HARMONIC WAVELET TRANSFORM SIGNAL DECOMPOSITION AND MODIFIED GROUP DELAY FOR IMPROVED WIGNER-VILLE DISTRIBUTION

S.V.Narasimhan[1] and B.K. Shreyamsha Kumar[2]
[1]Aerospace Electronics and Systems Division, National Aerospace Laboratory,
Bangalore - 560017, INDIA.
[2] Dept. of ECE, National Institute of Technology Karnataka (NITK),
Surathkal - 575025, INDIA.
E-mail: *svn@css.cmmacs.ernet.in*

Abstract - A new approach for the Wigner-Ville Distribution (WVD) based on signal decomposition by harmonic wavelet transform (SDHWT) and the modified magnitude group delay function (MMGD) has been proposed. The SDHWT directly provides subband signals and the WVD of these components are concatenated to get the overall WVD without using antialias and image rejection filtering. The SDHWT and the MMGD remove the existence of crossterms (CT) and the ripple effect due to truncation of the WVD kernel without applying any window, respectively. Since there is no time and frequency smoothing, the proposed method has a better performance in terms of both time and frequency resolution and desirable properties of a time-frequency representation (TFR) than the Pseudo WVD (PWVD). Further, it has a relatively better noise immunity compared to that of PWVD. In the WVD, for signal decomposition, the use of SDHWT, compared to that of a filter bank, provides almost similar results but has a significant (72%) computational advantage.

1. INTRODUCTION

The Wigner-Ville Distribution (WVD) was introduced to alleviate the tradeoff between time localization and frequency resolution (FR), found in the Short-Time Fourier transform, for processing the nonstationary signals. The WVD, at any time instant, is the Fourier Transform (FT) of the instantaneous autocorrelation (IACR) sequence of an infinite lag length and hence theoretically, it has infinite resolution both, in time and frequency. However, practically, it is the Pseudo WVD (PWVD) that is computed which considers IACR only for a finite number of lags. In the PWVD, to overcome the abrupt truncation effect, the IACR is weighted by a *window function* and for a given lag length, this deteriorates the FR. The WVD being quadratic in nature introduces crossterms for a multi-component signal. The CT makes the interpretation of the WVD difficult and it can be reduced by time smoothing only at the cost of time resolution.

In the last two decades, efforts have been made to: suppress CT effectively, improve the FR and to maintain the desired TFR properties [1]. In Choi-William's distribution, there is a tradeoff between CT suppression and the FR. In the Cone-Kernel, the CT suppression and the FR are achieved without much importance for the TFR properties.

In providing better FR for the WVD, the MMGD has been used (GDWVD) to remove the truncation effect viz., the ripple along the frequency axis, without using any *window function* [3]. However, the residual CTs left after time smoothing get enhanced due to the application of the MMGD and this requires a second time-smoothing.

Recently, a WVD based on signal decomposition (SD) approach realized by perfect reconstruction filter bank (PRFB) (FBWVD) along with MMGD to retain frequency resolution of rectangular window [4], has been proposed. The PRFB decomposes the multi-component signal into its constituents and on summation of their WVDs, results in a WVD *completely free* from CT. However, the signal decomposition (SD) and the IACR computation (at the original sampling rate) are computationally intensive.

The Harmonic Wavelet Transform (HWT) [5,6] can decompose and also reconstruct the signal without directly performing the decimation and interpolation operations. In HWT, these operations are built in and hence it is simple and computationally efficient.

In this paper, a new approach for the improved WVD (IWVD), which combines the *signal decomposition* by harmonic wavelet transform (SDHWT), for the removal of CTs and the *modified magnitude group delay function* (MMGD), for removing the ripple effect due to truncation of the WVD kernel, has been proposed.

0-7803-8674-4/04/$20.00 ©2004 IEEE

2. IMPROVED WIGNER-VILLE DISTRIBUTION (IWVD)

2.1 The Wigner-Ville Distribution:

For a signal $x(t)$, the WVD is defined as

$$W_x(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau/2)\, x^*(t-\tau/2) e^{-j\omega\tau} d\tau \qquad (1)$$

$r(t,\tau) = x(t+\tau/2)x^*(t-\tau/2)$ is the *Instantaneous-Autocorrelation* (IACR) function and * indicates conjugate operation. For computational purposes, it is necessary to weigh the signal by a window and this WVD is called *Pseudo Wigner-Ville Distribution WVD* (PWVD), $PW_x(t,\omega)$. The effect of the window is to *smear* the WVD along the frequency axis as the window eats away the correlation at higher lags and results in a poor FR.

For a composite signal with two components, the interference between these two components, due to quadratic operation introduces a third component, the *crossterm* (*CT*). The CT appears midway between two components of the signal. Its amplitude can be much larger than the individual components as it is proportional to product of the their amplitudes. As this is true with every pair of components in the signal, the CT makes the interpretation of the WVD difficult. But CT can be attenuated by smoothing the WVD along the time axis as it oscillates in time at a frequency equal to their frequency separation. The smoothing in time for CTs and in frequency for the lag window can be considered as a two-dimensional convolution of the WVD with a smoothing kernel [1]. The kernel determines the properties of the distribution [1]. Use of different smoothing kernels result in a class of distribution, called the *Cohen's class* [1]. Any smoothing will affect the properties of a time-frequency distribution and the choice of this kernel depends upon the application. The properties viz., marginality in frequency, group delay and frequency support; are not satisfied for common windows (other than rectangular) and this is due to smearing of the signal spectrum with that of the window. In the last two decades, significant research effort has been devoted in the choice of this smoothing function to provide: CT suppression, good time localization and FR and to have as many as the properties of a TFR [1].

2.2 HARMONIC WAVELET TRANSFORM (HWT) [5,6]:

Fig.1 illustrates the decomposition and reconstruction for an analytic signal $\{x(0), x(1), \ldots\ldots\ldots, x(15)\}$. If

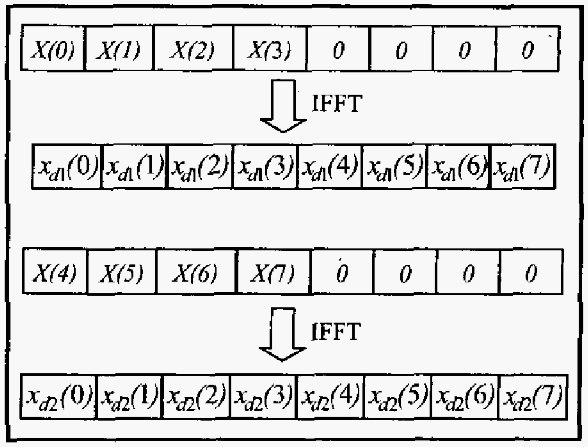

Fig. 1a Analytic signal decomposition using HWT.

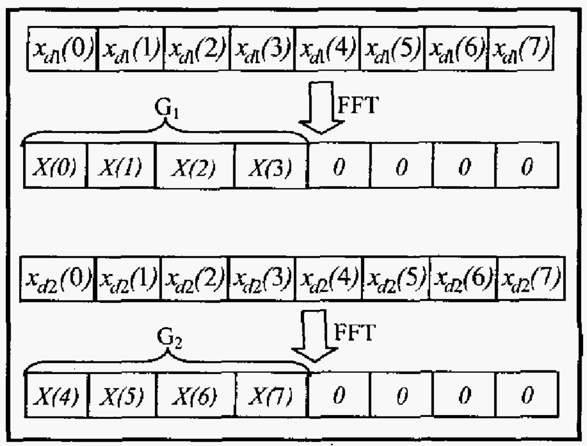

Fig. 1b Analytic signal reconstruction using HWT.

$\{X(0), X(1), \ldots\ldots\ldots, X(15)\}$ are the discrete Fourier transform (DFT) coefficients of the signal considered, then the DFT coefficients 0 to 7 will have non-zero values and from 8 to 15 will have zero values. Here the signal is decomposed into two bands. The inverse discrete Fourier transform of these bands give respective HWCs (Harmonic wavelet transform coefficients), which are decimated because of built-in decimation. The reconstruction from HWCs is just the reverse process of the decomposition as shown in Fig.1(b). Referring to the Fig.1b, G_1 and G_2 are concatenated and zeros are padded to recover the original spectrum.

2.3 MODIFIED MAGNITUDE GROUP DELAY FOR A COMPLEX SIGNAL (MMGD) [2, 3]:

If $x(n)$ is a minimum phase complex signal,

$$\ln|X(\omega)| = \sum_{n=0}^{\infty}[c_R(n)\cos\omega n + c_I(n)\sin\omega n] \quad (2)$$

$c(n) = c_R(n) + jc_I(n)$: cepstral coefficients
R : real part and I : imaginary part

and

$$\theta(\omega) = \sum_{n=0}^{\infty}[-c_R(n)\sin\omega n + c_I(n)\cos\omega n] \quad (3)$$

$\theta(\omega)$: the unwrapped phase.

The GD $\tau_m(\omega)$ for a complex signal is

$$\tau_m(\omega) = \frac{-\partial\theta(\omega)}{\partial\omega}$$

$$= \sum_{n=0}^{\infty} nc_R(n)\cos\omega n + nc_I(n)\sin\omega n$$

$$= (1/2)FT[nc(n) - nc^*(-n)] \quad (4)$$

If $nc(n)$ is conjugate symmetric, $\tau_m(\omega)$ is the FT of $nc(n)$. Since $nc(n)$ are derived from the magnitude, $\tau_m(\omega)$ is called as the magnitude GD for a complex signal (MGD).

In spectral estimation, the goal is to achieve lower bias, lower variance and high resolution. The variance / fine structure can be due to signal truncation effect or associated white noise or due to input white noise that drives a system in generating the signal or any of these combinations. These introduce zeros close to the unit circle and their effect cannot be removed by normal smoothing without the loss of FR. The modification suggested in [2, 3] removes these zeros close to the unit circle without disturbing the signal/ system poles retaining the FR.

The spectrum of a complex signal $x(n)$ is $X(\omega) = N(\omega)/D(\omega)$, where $D(\omega)$ corresponds to the spectrum of system or sinusoids and $N(\omega)$ that of the excitation or the associated noise. Because of zeros that are close to the unit circle, the MGD due to $N(\omega)$ will mask the MGD of $D(\omega)$ and this effect can be reduced by multiplying the MGD with $|N(\omega)|^2$. Hence, the modification of MGD requires the estimation of $|N(\omega)|^2$, which is given by,

$$|\tilde{N}(\omega)|^2 = \frac{|X(\omega)|^2}{|\overline{X}(\omega)|^2}$$

where, $|\overline{X}(\omega)|^2$ is the smoothed power spectrum obtained by the truncated cepstral sequence.
Therefore, the modified MGD (MMGD) is

$$\tau_{mo}(\omega) = \tau_m(\omega)|\tilde{N}(\omega)|^2 \quad (5)$$

2.4 IMPROVED WIGNER-VILLE DISTRIBUTION (IWVD):

In the proposed method, at each time instant, the CT free WVD slice is computed by signal decomposition using HWT (SDHWT) and this is used to derive the ripple/ truncation effects free power spectrum by the MMGD [3].

To avoid the occurrence of CTs due to the quadratic nature of the WVD, an analytic signal of a multicomponent signal is decomposed into its components using HWT to get respective HWCs. While computing HWCs, if FT coefficients are grouped in to M bands, it is decimated by M times and hence, only N/M time instants are retained for N original time instants. To preserve the original time instants, the grouped FT coefficients are zero padded to make its length equal to the length of the signal before taking inverse FT. This results in an interpolated HWCs by a factor M. However, at every input signal instant for computing the IACR, only every M^{th} sample is used, so that the effective lag length is decimated by a factor M and this reduces the computational complexity. Further, these IACR sequences are Fourier transformed to get the individual WVDs for the decomposed components. The complete WVD of the original signal is obtained by plugging back these individual component WVDs. The built-in decimation and interpolation operations and the absence of antialiasing and image rejection filters provide the signal decomposition with computational efficiency compared to that of a filter bank. The proposed method gives the 72.3% computational reduction in FLOPS compared to FBWVD method.

In MMGD,

$$\tilde{N}(\omega) = \frac{X(\omega)}{\overline{X}(\omega)} = \left[1 + \frac{\Delta(\omega)}{\overline{X}(\omega)}\right]$$

Here, $\Delta(\omega)$ represents the fluctuating part of $X(\omega)$. By multiplying $\tau_m(\omega)$ by $|\tilde{N}(\omega)|^2$, a spectrum that is free from fluctuations due to excitation or the associated noise with improved resolution is obtained. For a signal having a *flat spectral* characteristic, in the MGD $\tau_m(\omega)$, the contribution is only due to $\Delta(\omega)$. Therefore, a $\tau_{mo}(\omega)$ that is free from fluctuations, is given by

$$\tau_{mo}(\omega)=\tau_m(\omega)|\Delta(\omega)|^2 \qquad (6)$$

For the WVD, it is required to remove the ripple on the floor, which is equivalent to a flat spectral characteristic [3] and this is achieved by Eqn.(6). $\tau_m(\omega)$ is derived from the spectral magnitude (Eqns.(2) and (4)) of the truncated IACR and it is modified to get $\tau_{mo}(\omega)$ using Eqn.(6). The WVD at each time instant, for which the ripple free spectrum with better FR is obtained from $\tau_{mo}(\omega)$ by retracing the MMGD computation procedure in reverse order using Eqns. (4) and (2). For this, the cepstral sequence resulting from MMGD must be made conjugate symmetric.

Since there is no smoothing both in time and frequency domain, the desired properties of a TFR, viz., the marginals, instantaneous parameters and support; are obeyed relatively better than in the PWVD. Further, as the MMGD not only removes the zeros due to truncation but also due to associated noise and the SDHWT reduces/ removes the interaction between the noise components, they independently provide additional noise immunity. Hence the proposed method will have a relatively better noise performance than by the other methods.

3. Simulation Results

The results shown in the Figs. 2 and 3 for Frequency Shift Keying (FSK) and linear crossing chirp signals, respectively, illustrate that the proposed method retains the same performance of the FBWVD, in terms of CTs suppression and FR improvement (contour plots) with reduced computational complexity. With the IWVD, as the cross terms are not allowed to exist due to SDHWT prior to computing the WVD kernel, the original time resolution is retained. Also, the IWVD is very effective in removing the spurious spectral peaks due to noise along with CTs while preserving the FR (Figs.4 and 5). Figs.4 and 5 illustrates the performance comparison of IWVD with FBWVD for a chirp signal of 8dB SNR and a FSK of 3dB SNR, respectively. In all the above cases for computing FBWVD, total lag length of 33 points and DFT length of 128 points are used. In the proposed method, while computing WVD, the signal is decomposed into two subbands and for each of the subband; the reduced lag length of 17 points and DFT length of 64 points are used. The IWVD is computed from CT free WVD using MMGD, where the DFT of 128 points is used. For FSK and chirp signals, cepstral sequences having the initial 3 and initial 4 coefficients, respectively, are used in the estimation of $|\Delta(\omega)|^2$.

4. Conclusions

A WVD with improved performance (IWVD) based on SDHWT and MMGD was proposed. The HWT decomposes the signal in a simple and computationally efficient way as it only involves grouping of the DFT coefficients. Further, the IACR is computed at a decimated rate and the overall WVD slice is obtained by plugging back the FT of the component IACRs, in their corresponding positions. Thus it is free from antialias and image rejection filtering.

The SDHWT removes the very existence of crossterms and the modified magnitude group delay (MMGD) removes the autocorrelation truncation effects, without using any window function. Hence, the IWVD has both improved frequency and time resolution and obeys the desired TFR properties better, compared to the Cohen's class PWVD. Further, as the MMGD and SDHWT provide additional noise immunity, the IWVD has a better noise immunity. Its performance is almost similar to that of FBWVD but with a significant reduction (72%) in computational load.

5. References

[1] J. Jeong and W.J. Williams, "Kernel Design for reduced interference Distributions", *IEEE Trans. On Signal Processing,* Vol.40, No.2, 1992, 402-412

[2] B. Yegnanarayana and H.A. Murthy, "Significance of group delay functions in spectral estimation", *IEEE Trans. on Signal Processing,* Vol.40, No.9, 1992, 2281-2289.

[3] S.V. Narasimhan, E.I. Plotkin and M.N.S. Swamy, "Power spectrum estimation of complex signals and its application to Wigner-Ville distribution: A group delay approach", *Sadhana*, Vol.23, Part-I, 1998, 57-71.

[4] S.V. Narasimhan, M.B. Nayak, "Improved Wigner–Ville distribution performance by signal decomposition and modified group delay", *Signal Processing,* 83 (2003), 2523-2538.

[5] D.E. Newland, "Random Vibrations, Spectral & Wavelet analysis", *Longman Singapore Publishers Pvt Ltd,* Singapore, 3-rd edition, 1993.

[6] D.E. Newland, "Wavelet theory and Applications", *Proceedings of International congress on Air-and Structure-Borne Sound and Vibration,* Montreal, Canada, June 13-15,1994, pp.695-713.

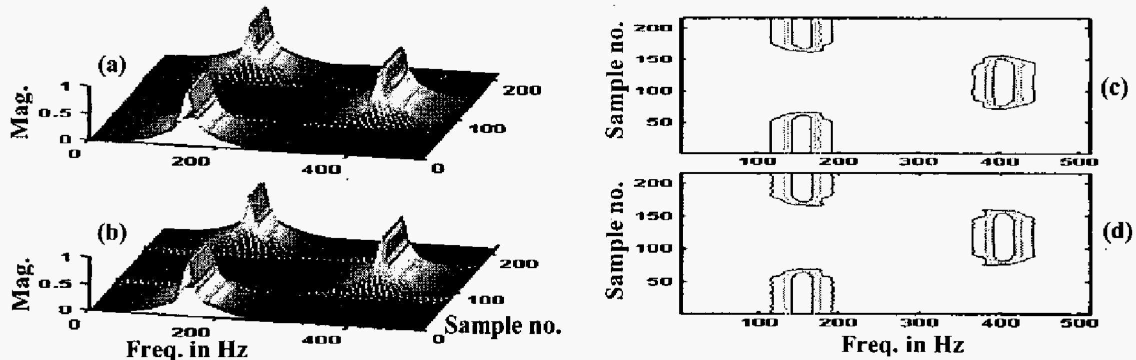

Fig. 2 TFR of FSK signal by (a) FBWVD, (b) IWVD, (c)-(d) are respective contour plots of (a) and (b).

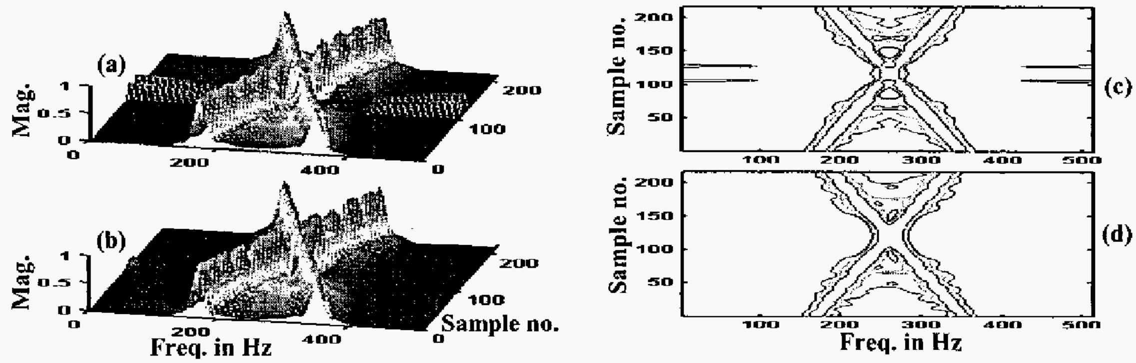

Fig. 3 TFR of crossing chirp signal by (a) FBWVD, (b) IWVD, (c)-(d) are respective contour plots of (a) and (b).

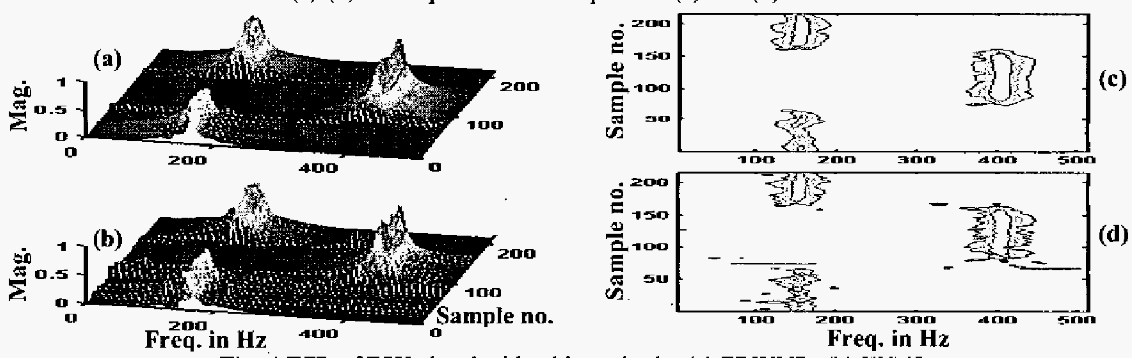

Fig. 4 TFR of FSK signal with white noise by (a) FBWVD, (b) IWVD, (c)-(d) are respective contour plots of (a) and (b).

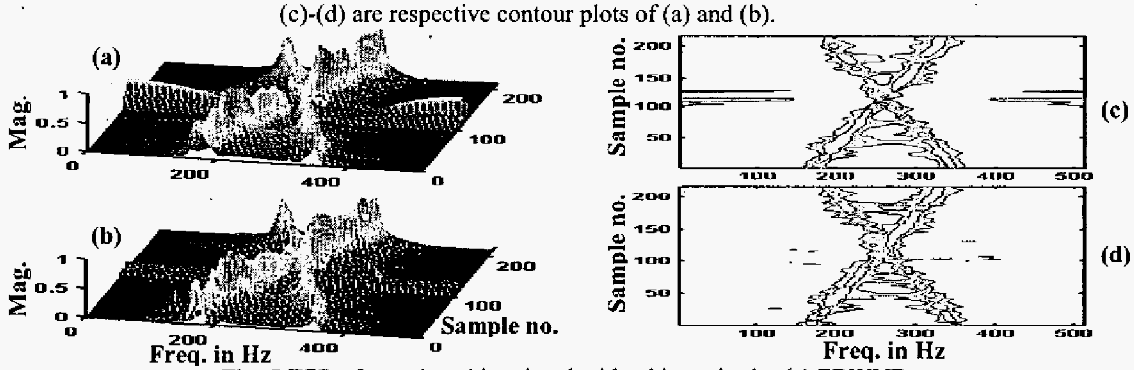

Fig. 5 TFR of crossing chirp signal with white noise by (a) FBWVD, (b) IWVD, (c)-(d) are respective contour plots of (a) and (b).