ESTIMATION OF EVOLUTIONARY SPECTRUM BASED ON STFT AND MODIFIED GROUP DELAY

S.V. Narasimhan[1] and S. Pavanalatha[2]
[1] Aerospace Electronics and Systems Division,
National Aerospace Laboratories, Bangalore-560017, INDIA
[2] Dept. of ECE, National Institute of Technology Karnataka, Surathkal-575025, INDIA
Email : *svn@css.cmmacs.ernet.in*

ABSTRACT

This paper proposes a new estimator for Evolutionary spectrum (ES) based on short time Fourier transform (STFT) and modified group delay (MGD). Here, the STFT enables crossterm suppression and the MGD preserves the frequency resolution of the rectangular window. It is applicable to deterministic and random signals generated by time varying systems. The proposed method provides signal to noise ratio enhancement due to the use of MGD. The results indicate that for linear chirp signals and for time varying random process, its frequency resolution is close to that of WVD and better than Evolutionary periodogram (EP) and STFT. Further, its noise immunity is better than those of EP and STFT.

1. INTRODUCTION

The STFT, Wigner Ville distribution (WVD) and ES are the three main approaches of time dependent spectral analysis. The EP has been proposed for the estimation of the Wold-Cramer ES. Dekta et al. [3] have shown that the EP, STFT and a class of Bilinear Distributions (BD) are estimators of ES. The BD suffers from the disadvantages like cross terms and negativity of the spectrum. In case of STFT, due to built in time domain smoothing, there are no cross terms and in the spectrogram, the magnitude square of the STFT ensures the positivity of time-frequency spectral representation. However, with the STFT there is a tradeoff between time and frequency resolutions. EP also produces positive spectra without any cross terms. As in STFT, even in EP for a given number of expansion functions, the resolution is controlled by the data length. Thus the STFT is similar EP. Further, for a given window length, the STFT achieves best frequency resolution when the window is rectangular but suffers from Gibbs ripple. Use of any window reduces the Gibbs ripple at the cost of frequency resolution [4]. Thus the STFT can become a better estimator of ES, if the Gibb's ripple can be removed without affecting the frequency resolution of the rectangular window.

The driving white noise input to a system, the associated white noise with a signal and the Gibbs ripple introduce zeros close to the unit circle. For spectral estimation, a modified group delay function (MGD) has been proposed [5]. The MGD basically removes the zeros close to unit circle without disturbing the poles of the system/signal and hence preserves the frequency resolution [4,5].

In this paper, a new estimator of evolutionary spectrum based on STFT and MGD, has been proposed. The STFT by itself suppresses the cross terms while the MGD preserves the frequency resolution of the rectangular window, and reduces the variance.

Wold-Cramer ES treats the nonstationary signal as the output of a linear time varying (LTV) system driven by a stationary white noise[2]. Hence, ES is expected to handle not only deterministic non-stationary signals like chirps but also random signals generated by time varying systems driven by stationary random noise. In fact, initially ES was proposed for handling random signals [1]. The proposed estimator can handle both types of signals.

2. EVOLUTIONARY SPECTRUM

The nonstationary signal analysis by ES was introduced by Priestly. According to this the spectrum is the output of a set of bandpass filters each being averaged in time[1]. But according to the Wold-Cramer ES, a nonstationary $x(n)$ is the output of a LTV system with a stationary input white noise [2]

$$x[n] = \int_{-\pi}^{\pi} H(n,\omega)\, e^{j\omega n}\, dZ(\omega) \qquad (1)$$

$Z(\omega)$ is a process with orthonormal increments and $H(n,\omega)$ is a process slowly varying in time. The Wold-Cramer Evolutionary spectrum is given by $Z(\omega)$

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$$S_{ES}(n,\omega) = |H(n,\omega)|^2 \quad (2)$$

From Eqns. (1) and (2), ES can be expressed as [3],

$$S(n,\omega) = \frac{1}{2\pi}\sum_k \sum_l \delta[l-n]\delta[k]$$

$$\int_{-\pi}^{\pi} H^*(l-k,\lambda)H(l+k,\lambda)\delta[\omega-\lambda]e^{-j(\omega-\lambda)2k}d\lambda$$

That is, the ES involves convolution in both time and frequency with delta functions and these provide localization both in time and frequency.

Dekta et al.[3] have shown that BD, STFT and EP are the estimators of ES. $\delta[l-n]\delta[k]$ functions of ES are approximated by the respective bases or window functions of EP, STFT and BD.

While the STFT and EP approximate the $\delta[\omega-\lambda]$ by $[1+e^{-j(\omega-\lambda)}]$, the BD approximates this term by a constant. The term $[1+e^{-j(\omega-\lambda)}]$ has a lowpass effect and hence reduces the cross term components in the estimate [3]. Absence of this term in BD results in cross terms.

Hence, compared to BD, *STFT has more resemblance to the EP and can be used as an estimator of ES. EP and STFT differ only in terms of window function, which decides the frequency resolution.* The EP provides a better frequency resolution than STFT. Improving the frequency resolution by some means can make STFT, a better estimator of ES.

3. THE MODIFIED GROUP DELAY [4, 5]

The variance / fine structure of a spectral estimate can be due to the driving white noise input of a system or due to white noise associated with the signal (sinusoids) or the signal truncation effects or any combination of these three which introduce spectral ripples. The spectrum of a white noise sequence or any spectrum with ripples has zeros close to the unit circle, in the Z-plane. The windowed and averaged periodogram reduce the variance only at the cost of spectral resolution. On the Z-plane, this can be interpreted as pulling the signal poles in addition to the zeros which cause ripple. The MGD removes the zeros which contribute to fine structure / variance without disturbing the signal/ system roots. *Therefore for ES estimation using STFT, to remove the effect of input white noise and to reduce the ripples due to rectangular windowing, the MGD approach is very appropriate.*

For a minimum phase *real* signal $x(n)$ [5],

$$\ln|X(\omega)| = \sum_{k=0}^{\infty} c(k)\cos(\omega k) \quad (3)$$

The GDF derived from the spectral magnitude is

$$\tau_p(\omega) = -\frac{\partial}{\partial\omega}[\theta(\omega)] = \sum_{k=1}^{\infty} kc(k)\cos(\omega k) \quad (4)$$

The modification [4,5] of GDF involves the estimation of these zeros and their removal in the GDF domain. If the signal $x(n)$ is real: generated by an all-pole system driven by a white noise or has sinusoids with white noise and further, its spectrum $X(\omega)= N(\omega)/D(\omega).D(\omega)$ corresponds to the system or sinusoids and $N(\omega)$ to the excitation or associated noise, respectively. For this case, the modified GD is

$$\tau_p(\omega) = \tau_{pN}(\omega) - \tau_{pD}(\omega) \quad (5)$$

and $\tau_{pD}(\omega)$ are the MGDs for $N(\omega)$ and $D(\omega)$[4,5], and are $\alpha_N(\omega)/|N(\omega)|^2$ and $\alpha_D(\omega)/|D(\omega)|^2$, respectively. $\alpha_N(\omega)$ and $\alpha_D(\omega)$ are their numerators. The $\tau_{pN}(\omega)$ will have large amplitude spikes due to very small values of $|N(\omega)|^2$ near the zeros (which are close to the unit circle) and this is not so with $\tau_{pD}(\omega)$, as the roots of $D(\omega)$ are well within the unit circle. Hence, in $\tau_p(\omega)$, the effect of input white noise or the associated noise or the truncation effects masks the system or the signal component, which is assumed to be an all-pole one. The effect of these zeros is reduced significantly by multiplying $\tau_p(\omega)$ (Eqn.(5)) by $|N(\omega)|^2$. Also, as the envelope of $|N(\omega)|^2$ is nearly flat, the significant features of $\tau_{pD}(\omega)$ continue to exist, with the $|N(\omega)|^2$ fluctuations superimposed on it. Hence, the modified GDF $\tau_{po}(\omega)$ is given by [4],

$$\tau_{po}(\omega) = \tau_p(\omega)|N(\omega)|^2 \quad (6)$$

The estimate of $|N(\omega)|^2$, $|\hat{N}(\omega)|^2 = |X(\omega)|^2 / |X_S(\omega)|^2$, $|X_S(\omega)|^2$ is the smoothed power spectrum obtained by the truncated cepstral sequence [4].

$$\hat{N}(\omega) = X(\omega)/X_s(\omega) = [1 + \Delta(\omega)/X_s(\omega)] \quad (7)$$

Here, $\Delta(\omega)$ is the fluctuating part of $X(\omega)$. For a signal having *flat spectral characteristic,* in the GD $\tau_p(\omega)$, the contribution is only due to $\Delta(\omega)$. Therefore, a GDF that is free from ripple on the flat floor is given by [4]

$$\tau_o(\omega) = \tau_p(\omega)|\Delta(\omega)|^2 \quad (8)$$

4. ESTIMATION OF ES BY STFT AND MODIFIED GD FUNCTION

The estimation of ES by STFT using a rectangular window, though free from cross terms suffers mainly from ripple effect. One of the contributions to the ripple effect is the Gibbs ripple effect due to abrupt data truncation. Others are driving input noise for a system output and the associated noise with a signal or system output. One of the obvious common solutions to remove the ripple effect due to above factors is to use a smooth window function, which will smear the ripple effect and make the spectrum smooth. But this smoothness or reduction in the variance of the spectrum is at the cost of frequency resolution. Hence the STFT though free from cross terms is not a good estimator of ES. Therefore it is required to remove the ripple effect without any loss of frequency resolution. This is achieved by the application of MGD.

For the signal under consideration first STFT is computed with a rectangular window of suitable length and at each instant of time MGD is computed. In STFT computation as higher lags that are having large variance are also included, and they contribute to the variance of the STFT estimate. To take care of this, autocorrelation coefficients for higher order lags, derived from spectrogram, are made zero and its FT is considered as estimate of spectrogram. This estimate is supposed to be a positive quantity as it represents the power spectral density (PSD). However the inevitable presence of the rectangular window may make the spectrogram values negative. Since GDF $\tau_p(\omega)$ involves logarithmic operation (Eqn. (3) and (4)), it is necessary to ensure that values of the slice of the spectrogram at each instant of time are positive. This is achieved by raising the floor level by scaling up the autocorrelation coefficient at the zeroth lag, sufficiently. Further, in computing [4] $\tau_p(\omega)$, the equivalent magnitude spectrum is computed from the spectrogram slice. For each TFR slice obtained from the modified GDF $\tau_o(\omega)$, the gain and base level are adjusted with respect to the original log spectrum.

For deterministic signals like, linear chirps or FSK signals, where only the Gibbs ripple on the floor is to be reduced, $\tau_p(\omega)$ is modified by multiplying with $|\Delta(\omega)|^2$. But for a time varying random process, $|N(\omega)|^2$ is used to get $\tau_o(\omega)$.

5. RESULTS AND CONCLUSIONS

For a linear chirp signal, for the proposed method of ES estimation, a rectangular window of length 16 and DFT length of 128 are used. The number of autocorrelation lags considered is 16. The $|\Delta(\omega)|^2$ estimate is obtained by considering the first 8 cepstral coefficients in computing the smoothed PSD. To avoid negative spectral values in spectrogram slice, the autocorrelation coefficient at zeroth lag has been lifted by a factor $U=10$. For STFT, 16 length hamming window is used. Fourier basis functions with M=5 are used for EP computation.

Fig.1a shows that compared to STFT and EP the proposed ES provides a high spectral resolution that is comparable to that of WVD *but without any cross terms*. For a signal with SNR of 6 dB (Fig.1b) in the new method the spurious peaks due to noise are minimum and also the frequency resolution is still maintained Here, the MGD is computed with $|\hat{N}(\omega)|^2$, as the spectrum is no more flat. For the random time varying signal, for the estimation of $|\hat{N}(\omega)|^2$, the first 5-cepstral coefficients are used. The ideal time varying AR spectrum and the corresponding results obtained with STFT, WVD, EP and proposed ES are shown in Fig. 1c. The proposed estimator can track the time varying AR with good resolution. Further, with an output SNR of 5dB (Fig.1d), the proposed algorithm has comparatively less unwanted peaks and can track time varying pattern with good frequency resolution.

In the proposed estimator for ES based on STFT and MGD, the STFT due to its built in averaging suppress the crossterms and the MGD preserves the frequency resolution of the rectangular window as it reduces the Gibbs ripple without using any window function. The MGD also provides signal to noise ratio enhancement as it removes the zeros close to the unit circle due to the associated noise. For linear chirp signals and for time varying random process its frequency resolution is close to that of WVD and better than EP and STFT. Further it has a better performance than the EP and STFT in the presence of noise.

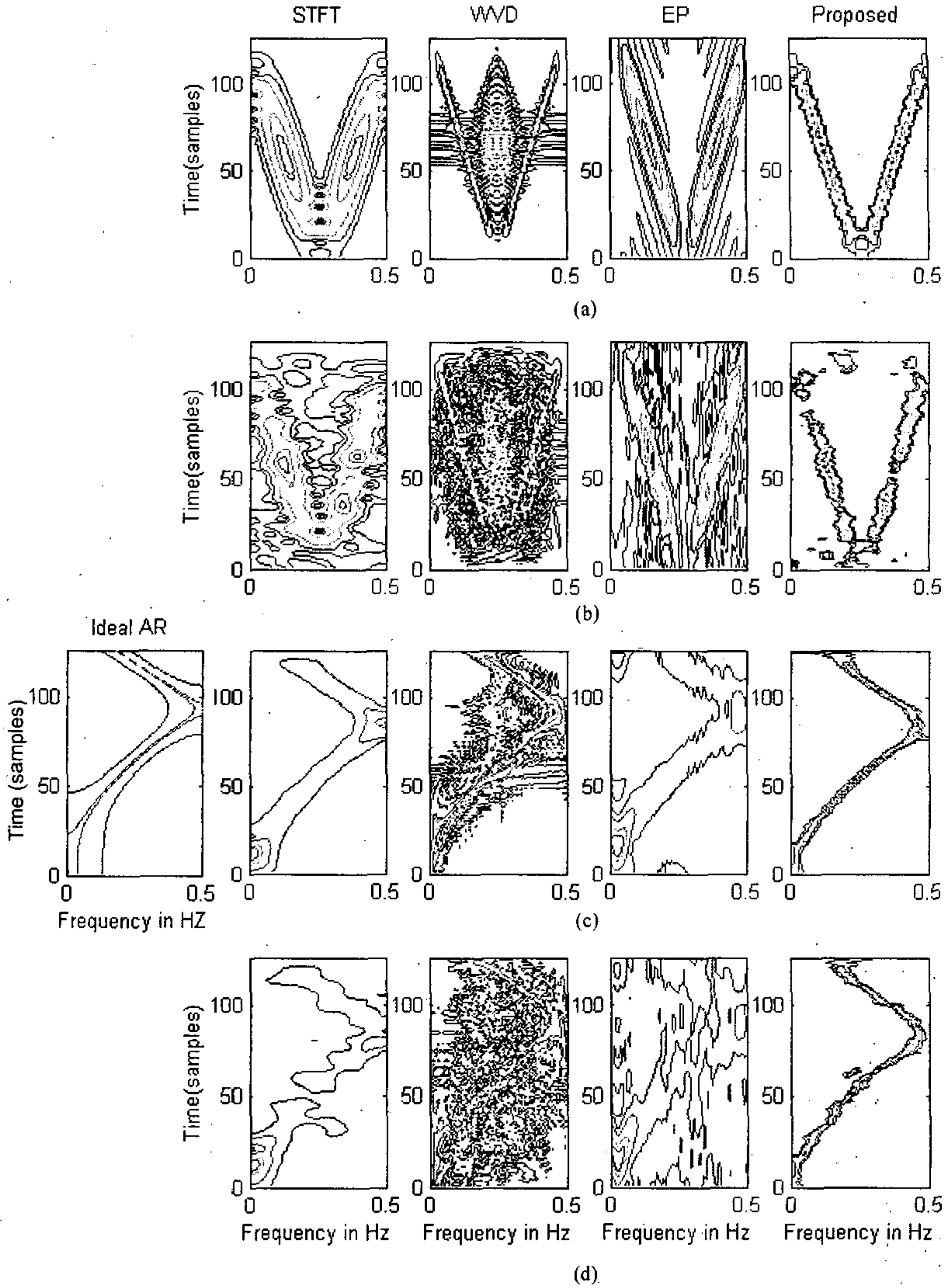

Fig.1. Contour plots for a chirp signal with (a) SNR= 0 dB and (b) SNR=6dB, and for a random signal with (c) SNR= 0 dB and (d) SNR=5dB.

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