A Comprehensive Analysis of Reactive Power Pricing in a Competitive Electricity Markets

Tukaram Moger
Department of Electrical and Electronics Engineering,
National Institute of Technology Karnataka Surathkal,
Mangalore -575 025, India.
E-mail: tukaram_moger@yahoo.co.in

Stanzin Dodjoo
Power Sector Northern Region,
Bharat Heavy Electricals Limited,
Leh, Ladakh, Jammu Kashmir, India.
E-mail: stanzindodjo@gmail.com

Abstract—Reactive power plays an important role in maintaining voltage stability, supporting real power transmission and overall system reliability. Thus, developing an accurate and feasible method for reactive power pricing is very significant in a deregulated electricity market. In this paper, the reactive power production cost of generator is represented as a nonlinear model that considers the corresponding economic loss if active power is not generated; that cost is the opportunity cost and the cost of reactive power production of the static capacitors are included in the objective function of total system operation cost. The methodology has been implemented using a modified optimal power flow is presented for calculation of the pricing of active and reactive power at each bus in competitive electricity markets. IEEE-14 bus system has been used for the computer study and the case studies done, shows the effects of various factors on the reactive power marginal price. Results demonstrate that the active and reactive power marginal prices give economical signals that could impel even more the participation of agents of competitive reactive power markets.

Keywords-- electricity market, optimal power flow, reactive power price, active power price, restructuring, open access

I. INTRODUCTION

Reactive power plays an important role in supporting the real power transfer. This support becomes especially important when an increasing number of transactions are utilizing the transmission system and voltages become a bottleneck in preventing additional power transfer. Establishing an appropriate price structure for reactive support is important both operationally and financially. Analyzing the costs involved is an indispensable part of determining the price. Thus planning and operation of the utilities are based on the economic principles of open-access markets. In this new environment, electric markets are essentially competitive. A fair and adequate method for allocating the costs may help the market participants make appropriate and efficient investments of reactive power sources, which can offer system operators more tools and can strengthen the system security. It is also realized that establishing an accurate pricing structure of reactive power can not only recover the costs of reactive power providers, but also provide economic information for real-time operations.

The real time pricing method of active power was established by Schweppe et al. [1]. They suggested spot pricing can help to improve production efficiency and yield maximum social benefits. In [2], real-time pricing of reactive power has been shown to perform better than the power factor penalty scheme in terms of providing incentives to all customers to reduce their consumption of reactive power irrespective of their power factor which is extended from the active power marginal pricing structure [1]. A comprehensive study of spot pricing and its implementation are reported in [3], [4]. In [5], it is pointed out that reactive power price should recover not only the operational cost, but also capital investments of capacitors. However, the reactive power production cost of generators is neglected. In [2], a detailed discussion on reactive power services is made and it is shown that the capital costs should be included in reactive power price. In [6], investigation is conducted on reactive power pricing by using the objective function of maximizing social benefit instead of minimizing the production cost. In [7], paper introduces opportunity cost as a reactive power production cost of generator but the computation of the cost is difficult.

In this paper, both active and reactive power production costs of generators and capital cost of capacitors are considered in the objective function of optimal power flow (OPF) problem. A program has been developed for the solution of the modified optimal power flow (MOPF) problem using constrained non-linear optimization function in MATLAB's optimization toolbox [8]. The study has been carried out on IEEE-14 bus system [9] and observed the impact of the change of objective function and system operating condition i.e. load power factor on both active and reactive power marginal prices have been studied in details to observe how these conditions influence both active and reactive power prices.

In section II, mathematical model of reactive power pricing is presented. Section III, presents detailed analysis and discussion on the results of various case studies, and conclusions are drawn in section IV.

II. AN OPF MODEL FOR REACTIVE POWER PRICING

Active and reactive power marginal prices are normally obtained through solving the OPF in which an objective

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function subject to a set of equality and inequality constraints is minimized.

A. Objective function

The objective is to minimize the overall production cost of active and reactive power which includes active power production cost of generators, reactive power production cost of generators and capital cost of capacitors. Therefore, the OPF objective function can be expressed as;

$$\min(F) = \sum_{i \in NG} (C_{pgi}(P_{gi}) + C_{qgi}(Q_{gi})) + \sum_{i \in C} C_{qci}(Q_{ci}) \quad (1)$$

where NG: number of generators; C: number of capacitors; $C_{pgi}(.)$, $C_{qgi}(.)$, $C_{qci}(.)$ are the cost of generator active power, generator reactive power also known as opportunity cost, cost of capacitor reactive power at i^{th} bus; P_{gi}, Q_{gi}, Q_{ci} are the generator active power production, generator reactive power production, capacitor reactive power output at i^{th} bus;

The cost of active power generation, which is the first item in (1), is approximated as a quadratic function where a, b and c are predetermined coefficients;

$$C_{pgi}(P_{gi}) = a + b\,P_{gi} + c\,P_{gi}^2 \quad (2)$$

The reactive power production cost of generator is called opportunity cost [7]. According to the loading capability diagram of a generator, reactive power output may reduce active power output capacity of generators which can at least serve as spinning reserve, therefore causes implicit financial loss to generators. Actually, opportunity cost depends on the real-time balance between demand and supply in the market, so it is difficult to determine the real value. For simplicity, we consider the opportunity cost approximately as;

$$C_{qgi}(Q_{gi}) = [C_{pgi}(S_{gi,\max}) - C_{pgi}(\sqrt{S_{gi,\max}^2 - Q_{gi}^2})] \cdot k \quad (3)$$

where $S_{gi,max}$: maximum apparent power of generator at i^{th} bus; k is the profit rate of active power generation, usually between 5% and 10%.

The third term of (1) is the equivalent investment cost of capacitors, which is expressed as their depreciation rate (the life-span of capacitor is assumed to be 30 years):

$$C_{qci}(Qci) = (Q_{ci} \times 11600) \div (30 \times 365 \times 24 \times h) \quad \$/MVArh$$

$$C_{qci}(Q_{ci}) = Q_{ci} \times 0.0598 \quad \$/MVarh \quad (4)$$

where h: the average usage rate of capacitor, set here as ¾ [10].

B. System operating constraints: The constraints are as follows:

1) *Linear constraints:*

The node voltage constraints:

$$V_{i,\min} \le V_i \le V_{i,\max} \quad (5)$$

Active and reactive power generation constraints:

$$P_{gi,\min} \le P_{gi} \le P_{gi,\max} \quad (6)$$

$$Q_{gi,\min} \le Q_{gi} \le Q_{gi,\max} \quad (7)$$

Capacitor output constraint:

$$Q_{ci,\min} \le Q_{ci} \le Q_{ci,\max} \quad (8)$$

2) *Nonlinear equality constraints: The constraints are as follows:*

Power flow equations:

$$P_{gi} - P_{Li} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \cos\delta_{ji} - B_{ij} \sin\delta_{ji}) = 0 \quad (9)$$

$$Q_{gi} - Q_{Li} + V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin\delta_{ji} + B_{ij} \cos\delta_{ji}) = 0 \quad (10)$$

$$Q_{gi} - Q_{Li} + Q_{ci} + V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin\delta_{ji} + B_{ij} \cos\delta_{ji}) = 0 \quad (11)$$

where NB: number of buses; P_{Li}, Q_{Li} are the real and reactive power demand at i^{th} bus; V_i: Voltage magnitude at i^{th} bus; G_{ij}, B_{ij}: Element of network admittance matrix [Y_{BUS}];

3) *Nonlinear inequality constraints: The constraints are as follows:*

Line power flow limits:

$$S_f(\delta, V) - S_{\max} \le 0 \quad (12)$$

$$S_t(\delta, V) - S_{\max} \le 0 \quad (13)$$

where S_f: apparent power in 'from bus'; S_t: apparent power in 'to bus'.

Because of the opportunity cost is present in the objective function, the following constraint is also considered. It is the limit for total apparent power generation which cannot be more than the generator capacity.

$$\sqrt{P_{gi}^2 + Q_{gi}^2} \le S_{gi,\max} \quad (14)$$

Based on the above mathematical model, the corresponding Lagrangian function of this optimization problem takes the form as;

$$L = \sum_{i \in NG} (C_{pgi}(P_{gi}) + C_{qgi}(Q_{gi})) + \sum_{i \in C} C_{qci}(Q_{ci})$$

$$- \sum_{i=1}^{NB} \lambda_{pi} [P_{gi} - P_{Li} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \cos\delta_{ji} - B_{ij} \sin\delta_{ji})]$$

$$- \sum_{i=1}^{NG} \lambda_{qgi} [Q_{gi} - Q_{Li} + V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin\delta_{ji} + B_{ij} \cos\delta_{ji})]$$

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$$- \sum_{i=1}^{NL} \lambda_{qli} [Q_{gi} - Q_{Li} + Q_{ci} + V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin \delta_{ji} + B_{ij} \cos \delta_{ji})]$$

$$+ \sum_{i=1}^{NG} \mu_{pgi,\min} (P_{gi,\min} - P_{gi}) + \sum_{i=1}^{NG} \mu_{pgi,\max} (P_{gi} - P_{gi,\max})$$

$$+ \sum_{i=1}^{NG} \mu_{qgi,\min} (Q_{gi,\min} - Q_{gi}) + \sum_{i=1}^{NG} \mu_{qgi,\max} (Q_{gi} - Q_{gi,\max})$$

$$+ \sum_{i=1}^{NG} \mu_{qci,\min} (Q_{ci,\min} - Q_{ci}) + \sum_{i=1}^{NG} \mu_{qci,\max} (Q_{ci} - Q_{ci,\max})$$

$$+ \sum_{i=1}^{Nl} \tau_{sf} (s_f - S_{f,\max}) + \sum_{i=1}^{Nl} \tau_{st} (s_t - S_{t,\max})$$

$$+ \sum_{i=1}^{NB} \upsilon_{i,\min} (V_{i,\min} - V_i) + \sum_{i=1}^{NB} \upsilon_{i,\max} (V_i - V_{i,\max}) \quad (15)$$

where L: Lagragian function; λ_{pi}: Lagrange multiplier on the active power equation at i^{th} bus; λ_{qgi}: Lagrange multiplier on the reactive power equation at i^{th} generator bus; λ_{qli}: Lagrange multiplier on the reactive power equation at i^{th} load bus; $\mu_{pgi,min}$: Lagrange multiplier on the min. active power generation limit at i^{th} bus; $\mu_{pgi,max}$: Lagrange multiplier on the max. active power generation limit at i^{th} bus; $\mu_{qgi,min}$: Lagrange multiplier on the min. reactive power generation limit at i^{th} bus; $\mu_{qgi,max}$: Lagrange multiplier on the max. reactive power generation limit at i^{th} bus; τ_{sf}: Lagrange multiplier on the apparent power flow limit in 'from bus' to in 'to bus'; τ_{st}: Lagrange multiplier on the apparent power flow limit in 'to bus' to in 'from bus'; $\nu_{i,min}$: Lagrange multiplier on the min. voltage level at i^{th} bus; $\nu_{i,max}$: Lagrange multiplier on the max. voltage level at i^{th} bus; $\mu_{qci,min}$: Lagrange multiplier on the min. reactive power generation limit from capacitor bank at i^{th} bus; $\mu_{qci,max}$: Lagrange multiplier on the max reactive power generation limit from capacitor bank at i^{th} bus;

The OPF solution is obtained by applying Kuhn-Tucker conditions for the minimization problem. According to microeconomics, the marginal prices for active and reactive powers at i^{th} bus are the marginal costs associated with the corresponding load flow equations when the OPF is solved as a non-linear programming problem. At a particular time, real price of real power and that of reactive power, at a bus-i are given by;

$$\rho_i{}^p = \frac{\partial L}{\partial P_i} = \lambda_{pi}, \qquad \rho_i{}^q = \frac{\partial L}{\partial Q_i} = \lambda_{qi}$$

where P_i, Q_i: Net injected real and reactive powers at i^{th} bus: λ_{pi}, λ_{qi}: Marginal costs are equal to the Lagragian multipliers of the corresponding power flow equations at the optimal solution point.

III. Simulation Analysis and Discussions

The IEEE-14 bus system has been used for the simulation. There are three capacitors installed on buses 5, 13 and 14. This result obtained from the separate algorithm for optimal location of capacitor units in a system which is not the scope of this paper.

In order to study the impact of reactive power cost factor and investment cost of capacitor bank on the marginal price of reactive power, six cases are studied:

Case I: The objective function is minimization of generator active power production cost. It includes only the first item of (1).

$$\min (F) = \sum_{i \in NG} C_{pgi}(P_{gi}) \quad (16)$$

Case II: The objective function is minimization of generator active power production cost. It includes only the first item of (1). In this case there are capacitors installed on specific load buses but their cost is not considered.

$$\min (F) = \sum_{i \in NG} C_{pgi}(P_{gi}) \quad (17)$$

Case III: The objective function is minimization of generator active power production cost and the overall capacitor cost. It includes the first and third item of (1). In this case capacitors are installed on specific load buses.

$$\min (F) = \sum_{i \in NG} C_{pgi}(P_{gi}) + \sum_{i \in C} C_{qci}(Q_{ci}) \quad (18)$$

Case IV: The objective function is minimization of generator active power production cost and the opportunity cost of generator real power production. It includes the first and second item of (1). In this case capacitors are not installed.

$$\min (F) = \sum_{i \in NG} (C_{pgi}(P_{gi}) + C_{qgi}(Q_{gi})) \quad (19)$$

Case V: The objective function is minimization of generator active power production cost and the opportunity cost of generator real power production. It includes the first and second item of (1). In this case capacitors are installed but cost is not considered.

$$\min (F) = \sum_{i \in NG} (C_{pgi}(P_{gi}) + C_{qgi}(Q_{gi})) \quad (20)$$

Case VI: The objective function is minimization of generator active power production cost, the opportunity cost of generator real power production and the overall capacitor cost.

$$\min (F) = \sum_{i \in NG} (C_{pgi}(P_{gi}) + C_{qgi}(Q_{gi})) + \sum_{i \in C} C_{qci}(Q_{ci}) \quad (21)$$

The simulation results for IEEE-14 bus system obtained from MOPF model for case I to VI are described in Table 1 and Fig. 1 to 9.

A. *Impact of Objective Function Variation*

The system result shown in Table 1 is for IEEE-14 bus system under normal operating condition. From Table 1 and Fig. 1 to 4, the following facts are observed;

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- Under normal circumstances, the active power marginal price (APMP) for all the different cases almost remains constant, the fluctuation in APMP at various buses have only small changes irrespective of the variations in the objective function (Fig. 1).
- The APMP is almost of the same order but the reactive power marginal price (RPMP) fluctuates significantly from bus to bus. Fig. 2 & 3 show the RPMP at different buses. As compared to the APMP the RPMP has more fluctuations. This shows that reactive power pricing is a more complicated process than for active power as it is required for both transmission of power in the lines as well as at the loads.
- The RPMP is very high in case IV (Fig. 3). It is because in case IV the opportunity cost is considered along with active power cost but there are no capacitors to support reactive power requirement. The opportunity cost of reactive power is much higher compared to the cost of reactive power support through capacitors.
- In case II, the capacitors are installed on bus no. 5, 13 and 14 but the cost are not considered because of which zero RPMP on these buses which can be interpreted as availability of free source. And when include the capacitor cost, the RPMP at these buses shoots up as in case III.

TABLE I. IEEE 14 BUS SYSTEM UNDER NORMAL OPERATING CONDITION FOR CASE I TO VI

	Bus No.	Case I	Case II	Case III	Case IV	Case V	Case VI
Generator output (MW, MVAR)	1 2 3 6 8	194.33, 0 36.72, 23.69 28.74, 24.13 0, 11.55 8.5, 8.27	194.45, 0 36.74, 7.81 28.88, 21.8 0, -3.3 8.09, 5.6	194.42, 0 36.73, 12.36 28.83, 22.39 0, -0.03 8.19, 6.02	191.86, 10 36.49, 2.49 29.98, 34.3 0, 11.29 9.8, 8.29	194.06, 0 36.7, 0.44 29.54, 14.14 0, -3.57 7.89, 3.17	192.94, 0.6 36.69, 0.54 29.54, 5.11 0, -2.69 8, 4.61
Capacitor output (MVAR)	5 13 14		22.7247 6.7503 5.7883	15.5535 5.3254 5.5186		42.6188 4.5206 6.1863	39.4794 3.6991 5.8857
Objective function value ($/hr)		8081.53	8076.4	8078.21	8106.42	8081.88	8084.96
Total active power cost ($/hr)		8081.64	8076.02	8076.53	8076.53	8078.92	8078.92
Total reactive cost of gens.($/hr)					28.33	3.31	3.31
Total cost of capacitors ($/hr)				1.55			2.89
Power loss (MW, MVAR)		9.287, 39.16	9.17, 39.03	9.173, 38.95	9.128, 38.51	9.181, 39.19	9.163, 39.1

B. *Impact of Load Power Factor Variation*

The power factor (pf) studies have been done for case VI, which has real power cost, capacitor cost and opportunity cost in its objective function. In this case, the power factor of the load is varied from 0.7 to 0.95 i.e. from high reactive power loading to near to unity power factor (upf) loading and its impact on RPMPs, voltage profiles and reactive power output of generators and capacitors have been studied and results are described in Fig. 5 to 9.

From the figures of case VI (load pf), the following facts are observed;

- When load power factor reduces from 0.95 to 0.7, the RPMP increases greatly while average price increases very slow. Therefore, RPMP can provide clear economic information to loads to improve their power factors (Fig. 5).
- When bus-3 reaches its minimum voltage of 0.95pu at lower power factor, the corresponding RPMP of bus-3 increase drastically this can act as an index of the urgency of the reactive power supply and voltage support on bus-3 (Fig. 5 & 6).
- As load power factor increase from 0.7 to 0.95, the reactive power output of the generators and capacitors start deceasing and hence objective function value also decreases. When the power factor is close to unity power factor, reactive power outputs of the (some) generators become negative. This means that the system has surplus reactive power. The corresponding RPMP is very small (Fig. 7, 8 & 9).

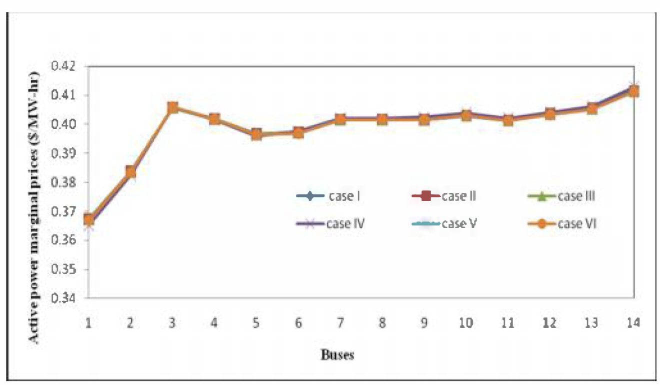

Figure 1. Active power marginal prices for case I to VI

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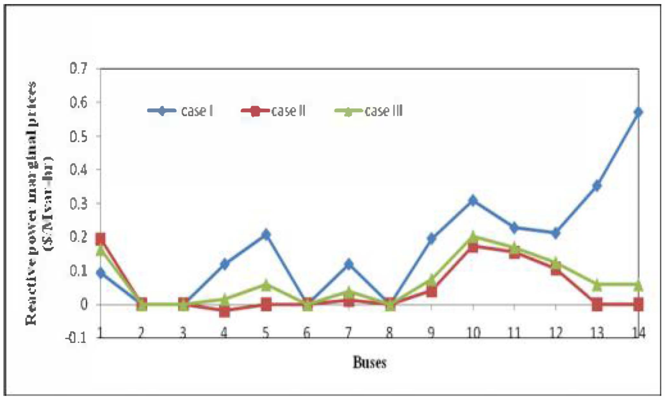

Figure 2. Reactive power marginal prices for case I to III

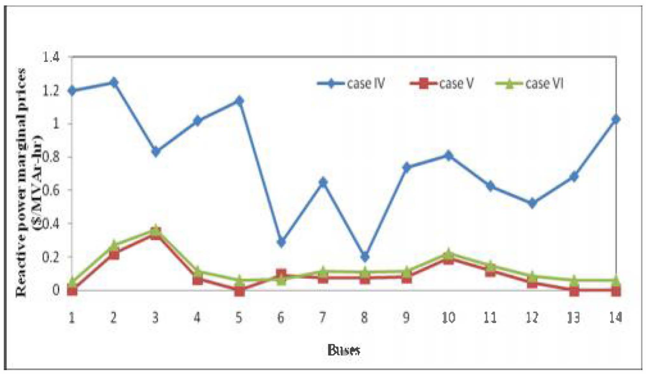

Figure 3. Reactive power marginal prices for case IV to VI

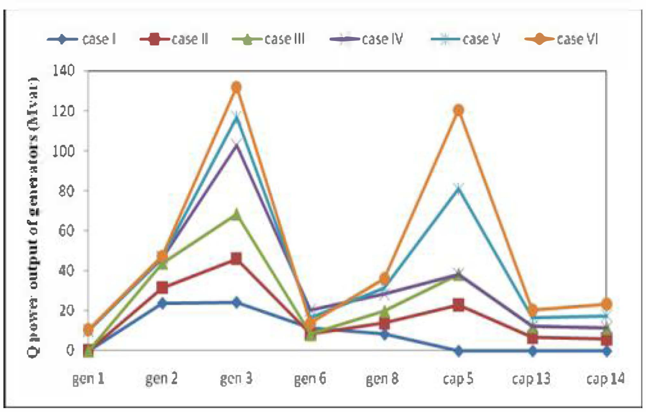

Figure 4. Reactive power output of generators and capacitors for case I to VI

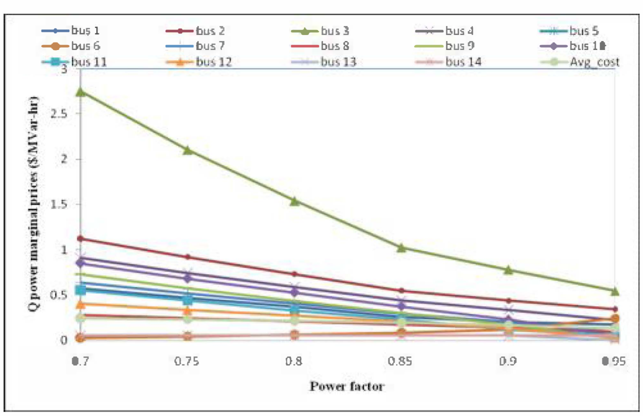

Figure 5. Reactive power marginal prices-load pf

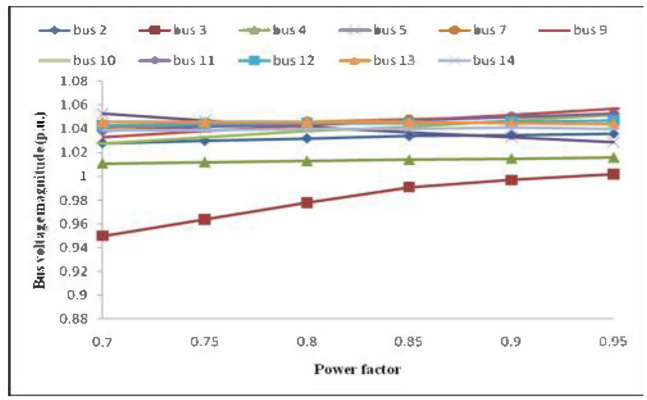

Figure 6. Voltage magnitude profiles-load pf (few buses)

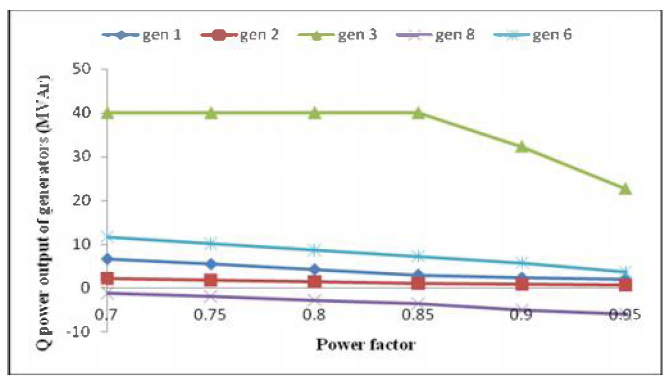

Figure 7. Reactive power output of generators-load pf

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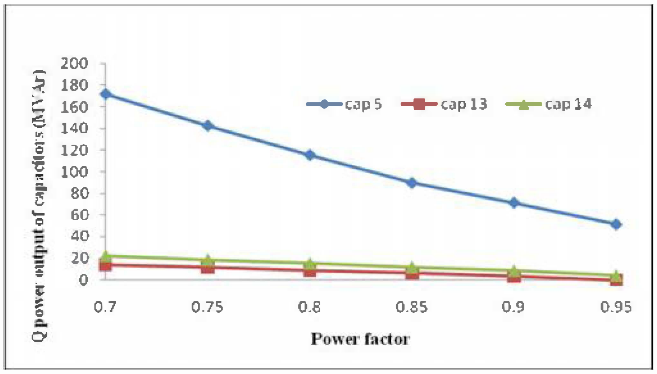

Figure 8. Reactive power output of capacitors-load pf

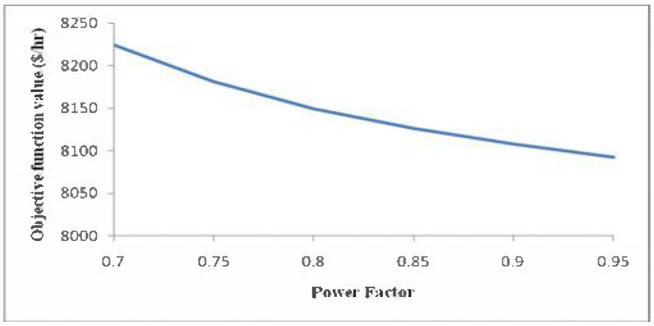

Figure 9. Objective function value-load pf

IV. CONCLUSIONS

A model for calculation of the active and reactive power marginal prices has been presented. This model used a methodology based on a modified optimal power flow in which the objective function minimizes the active and reactive power production cost. The impact of the change of objective function and system operating condition i.e. load power factor on both active and reactive power marginal prices have been studied. Based on the results obtained on the IEEE-14 bus system, the following main conclusions can be drawn;

- Based on various case studies, the active power marginal price can be studied independently without considering reactive power production cost.
- The capital investment of capacitors should be considered in reactive power pricing because of their noticeable impacts on reactive power marginal prices.
- Load power factor has significant impacts on reactive power marginal prices especially when some system operation limits are reached.
- Reactive power marginal price can serve as a system index related to the urgency of reactive power supply and system voltage support and an incentive to improve load power factor and reduce reactive power demand.
- The revenue based on reactive power marginal price is much higher than that based on reactive power average price. Therefore, some adjustment should be made in using reactive power marginal price.

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BIOGRAPHIES

Tukaram Moger (M, 10) received the B.E. degree in electrical & electronics engineering form Karnatak University, Dharwad, Karnataka, India, in 2001 and M.Tech. degree in electrical engineering from IIT Kanpur, in 2005. Since then, he has been associated with reputed institutions as a faculty member and currently, he is working as Assistant Professor in the Department of Electrical and Electronics Engineering, National Institute of Technology Karnataka (NITK) Surathkal, Mangalore, India. His research interests include computer aided power system analysis, reactive power optimization, voltage stability, power system operation and planning, power system restructuring/deregulation, and power electronics applications in transmission. He is a member of IEEE (USA), life member of Indian Society for Technical Education (ISTE) and System Society of India (SSI).

Stanzin Dodjoo received B.Tech. degree in electrical & electronics engineering from National Institute of Technology Karnataka (NITK) Surathkal, Mangalore, India. Currently, he is serving as a Enginer in the Department of Power Sector, Bharat Heavy Electricals Limited, Leh, Ladakh, Jammu Kashmir, India. His research interests include computer aided power system analysis, reactive power optimization, voltage stability, and hydro generator designs.

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