MIMO Radar with Spatial-Frequency Diversity for Improved Detection Performance

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***Abstract*- The Multiple Input Multiple Output (MIMO) radar concept exploits the independence between signals at the array elements unlike beamforming which presumes a high correlation between signals either transmitted or received by an array. Radar Cross Section (RCS) of a complex target varies with both transmitted frequency and target geometry. By widely separating transmit and receive antennas, MIMO radar systems observe a target simultaneously from different aspects resulting in spatial diversity, thus improving the detection performance. Also by utilizing different frequencies, independent RCS of the target can be observed, thus resulting in frequency diversity. In this paper, the spatial and the frequency diversities are studied together to bring out the combined benefits. The system proposed will not only have several antennas appropriately spaced but also several operating frequencies appropriately spaced, providing a better detection performance than conventional MIMO radar systems for the same transmission power. The simulation results exhibit a better detection performance of the proposed system as compared to MIMO radar systems with only spatial diversity.**

Keywords*- *RCS; Receiver Operating Characteristics (ROC); Chi-square distribution; Orthogonality; Scatterers.

I .INTRODUCTION

MIMO radars have been extensively studied [1] and studies have shown that by appropriately spacing the antennas, the uncorrelated target aspects can be seen from individual antenna simultaneously, thus resulting in spatial diversity. The spacing requirement between antennas for such a system is given by [1]

$$d_t \geq \lambda R/D \quad (1)$$

where 'd_t' is the spacing between the transmitting antennas, *R* is the range of the target from radar, 'λ' being the wavelength of operation and *D*, the target size.

Frequency diversity radar: This type of radar transmits different frequencies from one or more antennas in FDM fashion with frequencies so chosen as to exploit the frequency diversity [2]. Although the behavior of real targets can be quite complex, the change in frequency and angle required to decorrelate a target or clutter can be obtained using the relation given in [2], [3].

$$\delta F = c/2\mathcal{L}sin(\phi) \quad (2)$$

where *c* is the speed of light, $\mathcal{L}$ is the length of the target, and ϕ is the aspect angle between the target and the radar boresight. $2\mathcal{L}sin(\phi)$ is the length of the target projected along the radar boresight. This result does not depend on the transmitted frequency. Although the above result is based on a highly simplified target model, since most targets do not consist of a large collection of linearly distributed scatterers, this frequency step may not completely result in decorrelated returns from a target at all aspects. However, this value should be sufficient to avoid deep fading. To ensure better decorrelation, a more conservative separation of the frequency can provided if the front end bandwidth of the radar allows such separation.

MIMO Radar with Spatial and Frequency Diversity (MIMO-SF): This type of radar incorporates both, the spatial and the frequency diversity. It is shown in subsequent sections that it provides a superior detection performance compared to the MIMO radar with only spatial diversity for the same transmission power at the cost of extra hardware. Here we assume several antennas at the transmitter end and at the receiving end, the antenna spacing at the transmitter end being as per the formula given in (1) and the individual antenna would be transmitting a signal which is frequency division multiplexed with frequency spacing as per (2). Such a system is expected to have both the types of diversities, which is assumed to be independent of each other providing a better gain than the individual systems. Even though the idea of combining spatial and frequency diversity is not new in the field of communication, the idea is still fresh in the radar context and needs to be explored.

Section II recaps the general MIMO radar signal model given in reference 1 and also presents the transmitter architecture for MIMO-SF, Section III prepares the framework for comparison of conventional MIMO with MIMO-SF, Section IV presents the simulation results and compares the detection performance of conventional MIMO with MIMO-SF. Conclusion is brought out in Section V which discusses the dependence of the detection performance of MIMO-SF on SNR and probability of false alarm (P_{fa}).

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II. A GENERAL MIMO RADAR SIGNAL MODEL

The model discussed in this section [1], is assumed to have two uniform linear arrays of M antennas at the transmitter and N antennas at the receiver. Further, a far field complex target consisting of Q independent scatterers is assumed. Each scatterer is assumed to have isotropic reflectivity modeled by zero-mean, unit variance per dimension, independent and identically distributed Gaussian complex random variable ζq. The target is then modeled by the diagonal matrix

$$\Sigma = \left(1/\sqrt{2Q}\right) diag\left(\varsigma_0, \varsigma_1, \ldots\ldots, \varsigma_{Q-1}\right) \qquad (3)$$

where the normalization factor makes the target average RCS $E[trace(\Sigma\Sigma^H)] = 1$, independent of the number of scatterers in the model. Let ϕ_m be the aspect angle of the scatterer w.r.t. m^{th} transmits antenna (measured with respect to the normal to the arrays) and θ be the aspect angle of the scatterer w.r.t. the receiver antenna, Δ be the spacing between the scatterers such that $\Delta * Q = D$, the target length. d_t and d_r are the spacing between the transmit antenna elements and receive antenna elements respectively.

For simplicity, it is assumed that the target scatterers are laid out as a linear array, and that this array and the arrays at the transmitter and receiver are parallel. Fig.1 illustrates the model.

The signals radiated by the M transmit antennas impinge on the Q scatterers at angles $\phi_{m,q}$ $q = 0, \ldots, Q-1$ and $m = 0, \ldots, M-1$ (measured with respect to the normal to the arrays). Assuming that the length of the target array is small compared to the distance, the signal transmitted by the m^{th} transmit antenna arrives as a planewave at the target. Thus, the angle of arrival at the target is identical for all scatterers, $\phi_{m,q} = \phi_m$, ∀ q.

Transmitter architecture: The transmitter architecture for a MIMO radar system that can exploit both spatial & frequency diversity is depicted in Fig.2. As shown, there are M transmitting antennas in this design, and each antenna

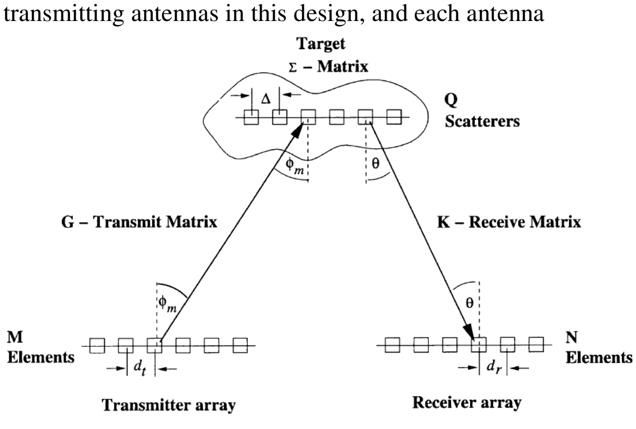

Figure 1. Bistatic radar scenario. The target consists of multiple scatterers organized in the form of a linear array. [1]

transmits F different carrier frequencies at the same time in FDM fashion. At the m^{th} transmit antenna, m = 1, . . ., M, the l^{th} carrier frequency, l= 1, . . .,F , is modulated signal $s_l(t)$. We consider MIMO spatial-frequency diversity radar with a uniform linear array with M transmitting antennas. The F carrier frequencies are chosen as $f_l = f_c + \left(l - \frac{F+1}{2}\right) * \delta F$, $l = 1, \ldots, F$, where f_c is the centre frequency.

The signal vector induced by the *m*-th transmit antenna, for the l^{th} frequency is given by

$$g_{m,l} = [1, e^{-j2\pi sin\theta_{m,2}\Delta_2/\lambda_l}, \ldots\ldots, e^{-j2\pi sin\theta_{m,Q}\Delta_Q/\lambda_l}]^T \qquad (4)$$

where λ_l is the wavelength corresponding to l^{th} frequency carrier & Δ_q is the spacing between the first and q-th scatterer. The signals are reflected by the scatterers, towards receiver array elements at angles $\theta_{n,q}$, n=0,1,……N-1, q=0,1,…….Q-1. Assuming that both the sizes of the target and the receiver array are small compared to the distance between them, it is found that $\theta_{n,q} = \theta$. The signals reflected by the scatterers have in the far field, relative phase shifts described by the vector $k(\theta,l)$

$$k(\theta, l) = [1, e^{j2\pi sin\theta\Delta/\lambda_l}, \ldots\ldots, e^{j2\pi sin\theta(Q-1)\Delta/\lambda_l}]^T \qquad (5)$$

A planewave signal arriving at the array at the angle θ excites the elements of the array with phase shifts given by the vector $a(\theta,l)$

$$a(\theta, l) = [1, e^{-j2\pi sin\theta d_r/\lambda_l}, \ldots\ldots, e^{-j2\pi sin\theta(N-1)d_r/\lambda_l}]^T \qquad (6)$$

With the vectors and the target matrix defined above, the received signals originating from the m^{th} transmitter, at l^{th} frequency and reflected by the target can be expressed as

$$r'_{m,l} = a(\theta, l)k^T(\theta, l)\Sigma g_{m,l} b_{m,l} s_{m,l} \qquad (7)$$

the term $b_{m,l}$ are complex variables representing the phase shifts between the signals coming from different transmitters due to the different propagating delays which is neglected in the MIMO scenario. The complex scalar $s_{m,l}$ represents the component of the sampled receive filter output due to the waveform transmitted by the m_{th} transmitter at l^{th} frequency. Organizing those scalars in the vector s_l=[s_{0l}, s_{1l} ,……s_{ml}………,s_{M-1l}], the received signal for a single frequency can be described as

$$r_l = a(\theta, l)k^T(\theta, l)\Sigma \sum_{m=0}^{M-1} g_{m,l} b_{m,l} s_{m,l} + v \qquad (8)$$

Here we represents the AWGN terms at the receiver. The effect of the vector k (θ,l) is to combine the signals coming from individual scatterers in the far field. As it is assumed that Σ consists of complex valued random scatterers, the effect of k(θ,l) can be embedded in Σ and can be replaced , without loss generality , with 1_Q=[1,1,…….1]^T . . The receive matrix is given by $K_l = a(\theta, l)1_Q^T$ and the channel matrix is given by $((1/\sqrt{2})a(\theta,l)\alpha_l^T)$ where the components $\alpha_{m,l}$ of the M x 1

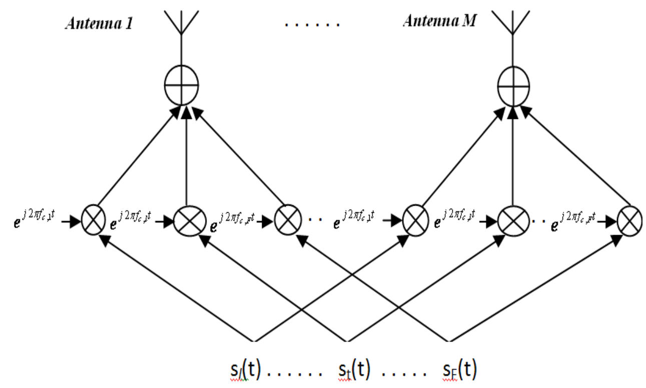

Figure 2. MIMO-SF radar transmitter design that can exploit both spatial & frequency diversity.

vector α_l, are previously introduced target fading coefficients for each target illuminating path $\alpha_{m,l} = \sqrt{2}1_Q^T\Sigma g_{m,l}$ The factor $\sqrt{2}$ is introduced here so that the $|\alpha|^2$ random variables have a χ^2_2 distribution. The vector α_l is given by $\alpha_{m,l} = \sqrt{2}1_Q^T\Sigma G_l^T$ for the frequency l=1,2,………F, Where the matrix G_l=[g_{0l}, g_{1l}, ……… g_{M-1l}], the one from the transmitter to the individual target scatterers. Thus, the vector α is given by α=[α_1, …α_l …………..,α_F]. Due to the orthogonality among transmit vectors, the $\alpha_{m,l}$ are uncorrelated. Moreover, the random variables $\alpha_{m,l}$ are zero mean, unit variance (per dimension), independent, identically distributed complex normal.

Assuming the proposed system, the received signal is given by [1]

$$r_l = (1/\sqrt{2})a(\theta, l)\sum_{l=0}^{F-1}\sum_{m=0}^{M-1}\alpha_{m,l}\, s_{m,l} + v \qquad (9)$$

The components $\alpha_{m,l}$ are target fading coefficients for each target illuminating path, due to the orthogonality among transmit vectors, the $\alpha_{m,l}$ are uncorrelated. Moreover, the random variables $\alpha_{m,l}$ are zero mean, unit variance (per dimension), independent, identically distributed complex normal.

III. PERFORMANCE COMPARISON

Neglecting the different range related gains in the proposed MIMO radar system for principal investigations, the test statistics for the proposed can be given [4] as below

$$\eta = \sum_{i=0}^{MNF-1}|r(i)|^2 = |\mathbf{r}|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \qquad (10)$$

Matched filtering is performed to extract the components of different transmitting antennas and different frequencies. The terms $|\alpha_{m,l}|^2$, are such that they are not only uncorrelated for different values of m=0,1,…M, n=0,1,..,N but also uncorrelated for different values of l=0,1,…F, thus the system for simplicity, can now be assumed to contain M*N*F independent channels having uncorrelated RCS coefficients. Hence their sum will have χ^2_{2MNF} distribution (Chi square distribution with 2*M*N*F degrees of freedom). This is due to the different, uncorrelated RCS presented by the target to the different elements of transmit, receive array and for different transmitting frequencies.

Now, the test statistics will have following distribution i.e.

$$\eta \sim \begin{cases} \frac{1}{2}\left(\sigma^2 + \frac{E}{MF}\right)\chi^2_{2MNF} & H_1 \\ \frac{1}{2}\,\sigma^2\;\chi^2_{2MNF} & H_0 \end{cases} \qquad (11)$$

Accordingly, for any false alarm rate, the following equality holds good

$$P_{fa} = P_r(\eta \geq \gamma\,/\,H_0) = P_r\left(\chi^2_{2MNF} \geq \frac{2\gamma}{\sigma^2}\right)$$

$$= 1 - F_{\chi^2_{2MNF}}\left(\frac{2\gamma}{\chi^2}\right) \qquad (12)$$

where 'γ' is the threshold. And the corresponding P_D is given by

$$P_d = P_r(\eta \geq \gamma\,/\,H_1) = 1 - F_{\chi^2_{2MNF}}\left(\frac{\sigma^2}{\sigma^2 + \frac{E}{MF}}F^{-1}_{\chi^2_{2MNF}}\left(1 - P_{fa}\right)\right) \qquad (13)$$

The expressions above clearly suggests an increased variance by factor F as compared to the conventional MIMO systems [1] which has only a single operating frequency i.e. F=1, always. This in turn result in a better detection performance since MIMO-SF has higher diversity by factor F, which is always greater than one ,in addition to the diversity in spatial domain.

Now we try to compare the two systems, a MIMO radar with spatial-frequency diversity (MIMO-SF) and conventional MIMO radar with only spatial diversity (MIMO-S). To illustrate we assume number of transmit antennas M=2, no of operating frequencies F=2 and no of receive antennas N=1.

MIMO radar with spatial-frequency diversity (MIMO-SF)

In this system we consider M=2, F=2. The received signal is given by

$$r = (1/\sqrt{2})(\alpha_{0.0}s_{0,0} + \alpha_{0.1}s_{0,1} + \alpha_{1.0}s_{1,0} + \alpha_{1.1}s_{1,1}) + v$$

$$|r|^2 = (1/\sqrt{2})\left|\alpha_{0.0}s_{0,0} + \alpha_{0.1}s_{0,1} + \alpha_{1.0}s_{1,0} + \alpha_{1.1}s_{1,1} + v\right|^2 \quad (14)$$

Here, it is assumed that the transmitted waveforms result in random and mutually independent components, $s_{m,l}$ of the sampled receive filter outputs.

Evaluating the expectation of the received signal power leads to

$$E\{|r|^2\} = E\{(1/\sqrt{2})|\alpha_{0.0}s_{0,0} + \alpha_{0.1}s_{0,1} + \alpha_{1.0}s_{1,0} + \alpha_{1.1}s_{1,1} + v|^2\}$$
$$= (1/4)(|\alpha_{0.0}|^2 + |\alpha_{0.1}|^2 + |\alpha_{1.0}|^2 + |\alpha_{1.1}|^2) + 2\sigma^2 \quad (15)$$

Where $E\{|s_l|^2\} = 1/2$ has been used. In the considered scenario, the estimate is based on a sufficiently large number of snapshots. The waveforms of each transmitter result in a different random output component for each snapshot. Thus, when the number of snapshots is sufficiently large, the sum over the received power may be approximated as

$$E\sum_{l=0}^{L-1}|r_l|^2 = E\left\{\sum_{l=0}^{L-1}\left|(1/\sqrt{2})(\alpha_{0.0}s_{0,0,l} + \alpha_{0.1}s_{0,1,l} + \alpha_{1.0}s_{1,0,l} + \alpha_{1.1}s_{1,1,l}) + v_l\right|^2\right\}$$
$$\approx 1/4\,(|\alpha_{0.0}|^2 + |\alpha_{0.1}|^2 + |\alpha_{1.0}|^2 + |\alpha_{1.1}|^2)L\hat{s}^2 + 2L\hat{\sigma}^2 \quad (16)$$

where $\hat{s}^2$ & $\hat{\sigma}^2$ indicate that these values are estimates of the (single) signal and noise power, which are random variables. In the remaining section of this paper, it is assumed that a sufficiently large number of snapshots are processed, and that therefore the power of the target component is determined by the (scaled) sum of the squared absolute values of the fading coefficients.

The fading in this case is due to the *sum* of $|\alpha_{0.0}|^2$, $|\alpha_{0.1}|^2$, $|\alpha_{1.1}|^2$, $|\alpha_{1.1}|^2$ and as the random variables $|\alpha_{m.l}|^2$, have a χ^2_2 distribution, and are i.i.d. (due to the orthogonality) their sum has a χ^2_8 (chi-square with 8 degrees of freedom) distribution. This is the result of different uncorrelated RCS presented by the target not only to the different elements of the transmitting array but also to the different operating frequencies used.

MIMO radar with spatial diversity (MIMO-S)

In this system we consider M=2, F=1. For a single operating frequency, the sum over the received power may be approximated as given below

$$E\sum_{l=0}^{L-1}|r_l|^2 \approx 1/4\,(|\alpha_0|^2 + |\alpha_1|^2)L\hat{s}^2 + 2L\hat{\sigma}^2 \quad (17)$$

The fading here is due to the *sum* of $|\alpha_0|^2$ and $|\alpha_1|^2$, and their sum has a χ^2_4 distribution.. This is a consequence of only spatial diversity being present. The higher degree of freedom in MIMO-SF results in the improved detection performance of the same compared to system having only spatial diversity i.e., MIMO-S, as shown in simulation results in following sections.

IV. SIMULATION RESULTS

The following simulations have been carried out by assuming two antennas at transmitter and a single antenna at receiver, i.e. M=2, N=1 and two operating frequencies have been chosen i.e. F=2. The spacing between the frequencies are taken to be 36 MHz which satisfies (2) and also the spacing between the antennas are assumed to produce independent realization of $\boldsymbol{\alpha}$. Hence the $|\boldsymbol{\alpha}_{m.l}|^2$ are assumed to be independent. The simulations compare the detection performance i.e. P_d vs. P_{fa} (Probability of Detection versus Probability of False Alarm) performance of MIMO-S radar with MIMO-SF radar, i.e. under the normalized conditions, for the same transmission power.

It is clearly seen from the Fig. 3 that MIMO-SF provides better performance over MIMO-S for certain portion of the Receiver Operating Characteristics (ROC) curves. The cross-over point is seen to be at $P_{fa}=10^{-5}$. The reason for the cross-over can be explained by the fact that even though the total power used is constant, as the number of individual diverse channels increase, noise power in the individual channel also increase. This effect is more predominant when the signal power is comparable to the noise power in the individual channels. In the current simulations for MIMO-SF we have more independent channels suffering equal noise characteristics due to more number of operating frequencies used as compared MIMO-S we have only single operating frequency hence at lower SNR values MIMO-S outperforms MIMO-SF.

Fig. 4, Fig. 5 and Fig. 6 show the dependency of the cross-over point on P_{fa} and amount of diversity. Fig. 4 and Fig.5 show P_d vs. SNR curve for P_{fa} of 10^{-5} and 10^{-6} respectively. It is clear from the curves that for P_{fa} of 10^{-5}, MIMO-SF outperforms MIMO-S for SNR greater than 20 dB ($10*\log_{10}100$) and for P_{fa} of 10^{-6}, MIMO-SF outperforms MIMO-S for SNR greater than 20.4 dB ($10*\log_{10}110$). It is thus observed that for stringent P_{fa} specifications, MIMO-SF outperforms MIMO-S for higher SNR.

Fig.6 shows the P_d vs. SNR curve for a P_{fa} of 10^{-5}, and with higher diversity. Here it is seen that that the cross-over occurs at SNR of 21.13 dB ($10*\log_{10}130$). Comparison of Fig.4 and Fig.6 shows that higher diversity requires higher SNR availability for MIMO-SF to outperform MIMO-S.

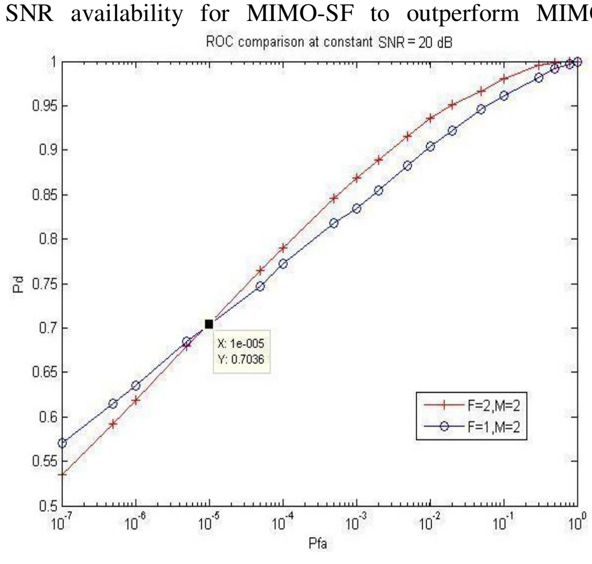

Figure 3. Receiver operating characteristics comparison at constant SNR =20dB, clearly showing improvement of MIMO-SF over MIMO-S

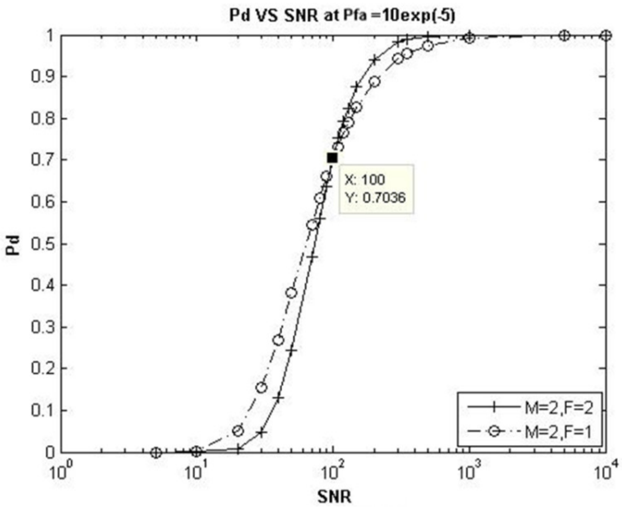

Figure 4. P_d vs. SNR characteristics comparison at constant $P_{fa} = 10^{-5}$, showing an improvement in case of MIMO-SF at better SNR but performance deterioration at lower SNR compared to MIMO-S with number of antennas=2, frequencies=2

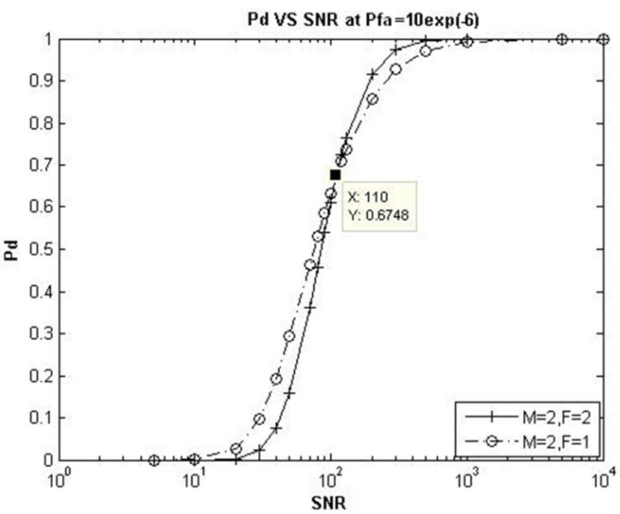

Figure 5. P_d vs. SNR characteristics comparison at constant $P_{fa} = 10^{-6}$, with number of antennas=2, frequencies=2

V.CONCLUSION

In this paper, the detection performance of MIMO radar with spatial-frequency diversity has been studied and has been compared with that of MIMO radar with only spatial diversity and it is shown that MIMO-SF outperforms MIMO-S for SNR higher than a particular value for a given P_{fa} or for P_{fa} higher than a particular value for given SNR for the same transmission power. It can be concluded that higher diversity and stringent P_{fa} requirement demands higher SNR for MIMO-SF to be a preferred choice. In this study it is assumed that the channel realizations are independent. Further work can be carried out to study the impact of correlated realization of the channels.

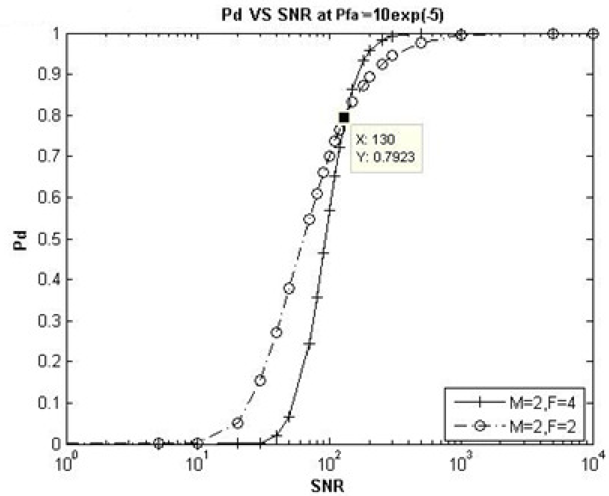

Figure 6. P_d vs. SNR characteristics comparison for higher diversity i.e. with number of antennas=2, frequencies=4, at constant $P_{fa} = 10^{-5}$ showing performance improvement only after an SNR threshold of 21.13, as compared to Figure 4

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