E+9	3.64E-7	5.53E-7	3.80E-7	11.87	7.91
16	7.77	25.73	144573	4.1E+9	3.63E-7	5.56E-7	3.80E-7	11.87	7.91
17	7.77	25.73	144573	4.1E+9	3.63E-7	5.57E-7	3.80E-7	11.89	7.93
18	7.75	25.8	143945	4.1E+9	3.62E-7	5.59E-7	3.79E-7	11.93	7.95
19	7.73	25.86	144503	4.1E+9	3.69E-7	5.61E-7	3.86E-7	11.98	7.98
20	7.73	25.86	146398	4.0E+9	3.82E-7	5.61E-7	3.97E-7	12.02	8.01
21	7.71	25.92	147565	3.9E+9	3.93E-7	5.63E-7	4.08E-7	12.05	8.04
22	7.71	25.92	149190	3.8E+9	4.07E-7	5.64E-7	4.20E-7	12.08	8.05
23	7.71	25.92	150676	3.8E+9	4.20E-7	5.64E-7	4.32E-7	12.10	8.07
24	7.70	25.99	151489	3.7E+9	4.31E-7	5.66E-7	4.43E-7	12.17	8.12
25	7.70	25.99	152718	3.6E+9	4.44E-7	5.66E-7	4.54E-7	12.18	8.12
26	7.70	25.99	153814	3.5E+9	4.57E-7	5.66E-7	4.67E-7	12.22	8.14
27	7.70	25.99	154780	3.5E+9	4.70E-7	5.67E-7	4.79E-7	12.24	8.16
28	7.68	26.05	155143	3.4E+9	4.82E-7	5.68E-7	4.89E-7	12.27	8.18
29	7.68	26.05	155869	3.3E+9	4.94E-7	5.68E-7	5.01E-7	12.31	8.21
30	7.68	26.05	156472	3.3E+9	5.07E-7	5.68E-7	5.12E-7	12.34	8.23
31	7.68	26.05	156953	3.2E+9	5.20E-7	5.68E-7	5.24E-7	12.35	8.24
32	7.68	26.05	157316	3.1E+9	5.32E-7	5.68E-7	5.35E-7	12.41	8.28
33	7.66	26.12	157156	3.1E+9	5.43E-7	5.70E-7	5.45E-7	12.44	8.30
34	7.66	26.12	157301	3.0E+9	5.57E-7	5.71E-7	5.58E-7	12.47	8.31
35	7.66	26.12	157334	2.9E+9	5.69E-7	5.71E-7	5.69E-7	12.53	8.35
36	7.66	26.12	157258	2.9E+9	5.81E-7	5.71E-7	5.80E-7	12.54	8.36
37	7.66	26.12	157074	2.8E+9	5.93E-7	5.71E-7	5.91E-7	12.57	8.38
38	7.66	26.12	156785	2.7E+9	6.05E-7	5.71E-7	6.02E-7	12.61	8.41
39	7.66	26.12	156394	2.7E+9	6.17E-7	5.71E-7	6.13E-7	12.63	8.42
40	7.66	26.12	155902	2.6E+9	6.29E-7	5.71E-7	6.24E-7	12.66	8.44
41	7.66	26.12	155313	2.6E+9	6.40E-7	5.71E-7	6.34E-7	12.67	8.45
42	7.64	26.18	154329	2.5E+9	6.50E-7	5.72E-7	6.44E-7	12.70	8.47

43	7.64	26.18	153562	2.5E+9	6.62E-7	5.72E-7	6.54E-7	12.71	8.48
44	7.64	26.18	152705	2.4E+9	6.73E-7	5.73E-7	6.65E-7	12.73	8.49
45	7.64	26.18	151759	2.4E+9	6.85E-7	5.74E-7	6.76E-7	12.73	8.49
46	7.64	26.18	150726	2.3E+9	6.96E-7	5.74E-7	6.86E-7	12.74	8.50
47	7.64	26.18	149611	2.3E+9	7.07E-7	5.74E-7	6.95E-7	12.75	8.50
48	7.64	26.18	148414	2.2E+9	7.17E-7	5.74E-7	7.05E-7	12.77	8.52
49	7.64	26.18	147138	2.2E+9	7.27E-7	5.74E-7	7.14E-7	12.77	8.51
50	7.64	26.18	145785	2.1E+9	7.38E-7	5.74E-7	7.23E-7	12.78	8.52

LIST OF PUBLICATIONS

1. Woju, Utino Worabo and Balu, A.S. (2020). "Time-dependent safety performance of reinforced concrete structures." *Journal of Engineering, Design and Technology,* Vol. 18 No. 5, https://doi.org/10.1108/JEDT-08-2019-0218.
2. Woju, Utino Worabo and Balu, A.S. (2020). "Fuzzy uncertainty and its applications in reinforced concrete structures." *Journal of Engineering, Design and Technology,* Vol. 18 No. 5, https://doi.org/10.1108/JEDT-10-2019-0281
3. Woju, Utino Worabo and Balu, A.S. (2020). "Time-dependent serviceability performance of the reinforced concrete structure." *ASCE- Journal of Structural Engineering (*Under review).
4. Woju, Utino Worabo and Balu, A.S. (2020). "Time-dependent failure possibility analysis of the reinforced concrete structure." *Reliability Engineering and System Safety* (Under review).

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Educational Background

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M.Sc.	2017	Structural Engineering (Department of Civil and Environmental Engineering)	Addis Ababa University (AAiT)
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Employment Condition: Lecturer at Wachemo University, Department of Civil engineering