Groundwater Modeling to Simulate Groundwater Levels Due to Interlinking of Rivers in Varada River Basin, India

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ABSTARCT

Using a two-dimensional Gelarkin finite element model, the present study characterizes groundwater flow in a Varada basin, a semi humid area of Karnataka, India. The model characterization involved taking field data by conducting pump tests describing an aquifer system. Geological geometry and the number of aquifers have been analyzed based on a large amount of geological, hydrogeological and topographical data. The aquifer properties are then transformed into input variables that the model code uses to solve governing equations of flow. The results of the field experiments showed that Varada basin is predominantly confined aquifer. For calibrating the numerical groundwater model, the groundwater flow was simulated in steady state. Results of study demonstrate a moderately high correlation between the observed and simulated groundwater level. In addition, the groundwater level and trend in the transient state has also been elucidated. The validated numerical groundwater model was used to predict the groundwater levels due to impact of recharge from the interlinking of rivers in the basin. The model result provides an excellent visual representation of groundwater levels, presenting resource managers and decision makers with a clear understanding of the nature of the interaction of groundwater levels with the proposed interlinking project. Results build a base for further analysis under different future scenarios for implementation.

Key Words: Finite element model, groundwater flow, Aquifer characterization, interlinking of rivers

1. INTRODUCTION

In the sub-humid plain of Varada basin, Karnataka, India, owing to the absence of reliable water resources, groundwater is the primary source of water supply for domestic, municipal, industrial and agricultural uses. In recent decades, increasing concerns over agricultural water use, surface water reliability and groundwater storage changes have increased demand for sustainable groundwater development and management. Studies across globe have shown that the groundwater level fluctuation and long term trends depend on the groundwater recharge, which is a function of precipitation, evapotranspiration, and pumped water (10; 6]. Finite element groundwater model is the most efficient and powerful numerical method to analyze groundwater flow [5;2;1]. This most difficult part of the groundwater modeling process requires large amount of geological, hydrogeological and topographical data, which are usually expensive and hard to get [6;9]. Development of groundwater model requires aquifer layers, geological maps, and boreholes information[7]. In this study, the hydrogeological database was built and a two-dimensional finite element model was developed to simulate the groundwater flow in both steady and transient state [4; 11]. The present study aimed to simulate the groundwater levels in Varada basin if 20% of the water which is being transported through Varada river is utilized for irrigation. The calibrated model was used to predict scenarios of groundwater levels due to the impact of recharge from interlinking of rivers in the basin for sustainable development.

2. MATERIALS AND METHODS

2.1 STUDY AREA DESCRIPTION

The site chosen for this study is located in the central part of Karnataka state, India shown in figure 1. The Varada river basin lies between latitude 14 –15° 15' N and longitude 74° 45' to 75° 45' E in the Karnataka state, India The Varada basin has a sub-humid and semi-arid climate, with an average temperature of 26.8^{o}. The normal rainfall over the basin varies from 2,070 mm in the western ghats to 775 mm in the plateau region spread between June to October. There is no rain during the rest of year and occasionally the basin experiences south eastern rainfall for very short period of time in December. The Varada basin has been continuously monitored for rainfall, stream flow, groundwater level, groundwater temperature and electrical conductivity, etc., for about three decades. Such continuously monitored datasets can be helpful in the detailed conceptualization of the hydro-geological setup of the basin.

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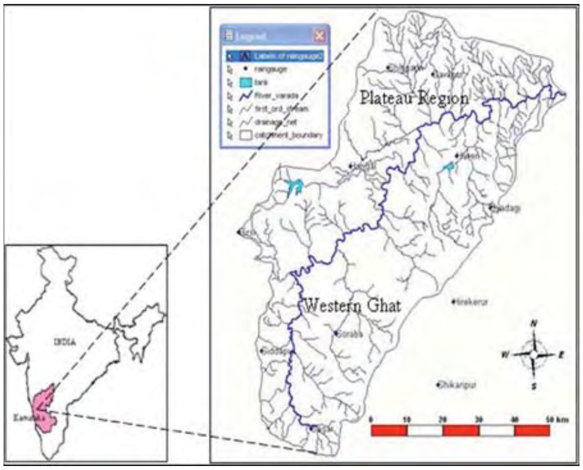

Figure 1. Study Area [8]

There is an interlinking of peninsular India river proposal from government of India to transfer surplus water from Bethi river to water deficit areas such as Raichur through Varada river. The water which is being transported to Raichur district is not to be utilized in Varada river basin even though it is water scarce. One of the interlinks shown in figure 2 called Bedthi- Varada river link was considered. Both rivers are originating from western ghats, but Bedthi rivers flows towards west and joins Arabian sea and Varada river flows towards North east and joins Thungabhadra, Krishna and then Bay of Bengal sea.

Figure 2. Bedthi- Varada river link (Source: NWDA, 1994)

2.2 GOVERNING EQUATIONS

The groundwater flow modeling methodology given by American Society for Testing Materials (ASTM) was used. The following governing equations were used

For steady state condition:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) \pm G(x, y)=0 \qquad (1)$$

For transient condition:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right)=S \frac{\partial h}{\partial t} \pm G(x, y, t) \qquad (2)$$

where T_x and T_y are the x and y – direction transmissivities respectively (m^2/day)

h- Piezometric head (m), S – Storage coefficient (dimensionless).

G(x,y,t) – Pumping/Recharge intensity expressed as a depth (m^3/day), t – Time (days).

x & y – Coordinate axes.

2.3 INITIAL CONDITION

The initial water level is provided as initial condition.

$$h(x_i, y_i, 0)=h\ (x_i, y_i) \text{ in } \Omega \qquad (3)$$

Where, $h(x_i,\ y_i)$ is spatially varying piezometric heads.

2.4 BOUNDARY CONDITION

$$h=\bar{h}(x, y, z, t) \quad \text{on} \qquad (4)$$

and

$$k_x \frac{\partial h}{\partial x} l_x+k_y \frac{\partial h}{\partial y} l_y+k_z \frac{\partial h}{\partial z} l_z=q(x, y, z, t)$$

on A2 (5)

where h - ….and l_x, l_y, l_z are the direction cosines between the normal to the boundary surface and the coordinate axes; A1 represents those part of the surface where *h* is known and is therefore specified. For the remaining part of boundary referred to as A2; q is prescribed flow rate per unit area across of the boundary. For the general case of transient flow with phreatic surface moving with a velocity V_n normal to its instantaneous configuration, the quantity of flow entering its unit area is given by

$$q=V_n S+I * l_x \qquad (6)$$

where S is the specific yield coefficient relating the total volume of material to the quantity of fluid which can be drained. I is the infiltration or evaporation.

3. DISCRETIZATION OF STUDY AREA

The general principle is the that where accurate results are required and relatively large changes of the water table occur or expected, the mesh size should be finer, where no such accuracy is required and only minor water table changes occur or expected, the mesh size can be coarse (Thomas, 1973). In this study, the groundwater basin was discretised into 329 two dimensional triangular elements with 196 nodes (Fig 3) having finer discretization in the western ghat region and coarser mesh in the plateau region. Nodes were assigned for observation wells to calibrate and compare the results with the water level recorded.

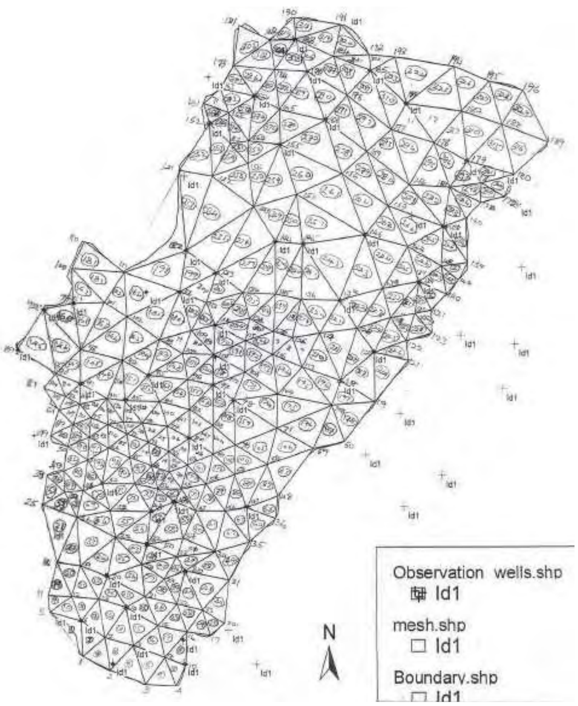

Fig 3. Finite element mesh of Varada river basin

3.1 FINITE ELEMENT FORMULATION

The finite element solution of equations (1 & 2) with boundary conditions (3 & 4) is derived using Gelarkin's weighted residuals method.

The variable *h* is approximated as

$$h = \sum_{i=1}^{n} N_i h_i \qquad (7)$$

over the domain; where N_i are the interpolation functions; h_i are the nodal values of *h*;

n is the number nodes

The application of Galarkin method to the steady state equation yields following integral equation:

$$\int_D R N_i \, d\Omega = 0; \qquad i = 1, 2, \ldots n \quad (8)$$

in which

$$R = \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial}{\partial z} \right) \right] \sum_{1}^{n} N_i h_i \qquad (9)$$

where Ω refers to the volume of flow domain

By applying Green's theorem, equation (8) can be modified to

$$\int_D \left(\frac{\partial N_m}{\partial x} \sum_1^n k_x \frac{\partial N_i}{\partial x} + \frac{\partial N_m}{\partial y} \sum_1^n k_y \frac{\partial N_i}{\partial y} + \frac{\partial N_m}{\partial z} \sum_1^n k_z \frac{\partial N_i}{\partial z} \right) h_i \, d\Omega$$

$$- \int_A N_m \left(\sum_1^n k_x \frac{\partial N_i}{\partial x} l_x + \sum_1^n k_y \frac{\partial N_i}{\partial y} l_y + \sum_1^n k_z \frac{\partial N_i}{\partial z} l_z \right) h_i \, dA = 0 \qquad (10)$$

where A refers to external surface area. Equation (4.43) leads to a system of simultaneous equations which can be expressed as

$$[P]\{h\} = \{F\} \qquad (11)$$

where [P] – conductivity matrix

{h}- vector of nodal values

{F} – Load vector

$$P_m = \sum \int_E \left(\frac{\partial N_m}{\partial x} k_x \frac{\partial N_i}{\partial x} + \frac{\partial N_m}{\partial y} k_y \frac{\partial N_i}{\partial y} + \frac{\partial N_m}{\partial z} k_z \frac{\partial N_i}{\partial z} \right) d\Omega \qquad (12)$$

and

$$F_m = \sum \int_{SE} N_m q \, dA \qquad (13)$$

where E denotes an element; SE refers to elements with an external surface. The element equations are assembled into global system of equation. The prescribed boundary conditions are inserted at this stage and the solution is obtained using Gauss elimination routine.

3.2 DEVELOPMENT OF TRANSIENT MODEL

Rewriting the equation (2) describing linearised unsteady groundwater flow

$$T \left\{ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right\} - S \frac{\partial h}{\partial t} = \pm G(x, y, t) \qquad (14)$$

To solve this equation, homogeneous and isotropic domain with boundary Γ in the time interval $(0, t_n)$ is assumed. Both an essential and natural boundary conditions are imposed on the boundary.

$$h(x, y, t) = h_0(x, y, t) \text{ on } \Gamma_1 \qquad (15)$$

$$T \left(\frac{\partial h}{\partial x} n_x + \frac{\partial h}{\partial y} n_y \right) + q_0 = 0 \text{ on } \Gamma_2 \qquad (16)$$

where n_x and n_y are directional cosines of the outward normal to Γ.

The following initial condition must be imposed on the domain Ω

$$h(x,y,0)=H(x,y) \text{ in } \Omega \qquad (17)$$

Applying the Galerkin method to equation (14)

$$\int_G N_i\left\{T\left[\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right]-S\frac{\partial h}{\partial t}\right\}d\Omega=0$$

i=1,2,3….n_u (18)

Where n_u is number of nodes in the finite element mesh and N_i are the applied local shape functions. Now applying Green's theorem yields

$$\int_G T\left\{\frac{\partial N_i}{\partial x}\frac{\partial h}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial h}{\partial y}\right\}d\Omega+\int_{\Gamma_2} N_i q_0 d\Gamma+\int_G N_i S\frac{\partial h}{\partial t}d\Omega=0 \qquad (19)$$

The resulting system can be conveniently written in matrix form:

$$[P]\{h\}+[L]\left\{\frac{\partial h}{\partial t}\right\}=-\{F\} \qquad (20)$$

Where [P]- conductivity matrix; [L]- storativity matrix.

The elements of the matrices are given as

$$p_{ij}=\int_{G^e} T\left\{\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right\}dG \qquad (21)$$

$$l_{ij}=\int_{G^e} SN_iN_j dG \qquad (23)$$

l_{ij}= (1/6) A.S (for i=j); =(1/12) A.S (i≠j)
where A is the area of triangle

Considering forward difference scheme for the time derivative term in the equation (20)

$$\frac{\partial h}{\partial t}=\frac{h_{t+1}-h_t}{\Delta t} \qquad (24)$$

Also, considering the system of equations with a time stepping scheme is introduced with a factor θ. The solution accuracy and the numerical stability depends the choice of values of θ, is of decisive significance. Most frequently 1, ½, or 0 are substituted for θ. Equation (20) thus obtains the form:

Case (i): θ =1, Backward scheme;

$$(L+P_{t+1}\Delta t)h_{t+1}=Lh_t-F_{t+1}\Delta t \qquad (25)$$

Case (ii): θ =1/2, central (Crank-Nicolson) scheme

$$\left(L+\frac{1}{2}P_{t+1}\Delta t\right)h_{t+1}=\left(L-\frac{1}{2}P_t\Delta t\right)h_t-\frac{1}{2}(F_t+F_{t+1})\Delta t \qquad (26)$$

Case (iii): θ =0, forward scheme;

$$Lh_{t+1}=(L-P_t\Delta t)h_t-F_t\Delta t \qquad (27)$$

In the present study, an implicit scheme with θ = 1/2, (Crank-Nicholson scheme) was adopted. The model was operated on a monthly basis to suit the availability of data. A computer code was developed in visual C++ for the entire process. The results are presented in GIS platform (Arc View GIS; ESRI, 2004) for better visualization.

3.3 MODEL CALIBRATION

In the present study, trial and error calibration[2] procedure is adopted. Initially, the aquifer parameters such as T and S are assigned based on the field test results. The simulated and measured values of piezometric heads were compared by adjusting the model parameters to improve the fit. For the second and third parameters, an empirical equation has been used to compute recharge to groundwater depending on the depth of rainfall as discussed earlier. For the rest of parameters, following generalities of transient calibration[3] were followed.

3.4 CONVERGENCE CRITERIA

A modeler must decide what levels of accuracy are appropriate for comparative assessment of alternatives. This can be achieved by imposing convergence criterion and tolerance limits in the model to stop the number of iterations. The following convergence criterion is used in the present study.

$$\frac{\sqrt{\sum_{j=n}^{n} h_i^2}-\sqrt{\sum_{j=1}^{n} h_{i-1}^2}}{\sqrt{\sum_{j=1}^{n} h_i^2}} \le \varepsilon \qquad (28)$$

where i = iteration index, j = no. of nodes,
ε = tolerance limit (0.001)

4. RESULTS AND DISCUSSIONS

4.1 STEADY STATE CALIBRATION

The aquifer condition of January 1993 was assumed to be the initial condition for the steady state model calibration. Minimizing the difference between the computed and the observed water level for each observation point started the steady state model calibration. A number of trail runs were made by varying transmissivity values of the aquifers. The model showed a good agreement between simulated and observed water levels. The study area is having two distinct hydro-geological formations and hence divided into two zones. The calibrated transmissivity (T) values for the western ghat zone was found to be 150 m^2/day and plain area zone was found to be 100m^2/day.

4.2 TRANSIENT STATE CALIBRATION

The transient calibration was carried out for the period January 1993 to December 1998. The hydraulic conductivity values, boundary conditions and the water levels, arrived through the steady state model calibration were then used as the initial condition in the transient model calibration. A numbers of trial runs were made by varying the storage coefficient (S) values so that a reasonably good match was obtained between computed and observed water levels. Forty seven observation wells were selected as the fitting wells after consideration of their data availability and distribution in the region. The calibrated storage coefficient values for western ghat zone and plain area zone were found to be 0.0025 and 0.0063 respectively.

4.3 MODEL PERFORMANCE & VALIDATION

The performance of the calibrated model could be quantified by a number of statistics comparing the observed and simulated hydraulic heads (table 1).

Table 1. Statistical measures between observed and simulated groundwater levels

Error Measures	Well Location (nodes)						
	6	14	43	105	139	175	185
ME	-0.08	-0.31	0.47	0.55	-0.21	-0.43	-0.08
RMSE	0.65	0.46	0.73	0.78	0.56	0.76	0.69
R^2	0.83	0.89	0.89	0.89	0.91	0.78	0.86

In the present study, an attempt was made to utilize about 20% of diversion water in the Varada basin development. In this connection, the groundwater levels are simulated by considering an input of 60.5 million m^3 as recharge through the proposed canal link and three irrigation tanks in the Varada basin. There is an increase in groundwater recharge due to an additional irrigable area shown in table 2 with node numbers. The model predicts a significant increase the groundwater level with this proposed scheme compared to 2003. Model results showed that the ground water levels are increased by 10 – 15 m from the proposed interlink of rivers.

Table 2. Simulated groundwater levels of Bedti-Varada interlinking

Node No.	Groundwater level in meters (above msl)			
	January		September	
	Scenario-3 2010	Bedti-Varada link 2011	Scenario – 3 2010	Bedti-Varada link 2011
85	558.234	570.621	589.19	589.19
86	538.953	550.958	567.761	567.761
95	511.321	528.559	564.517	564.517
96	504.456	519.383	551.587	551.587
97	525.947	537.044	541.06	541.06
104	524.318	539.119	568.477	568.477
105	529.975	537.397	541.9	541.9
106	515.98	524.781	538.701	538.701
114	525.138	536.998	543.685	543.685
116	512.764	521.08	537.267	537.265
124	513.936	526.888	557.431	557.431
126	486.114	497.367	528.632	528.632
127	488.154	493.479	512.682	512.665
135	521.975	529.465	545.621	545.621
136	508.153	514.314	531.961	531.962
137	497.623	502.649	520.753	520.753
138	513.334	519.653	536.154	536.15
139	572.631	575.459	570.943	570.944
144	523.453	533.43	561.029	561.041
145	517.293	526.128	556.68	556.684
146	538.079	541.932	564.607	564.457
147	538.281	548.672	585.782	585.809
148	563.432	568.477	592.388	592.382
156	566.391	567.389	579.551	579.277

157	580.355	583.147	595.419	595.474
166	568.619	572.791	579.763	579.651
167	576.781	581.915	589.406	590.001
168	590.715	595.177	609.763	609.668

5. CONCLUSIONS

An attempt was made in the present study to simulate response of groundwater levels for interlinking of rivers through finite element method. The following conclusions may be drawn from the present study:

1. The numerical solution was effective and accurate enough to simulate the aquifer system with mean error ranging between -0.43 to 0.55 and the correlation coefficient between from 0.78 to 0.91.
2. The proposed Bedti-Varada link system could augment the groundwater/surface water system of the surrounding region significantly even if a minimum utilization of 20% of total transferable amount of 242 Mm^3 is considered.
3. Finite element method is considered as an accurate numerical scheme which can be applied effectively for groundwater flow problems.

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